Randomization Devices and the Elicitation of Ambiguity Averse Preferences

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Abstract

In random incentive mechanisms agents choose from multiple problems and a randomization device determines which problem is operative for payment. Agents are assumed to act as if they faced each problem on its own. While this approach is valid when agents are expected utility maximizers, ambiguity averse agents may use the randomization device to hedge, and thereby contaminate the data.

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1 Introduction

Experimental economists often ask subjects to simultaneously choose from several different problems. One of these problems is then randomly drawn; the subject's choice from this problem determines the outcome of the experiment. The agent might, for example, be asked to report choices from six different sets of bets, with the experimenter then rolling a die to determine which of the six choices is payoff-relevant. Any experimental design which uses a randomization device to elicit choices from several problems is a **random incentive mechanism**.

If a subject's choice from each separate problem is identical to his choice from the same problem when it appears as part of a random incentive mechanism, the mechanism has many advantages over separate single choice experiments. Large sets of data can be elicited with one payment. The subject does not accumulate payments over the course of the experiment, so the separate choices are not affected by what the agent has earned (or lost) earlier in the experiment. More importantly, to check for regularities in an agent's behavior we must elicit his choices from various problems; a single choice experiment carries no information about the consistency of an agent's choices. But if an agent's behavior in a random incentive mechanism differs from his behavior in separate choice situations it is not clear how one should interpret the data generated by the mechanism.

Random incentive mechanisms have been used widely in the experimental literature on ambiguity aversion (see for instance Camerer and Weber [2], Halevy [6] and Ahn et al. [1]). However, there are no theoretical results on the incentives for ambiguity averse agents to reveal their true preferences in these mechanisms. Will the choices of ambiguity averse agents in random incentive mechanisms coincide with their choices in separate single choice problems?

The present study argues that random incentive mechanisms stand on shaky ground when agents are ambiguity averse. Ambiguity aversion entails a preference for hedging: ambiguity averse agents will typically prefer some randomization over a set of ambiguous acts to any of the acts individually. Consider a mechanism that is designed to elicit preferences over acts that are conditioned on a set of possibly ambiguous events. All acts that the agent can choose from in the mechanism are conditioned on events in this set. If this set of events is independent of the randomization device then the agent can use the randomization device as a hedging device. Preference reversals, where agents behave differently in a random incentive mechanism than in single choice experiments, are bound to happen. Let me give an example of a plausible ambiguity averse preference where such hedging occurs.

Example: an urn and a coin. There is an urn filled with 30 blue balls and 60 green and red balls in unknown proportion. We are interested in an agent's preferences over "urn-acts" $f = (f(B), f(G), f(R))$ where $f(B), f(G)$ and $f(R)$ denote the agent's utility-payoffs in the events B, G, and R that a blue, green, or red ball is drawn.[1] Let the agent choose among a "blue act" that delivers utility 5 when a blue ball is drawn from the urn, a "green act" that delivers utility 9 when a green ball is dawn and a "red act" which also delivers 9 when a red ball is drawn. We can represent these acts as $blue\colon = (5, 0, 0)$, $green\colon = (0, 9, 0)$ and $red\colon = (0, 0, 9)$.

Assuming our agent believes that a blue ball is drawn from the urn with probability $\frac{1}{3}$ the preference $blue \succ green \sim red$ is inconsistent with expected utility theory. If our agent was an expected utility maximizer he would have to believe that either R or G occurs with a probability of at least $\frac{1}{3}$. Consequently his preferred act among *red* and *green* would have to deliver an expected utility of at least $\frac{1}{3} \times 9$ whereas *blue* delivers only $\frac{1}{3} \times 5$. But an ambiguity averse agent might well prefer the objective lottery *blue* to the acts *green* and *red* that leave winning probabilities uncertain. Would an ambiguity averse agent with the preference $blue \succ green \sim red$ reveal this preference in a random incentive mechanism?

To pose this question concretely, let us assume that the agent's preference $\succsim$ over acts f is represented by a maxmin expected utility $U(f) = \min_{\pi \in C} \sum_{\Omega} f(\omega)\pi(\omega)$. Unlike an expected utility maximizer this agent holds a set of beliefs C on the state space Ω, not a single prior. He calculates an expected utility with respect to every prior in the set C and evaluates his overall utility as the lowest among these. Evaluations thus depend on the most pessimistic prior in C. Consistently with our earlier assumption that the agent believes a blue ball is drawn with probability $\frac{1}{3}$, let $\pi(B) = \frac{1}{3}$ hold for all $\pi \in C$. He is, however, unsure with which probability a green ball is drawn and perceives the event G

[1] This is a deviation from the more common assumption that acts map to lotteries over outcomes. Given that we assume an expected utility representation on lotteries over outcomes we can derive acts f which directly map states to utilities from more basic acts g which map to lotteries over outcomes by letting $f(\omega) = u(g(\omega))$ hold for every state ω.

that a green is drawn as ambiguous. The same holds for the event R. Specifically, let $\pi(G)$ either equal $\frac{1}{9}$ or $\frac{5}{9}$, implying that $\pi(R)$ also equals either $\frac{1}{9}$ or $\frac{5}{9}$.[2] So our agent evaluates any urn-act by either $\pi^{red} = (\frac{1}{3}, \frac{1}{9}, \frac{5}{9})$ or $\pi^{green} = (\frac{1}{3}, \frac{5}{9}, \frac{1}{9})$ where the components of these vectors denote the probabilities of the events B, G and R respectively. Since our agent assigns probability $\frac{1}{3}$ to a blue ball, his utility of the act *blue* is $\frac{1}{3} \times 5$. His utility of *green* is just $\pi^{red}(G)9 = 1$, given that π^{red} is the most pessimistic prior in C to evaluate *green*. Similarly, the agent's utility of *red* is $\pi^{green}(R)9 = 1$. In sum, our agent prefers *blue* to both *green* and *red*.

Now let's construct a random incentive mechanism to elicit these preferences. First, let us ask our agent to list choices from the two problems $S_H :\ = \{blue, green\}$ and $S_T :\ = \{blue, red\}$. Let us then use a fair coin as the randomization device to determine which of these two choices is operative for payment. The agent is paid according to his choice from the set S_H if heads comes up, otherwise he is paid according to his choice from S_T.[3]

To model the agent's behavior we need to specify his preferences over acts that are not only conditioned on the on the events B, G and R but also on the event H that the coin comes up heads and the complementary event T. Since the color of the ball and the side of the coin are the only payoff relevant facts, let us define any state ω in the state space Ω as the intersection of a coin- and an urn-event. For example, the state ω with $\{\omega\} = H \cap B$ is the unique state at which the coin comes up heads and a blue ball is drawn from the urn. Assuming that the agent assigns probability $\frac{1}{2}$ to each coin-outcome, let C consist of the priors defined by the following two matrices:

	B	G	R
H	$\frac{1}{6}$	$\frac{1}{18}$	$\frac{5}{18}$
T	$\frac{1}{6}$	$\frac{1}{18}$	$\frac{5}{18}$

	B	G	R
H	$\frac{1}{6}$	$\frac{5}{18}$	$\frac{1}{18}$
T	$\frac{1}{6}$	$\frac{5}{18}$	$\frac{1}{18}$

[2]The set C is not convex, so the representation U deviates from the maximin expected utility model of Gilboa and Schmeidler [4], which requires that C be a convex and compact set. However, the analysis of the example goes through unchanged if we replace the set C defined here with its convex hull.

[3]This example was inspired by the experimental setup in Ahn et al. [1]. Similarly to the example presented here the subjects in Ahn et al.'s [1] experiment could choose acts conditioned on three different events, one of which was known to have probability $\frac{1}{3}$. Nothing was known about the probabilities of the other two events. However, Ahn et al. [1] did not just elicit choices from two sets but from a large number of sets. Moreover, each of the sets contained vastly more options than S_H and S_T. Indeed Ahn et al. [1] elicited a particularly rich set of data by presenting agents with budgets sets.

According to C, the agent evaluates any urn-act at π^{red} or at π^{green} and he assigns a probability of $\frac{1}{2}$ to the coin coming up heads. The coin and the urn are independent according to any prior in C. The probability of heads and a red ball $\pi(H \cap R)$ is, for example, equal to the product of $\pi(H)$ and $\pi(R)$ for all $\pi \in C$.

The agent prefers choosing *green* from S_H and *red* from S_T to all other choices within the random incentive mechanism. For any prior π the expected utility of this plan equals $\frac{1}{2}\pi(G)9 + \frac{1}{2}\pi(R)9$. Since $\pi(G) + \pi(R) = \frac{2}{3}$ holds for any $\pi \in C$ this expected utility equals $3 = \frac{1}{2}\pi(G)9 + \frac{1}{2}(\frac{2}{3} - \pi(G))9$ for any prior π in C. Consequently the maxmin expected utility of this plan also equals 3. On the other hand, choosing *blue* out of both sets yields a utility of only $\min_{\pi \in C} \pi(B)5 = \frac{5}{3}$ to our agent. The remaining two options (choosing *blue* from exactly one of the two sets) deliver a yet lower utility. In sum, there is a preference reversal. While the agent prefers *blue* to *green* and *red* it is optimal for him to choose *green* from S_H and *red* from S_T in the random incentive mechanism. □

The main result of the paper, Theorem 1, shows that the preceding example is no accident. Preference reversals must occur when agents are ambiguity averse. To make this point I consider the two most popular models of ambiguity averse preferences: the maxmin expected utility model of Gilboa and Schmeidler [4] and the smooth model of Klibanoff, Marinacci and Mukherji [10]. I fix a randomization device $\mathcal{D}$, defined by a set of "coin-events", and a set of - possibly ambiguous - events $\mathcal{A}$ and assume that $\mathcal{D}$ and $\mathcal{A}$ are independent. I assume that the agent is strictly ambiguity averse with respect to acts that are conditioned on $\mathcal{A}$. Then I consider the set of random incentive mechanisms in which the agent gets to choose from sets of acts that are conditioned on events in $\mathcal{A}$. In all these mechanisms the randomization device $\mathcal{D}$ determines which of the agent's choices is operative for payment. Theorem 1 shows that the agent's preference must exhibit a reversal in some mechanism in this set. Such reversals can only be ruled out if the agent is an expected utility maximizer with respect to the acts under study.

To put the example into perspective it helps to go back to Schmeidler's [12] original insight on the representation of ambiguity averse preferences, where he explains that "intuitively, uncertainty aversion means that 'smoothing' or averaging utility distributions makes the decision maker better off." This is exactly what happens here: our experimental subject uses the coin to average out the two uncertain acts *green* and *red*. On its own each of these acts delivers utility 9 in one uncertain event and 0 otherwise. But in the compound act according to which *green* is played if the coin comes up heads and *red* is played if tails, the event R is neither as advantageous as it is under *red* nor as unfavorable as it is under *green*. If the event R occurs the compound act delivers utility 9 with tails and 0 with heads and vice versa for G. The coin, therefore, averages out the utilities delivered by the two urn-acts *green* and *red*. In the present case, this averaging or hedging is so efficient that the agent's choices in the random incentive mechanism differ from his choices in the two single choice experiments.

My arguments share some similarity with the Karni and Safra's [8] and Holt's [7] analysis preference reversals in random incentive mechanisms. These two studies take some empirically documented reversals (Lichtenstein and Slovic [11] and Grether and Plott [5]) of preferences over lotteries as their starting point and claim that such reversals should be expected when agents have rank-dependent preferences. Similarly to the present paper, Karni and Safra [8] and Holt [7] argue that, without the assumption of expected utility preferences, an agent's behavior in the random incentive mechanism as a whole need not be indicative of the agent's behavior in single choice experiments. However Karni and Safra [8] and Holt [7] do not address the question whether random incentive mechanisms truthfully elicit the preferences of ambiguity averse agents; their studies only consider the case of objective lotteries. To make sure that my results are driven by ambiguity aversion alone, I assume that preferences over objective lotteries have expected utility representations.

The fact that some experimental studies, such as Stahl [13] on ambiguity aversion find rather inconclusive results can be viewed as empirical motivation for my study. If experimental subjects use the randomization device as a hedging device the full extent of their ambiguity aversion will not be visible in the data. If others subjects do not hedge, the empirical picture might turn out very had to analyze.

2 Definitions

2.1 Basics

The agent has a complete and transitive preference $\succsim$ over acts which are functions from a finite state space Ω to $\mathbb{R}$. Under the act f the agent obtains utility $f(\omega)$ in state ω. So the acts under study here differ from Anscombe-Aumann acts which map a state space to objective lotteries over outcomes. But, if we assume an expected utility representation u over objective lotteries, then we can map any Anscombe-Aumann act g to an act $f : \Omega \to \mathbb{R}$ by letting $f(\omega) :=u(g(\omega))$ for all $\omega \in \Omega$. A **constant act** maps every state to the same utility level $x \in \mathbb{R}$. As a shorthand a constant act is also denoted x. The constant act x_f which is indifferent to an act f is the **certainty equivalent** of f. For any pair of acts f, g and event $E \subset \Omega$, I define the compound act $f_E g$ such that $f(\omega)$ is agent's payoff if $\omega \in E$ and $g(\omega)$ is the payoff if $\omega \notin E$. If f and g are constant acts then $f_E g$ is a **bet (on E)**. In terms of the introductory example the compound act $green_H red$ is the act in which payoffs are determined by *green* in case of heads and by *red* in case of tails. The acts *blue*, *green* and *red* are all bets.

Fix some partition $\mathcal{P}$ of Ω. The act f is called a $\mathcal{P}$-act if any two states belonging to the same event in the partition $\mathcal{P}$ yield the same utility level. For any $\mathcal{P}$-act f and any $E \in \mathcal{P}$ I write $f(E)$ for $f(\omega)$ if $\omega \in E$. Any union of events in $\mathcal{P}$ is called a $\mathcal{P}$-event. The complement of some event E is denoted $\overline{E}$. The set of probability measures on some finite set S is denoted ΔS. So $\Delta\Omega$ is the set of priors on Ω. For any $\pi \in \Delta\Omega$, the marginal distribution on $\mathcal{P}$ (the restriction of π to $\mathcal{P}$-events) is denoted $\pi_{\mathcal{P}}$. The conditional distribution of $\mathcal{P}$-events when conditioning on some event E is denoted $\pi_{\mathcal{P}}(\cdot \mid E)$.

2.2 Random incentive mechanisms

The experimenter is interested in an agent's preference over the set of $\mathcal{A}$-acts, where some events in the partition $\mathcal{A}$ might be ambiguous. The experiment uses a **randomization device** $\mathcal{D}$. Formally $\mathcal{D} := (D_1, \cdots, D_n)$ is another partition of the state space Ω. In terms of the introductory example we have $\mathcal{D} = (H, T)$ and $\mathcal{A} = (B, G, R)$. Since the events in the partitions $\mathcal{A}$ and $\mathcal{D}$ are the only ones that matter to the present study, it is without loss of generality to assume that any singleton subset of the state space is the intersection of a $\mathcal{D}$-event and an $\mathcal{A}$-event: each $\omega \in \Omega$ is identified with a pair of events $A \in \mathcal{A}$ and $D_i \in \mathcal{D}$ such that $\{\omega\} = A \cap D_i$. For any act $f : \Omega \to \mathbb{R}$ and any $i \in \{1, \cdots, n\}$ define an $\mathcal{A}$-act $f[i]$ by $f[i](A) := f(A \cap D_i)$ for all $A \in \mathcal{A}$. There is a one-to-one-correspondence between acts $f : \Omega \to \mathbb{R}$ and lists $(f[1], \cdots, f[n])$ of $\mathcal{A}$-acts. The list $(f[1], \cdots, f[n])$ of $\mathcal{A}$-acts uniquely defines the act $f : \Omega \to \mathbb{R}$ through $f(A \cap D_i) = f[i](A)$ for all $A \in \mathcal{A}$ and $i \in \{1, \cdots, n\}$.

Random incentive mechanisms, designed to elicit preferences over $\mathcal{A}$-acts, are constructed as follows. The agent is presented with n sets $S_1, \cdots, S_n$ of $\mathcal{A}$-acts. He is asked to choose one act $f^*[i]$ from each S_i. Then the randomization device $\mathcal{D}$ selects the choice that determines the agent's payoff. If D_i is drawn the agent is paid in accordance with his choice $f^*[i] \in S_i$. The agent's choices determine the act $f^* : \Omega \to \mathbb{R}$ that corresponds to the list $(f^*[1], \cdots, f^*[n])$. A random incentive mechanism that uses $\mathcal{D}$ to elicit preferences over $\mathcal{A}$-acts is denoted $S\colon = (S_1 \times \cdots \times S_n)$. An act $f : \Omega \to \mathbb{R}$ is an element of S if $f[i] \in S_i$ for all i.

Do the choices of an agent in a random incentive mechanisms truthfully represent his preference? That is, does an agent's choice from each S_i in a random incentive mechanism reveal what he would choose if he only had that one choice? The preference $\succsim$ **does not exhibit a preference reversal** in the random incentive mechanism S if

$$f^* \succsim f \text{ for all} f \in S \Leftrightarrow f^*[i] \succsim f[i] \text{ for all } f[i] \in S_i \text{ and } 1 \leq i \leq n.$$

So $\succsim$ does not exhibit a preference reversal in S if the agent chooses the same $f^*[i]$ from the set S_i, whether he faces just that choice or has to choose from the entire list $(S_1, \cdots, S_n)$. Conversely, $\succsim$ exhibits a **preference reversal** in S if the preceding equivalence is violated. So S exhibits a preference reversal if the agent's optimal choices within the mechanism differ from his optimal choices in the separate choice problems. If $\succsim$ does not exhibit a preference reversal in any mechanism (that uses the randomization device $\mathcal{D}$ to elicit preferences over $\mathcal{A}$-acts) then $\succsim$ is **transparent**.

2.3 Representations

A representation U of the preference $\succsim$ is a **MMEU representation** (maxmin expected utility representation, Gilboa and Schmeidler [4]) if there exists a convex and compact set of beliefs C on Ω such that $U(f) = \min_{\pi \in C} \sum_{\Omega} f(\omega)\pi(\omega)$. A representation V is **smooth** (Klibanoff, Marinacci and Mukherji [10]) if there exists a concave function $\phi : \mathbb{R} \to \mathbb{R}$ and a probability measure μ on the set of priors $\Delta\Omega$ such that $V(f) = \int_{\Delta\Omega} \phi\Big(\sum_{\Omega} f(\omega)\pi(\omega) \Big) d\mu(\pi)$. A smooth representation is often viewed as a two-stage procedure: in the first stage nature uses the distribution μ to determine the distribution π from which she then draws the state ω in the second stage. Just as risk aversion is expressed through the curvature of a utility functional u that maps monetary outcomes to utilities, ambiguity aversion is expressed through the curvature of ϕ. Given that acts directly map states to utilities in this paper, a MMEU representation is defined entirely through the set of beliefs C, and a smooth representation is defined entirely through the prior over priors μ and the function ϕ.

I assume from now on that $\succsim$ either has a MMEU or a smooth representation. A preference (that has such a) is **strictly ambiguity averse** if it does not have an expected utility representation.

2.4 Independence

Two events E_1 and E_2 are **(stochastically) independent** if the probability of both events occurring is equal to the product of their probabilities, $\pi(E_1 \cap E_2) = \pi(E_1)\pi(E_2)$. Two partitions $\mathcal{P}_1$ and $\mathcal{P}_2$ are (stochastically) independent if any pair of $\mathcal{P}_1$- and $\mathcal{P}_2$-events are stochastically independent. So $\mathcal{P}_1$ and $\mathcal{P}_2$ are independent according to π if $\pi(E_1 \cap E_2) = \pi(E_1)\pi(E_2)$ holds for any $\mathcal{P}_1$-event E_1 and any $\mathcal{P}_2$-event E_2.

Since the representations we consider involve multiple priors on Ω we cannot use the classic notion of independence to define $\mathcal{D}$ and $\mathcal{A}$ as independent. Instead we need a behavioral concept of independence, which I define following Klibanoff [9]. Fix two events E_1 and E_2 and a bet b, that delivers 1 if E_2 occurs and 0 otherwise. Consider the agent's preference over x_b, the certainty equivalent of b, and the compound act $b_{E_1}x_b$ according to which the agent gets to play the bet b if E_1 occurs and receives x_b if E_1 does not occur. If the agent is an expected utility maximizer with a prior π according to which E_1 and E_2 are independent then he must be indifferent between $b_{E_1}x_b$ and x_b. The reason is that the two acts only differ in the event E_1, and, due to the independence of E_1 and E_2, the preferred outcome of the bet b is just as likely under $\pi(\cdot \mid E_1)$ as it is under π.

It is therefore reasonable to say that the events E_1 and E_2 are "independent" for an agent if $b_{E_i}x_b \sim b$ holds for all bets b on E_j and all $\{i, j\} = \{1, 2\}$. If the indifference $b_{E_1}x_b \sim b$ holds for a bet b on E_2 then the value that the agent assigns to b does not depend on E_1 occurring or not. So the agent cannot consider his preferred event under b, be it E_2 or $\overline{E}_2$, to be correlated with E_1. The following definition generalizes this idea to the case of two partitions $\mathcal{P}_1$ and $\mathcal{P}_2$ on Ω.

Definition 1 *Two partitions $\mathcal{P}_1$ and $\mathcal{P}_2$ on Ω are* ***(behaviorally) independent*** *according to $\succsim$ if $f \sim f_E x_f$ holds for all pairs of a $\mathcal{P}_i$-event E and a $\mathcal{P}_j$-act f, with $\{i, j\} = \{1, 2\}$.*

This definition applies the intuition developed above to any combination of a $\mathcal{P}_i$-event and a $\mathcal{P}_j$-act f, not just bets. Klibanoff [9] shows that behavioral independence reduces to the classical definition when the agent is an expected utility maximizer.

3 An Impossibility Result

Independent randomization devices offer agents the opportunity to hedge. When the agent does hedge, his choices in the mechanism appear to be more ambiguity accepting than the choices he would make if he faced the problems on their own. Consequently any strictly ambiguity averse preference exhibits a reversal in some random incentive mechanism with an independent randomization device. For the statement of the following theorem remember that the preference $\succsim$ is defined over all acts $f : \Omega \to \mathbb{R}$, where each state $\omega \in \Omega$ can be represented as the intersection of a $\mathcal{D}$-event with a $\mathcal{A}$-event and the randomization device $\mathcal{D}$ and $\mathcal{A}$ are independent. The preference $\succsim$ either has a smooth or a MMEU representation. A preference that does not exhibit a reversal in any random incentive mechanism that uses $\mathcal{D}$ to elicit preferences over $\mathcal{A}$-acts is said to be transparent.

Theorem 1 *If $\succsim$ is transparent, then the restriction of $\succsim$ to $\mathcal{A}$-acts, must have an expected utility representation.*

The proof of the theorem is by contradiction. The following three properties of an ambiguity averse preference $\succsim$ cannot be reconciled: transparency, strict ambiguity aversion with respect to $\mathcal{A}$-acts, and the independence of $\mathcal{D}$ and $\mathcal{A}$. In the next section I illustrate the clash of these properties with the example of the coin and the urn. The proof is in the Appendix.

4 The coin and the urn

Let the preference $\succsim$ be transparent. First I assume that $\succsim$ has a MMEU representation and coincides on the set of urn-acts with the ambiguity averse preference postulated in the Introduction. To illustrate Theorem 1, I show that the coin and the urn then cannot be independent according to $\succsim$. After repeating this exercise for the case of a smooth representation, I sketch the proof of Theorem 1.

Assume that the agent's preference over acts conditioned on the coin and the urn have a MMEU representation $U^I(f) = \min_{\pi \in C^I} \sum_\Omega f(\omega)\pi(\omega)$. Define four priors,

$\pi^{coin} \times \pi^{red} :=$

	B	G	R
H	$\frac{1}{6}$	$\frac{1}{18}$	$\frac{5}{18}$
T	$\frac{1}{6}$	$\frac{1}{18}$	$\frac{5}{18}$

$\pi^{mix1} :=$

	B	G	R
H	$\frac{1}{6}$	$\frac{5}{18}$	$\frac{1}{18}$
T	$\frac{1}{6}$	$\frac{1}{18}$	$\frac{5}{18}$

$\pi^{coin} \times \pi^{green} :=$

	B	G	R
H	$\frac{1}{6}$	$\frac{5}{18}$	$\frac{1}{18}$
T	$\frac{1}{6}$	$\frac{5}{18}$	$\frac{1}{18}$

$\pi^{mix2} :=$

	B	G	R
H	$\frac{1}{6}$	$\frac{1}{18}$	$\frac{5}{18}$
T	$\frac{1}{6}$	$\frac{5}{18}$	$\frac{1}{18}$

and let $C^I := \{\pi^{coin} \times \pi^{red}, \pi^{coin} \times \pi^{green}, \pi^{mix1}, \pi^{mix2}\}$.[4] Fix two arbitrary urn acts f and g and assume that the minimum of $\sum_{\omega\in\Omega}(f_H g)(\omega)\pi(\omega)$ over all $\pi \in C^I$, $U^I(f_H g)$, is attained at π^*. Since any prior π in C^I assigns probability $\frac{1}{2}$ to heads, we have

$$\sum_{\omega\in\Omega}(f_H g)(\omega)\pi^*(\omega) = \frac{1}{2}\sum_{A\in\mathcal{A}} f(A)\pi^*(A \mid H) + \frac{1}{2}\sum_{A\in\mathcal{A}} g(A)\pi^*(A \mid T).$$

It is easily checked that the MMEU of any urn-act (including f and g) is attained either at π^{red} or π^{green}. This fact together with the observation that for any $\pi \in C^I$ the distribution of urn outcomes when conditioning on heads or tails is either π^{red} or π^{green}, implies that $U^I(f)$ is a lower bound for the expected utility of f according to $\pi^*(\cdot \mid H)$: $\sum_{A\in\mathcal{A}} f(A)\pi^*(A \mid H) \geq U^I(f)$. By the same logic, $\sum_{A\in\mathcal{A}} g(A)\pi^*(A \mid T) \geq U^I(g)$ also holds.

In fact neither of these two inequalities can be strict. For suppose that, say, the first inequality is strict and for concreteness imagine that according to π^* the conditional distribution of urn outcomes given heads is π^{red} whereas $U^I(f)$ is attained at π^{green}. Then we could find another distribution $\pi' \in C^I$ where $\sum_{\omega\in\Omega}(f_H g)(\omega)\pi'(\omega)$ is strictly less than the value achieved at π^*, namely the distribution π' whose conditional distribution given

[4] Just like the first set of beliefs defined in the introductory example, this is not a convex and compact set. The analysis goes through unchanged if we replace C^I by it's convex hull $co(C^I)$ which is also compact. As an aside, note that $co(C^I)$ is rectangular with respect to the filtration $\{H, T\}$ in the sense of Epstein and Schneider [3]. This means that full Bayesian updating with respect to H and T is dynamically consistent. What is not implied by dynamic consistency but by independence is that the set of posteriors on urn outcomes when updating with respect to H or T is identical to the set of priors on urn outcomes. The priors $\pi^{coin} \times \pi^{green}$ and $\pi^{coin} \times \pi^{red}$ can be viewed as the product measures of π^{coin} and π^{green} and π^{red} respectively, where π^{coin} is the marginal distribution on the coin, π^{coin} with $\pi^{coin}(H) = \frac{1}{2}$.

heads is π^{red} and whose conditional distribution given tails coincides with that of π^*.[5] So we have $U^I(f_H g) = \frac{1}{2}U^I(f) + \frac{1}{2}U^I(g)$.

Now fix two arbitrary sets of urn acts S_T and S_H and consider the agent's optimal choice in the random incentive mechanism $S = (S_H \times S_T)$, where the agent is paid according to his choice from S_H in case of heads and according to his choice from S_T in case of tails. Given the above equality we have

$$\max_{f_H g \in S} U^I\big(f_H g\big) = \frac{1}{2}\max_{f \in S_H} U^I(f) + \frac{1}{2}\max_{g \in S_T} U^I(g).$$

It is optimal for the agent to choose $f^*_H g^*$ in the mechanism if and only if f^* is his optimal choice from the set associated with heads and g^* his optimal choice from the set associated with tails. The preference represented by U^I is indeed transparent.

I will show next that the agent's preference can only be transparent if π^{mix2}, whose conditional distributions of urn outcomes given heads and tails differ, belongs to the agent's set of beliefs. To see this let U' be the MMEU with the belief set $C^I \setminus \{\pi^{mix2}\}$. It is easily checked that

$$U'(green) = \sum_{\omega \in \Omega} green(\omega)\pi^{red}(\omega) = U'(red) = \sum_{\omega \in \Omega} red(\omega)\pi^{green}(\omega) = 1.$$

However $U'(green_H red) > 1$ holds given that the agent holds no belief such that the conditional distribution of urn outcomes given heads is π^{red} and the conditional distribution of urn outcomes given tails. For no prior does the agent simultaneously evaluate *green* and *red* in the compound act $green_H red$ by their respective minimizing distributions of urn outcomes π^{red} and π^{green}. The agent with the belief set $C^I \setminus \{\pi^{mix2}\}$ is able to hedge. To see that the preference represented by U' exhibits a reversal in some mechanism, define the constant act x such that $U'(green_H red) > x > 1$ and consider the mechanism in which the agent can choose between *green* and x if heads and between *red* and x if tails. In this mechanism the agent chooses *green* from the first set and *red* from the second, even though he would choose x over either one of these two acts in separate single choice experiments. In sum π^{mix2} (and by the same arguments mutatis mutandis π^{mix1}) has to be in C^I for $\succsim$ to be transparent.

But the presence of π^{mix1} and π^{mix2} in C^I prevent the coin and the urn from being independent. To see this define a bet b on the coin which yields 1 if the coin comes up

[5] The crucial feature of C^I is that for any two priors on urn outcomes $\pi^1, \pi^2 \in \{\pi^{red}, \pi^{green}\}$, there exists a prior in C^I such that $\pi_{\mathcal{A}}(\cdot \mid H) = \pi^1$ and $\pi_{\mathcal{A}}(\cdot \mid T) = \pi^2$ both hold.

heads and -1 otherwise. So we have $b \sim 0$. For the coin and the urn to be independent the act $b_G 0$ would have to be indifferent to 0. But $U^I\big(b_G 0\big)$ equals

$$\min_{\pi \in C^I} \big(\pi(H \cap G) - \pi(T \cap G)\big) = \pi^{mix2}(H \cap G) - \pi^{mix2}(T \cap G) = -\frac{2}{9} < 0.$$

The urn and the coin are not independent. What is noteworthy is that the argument for the transparency of $\succsim$ requires the existence of a prior $\pi \in C^I$ with $\pi_{\mathcal{A}}(\cdot \mid H) = \pi^{red}$ and $\pi_{\mathcal{A}}(\cdot \mid T) = \pi^{green}$. The prior π^{mix2} has exactly that feature. But this feature entails that under π^{mix2} a green ball is more likely in case of heads than in case of tails, a violation of independence.

Preferences with a smooth representation fare no better. To see this assume that $\succsim$ is represented by $V^I(f) = \int_{\Delta\Omega} \phi\Big(\sum_{\Omega} f(\omega)\pi(\omega)\Big) d\mu^I(\pi)$ with $\phi(0) = 0$ and, to reflect the agent's ambiguity aversion, ϕ strictly concave. Let μ^I assign probability $\frac{1}{4}$ to each one of measures $\pi^H \times \pi^{red}, \pi^H \times \pi^{green}, \pi^T \times \pi^{red}$ and $\pi^T \times \pi^{green}$ with[6]

$\pi^H \times \pi^{red}$: =

	B	G	R
H	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{5}{9}$
T	0	0	0

$\pi^H \times \pi^{green}$: =

	B	G	R
H	$\frac{1}{3}$	$\frac{5}{9}$	$\frac{1}{9}$
T	0	0	0

$\pi^T \times \pi^{red}$: =

	B	G	R
H	0	0	0
T	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{5}{9}$

$\pi^T \times \pi^{green}$: =

	B	G	R
H	0	0	0
T	$\frac{1}{3}$	$\frac{5}{9}$	$\frac{1}{9}$

Since all uncertainty about the coin is resolved in the first stage ($\pi(H) \in \{0, 1\}$ for all $\pi \in supp(\mu^I)$) the agent is ambiguity neutral with respect to bets on the coin. The agent considers heads and tails to be equally likely as μ^I assigns probability $\frac{1}{2}$ to the events $\{\pi \mid \pi(T) = 1\}$ and $\{\pi \mid \pi(H) = 1\}$. The agent's beliefs on the urn do not depend on the throw of the coin: π^{red} and π^{green} are equally likely according to μ^I, whether the agent conditions on heads or tails, or does not condition at all. The utility $V^I(f_H g)$ of any choice $f_H g$ in the mechanism can be calculated as:

[6] We can think of π^H and π^T as two marginal distributions on coin outcomes such that heads comes up with certainty under π^H and vice versa for π^T.

$$\int_{\Delta\Omega} \phi\Big(\sum_{\Omega} (f_H g)(\omega)\pi(\omega)\Big) d\mu^I(\pi) =$$
$$\mu^I\big(\{\pi \mid \pi(H)=1\}\big) \int_{\Delta\Omega} \phi\Big(\sum_{A\in\mathcal{A}} f(A)\pi(A)\Big) d\mu^I\big(\pi \mid \pi(H)=1\big) +$$
$$\mu^I\big(\{\pi \mid \pi(T)=1\}\big) \int_{\Delta\Omega} \phi\Big(\sum_{A\in\mathcal{A}} g(A)\pi(A)\Big) d\mu^I\big(\pi \mid \pi(T)=1\big) =$$
$$\tfrac{1}{2} \int_{\Delta\Omega} \phi\Big(\sum_{A\in\mathcal{A}} f(A)\pi(A)\Big) d\mu^I(\pi) + \tfrac{1}{2} \int_{\Delta\Omega} \phi\Big(\sum_{A\in\mathcal{A}} g(A)\pi(A)\Big) d\mu^I(\pi) =$$
$$\tfrac{1}{2}V^I(f) + \tfrac{1}{2}V^I(g).$$

The first equality equality splits the integral over all measures π into two parts: first an integral over the measures with $\pi(H) = 1$ then an integral over the remainder. The second equality holds since nature chooses a distribution π with $\pi(H) = 1$ with probability one half and since the distribution over distributions of urn outcomes does not depend on the conditioning events $\{\pi \mid \pi(H) = 1\}$ and $\{\pi \mid \pi(T) = 1\}$. Having established $V^I(f_H g) = \frac{1}{2}V^I(f) + \frac{1}{2}V^I(g)$, the proof that V^I is transparent can be transferred from the MMEU-case.

There is only one smooth representation that on the one hand coincides with V^I on the set of urn-acts and on the other hand assigns probability one half to either side of coin: V^I. The fact that the agent is ambiguity neutral with respect to bets on the coin requires that all uncertainty about the coin is resolved at the first stage. Moreover the conditional distribution over distributions of urn outcomes given heads must be identical to the unconditional distribution over distributions of urn outcomes. If not, then the agent's preferences over the choice set associated with heads in a random incentive mechanism would differ from the agent's preferences over urn acts. Given this difference the preference would exhibit a reversal in some mechanism. By the same logic, the conditional distribution over distributions of urn outcomes given tails must also be identical to the unconditional distribution over distributions of urn outcomes.

The problem is, here as it was with the MMEU representation U^I, that the coin and the urn are not independent according to V^I. To see this define a bet b on the coin which yields 1 if the coin comes up heads and x otherwise. Assume moreover that b is indifferent to 0. So we have $b :\, = 1_H x \sim 0$. To see that $b_G 0$ is not indifferent to 0 calculate $V^I(b_G 0)$

as

$$
\begin{aligned}
&\mu^I\Big(\{\pi \mid \pi(H)=1\}\Big)\Big(\frac{1}{2}\phi\big(\pi^{red}(G)+0\pi^{red}(\overline{G})\big)+\frac{1}{2}\phi\big(\pi^{green}(G)+0\pi^{green}(\overline{G})\big)\Big)+\\
&\mu^I\Big(\{\pi \mid \pi(T)=1\}\Big)\Big(\frac{1}{2}\phi\big(x\pi^{red}(G)+0\pi^{red}(\overline{G})\big)+\frac{1}{2}\phi\big(x\pi^{green}(G)+0\pi^{green}(\overline{G})\big)\Big)>\\
&\quad\frac{1}{2}\Big(\frac{1}{2}\big(\phi(1)\pi^{red}(G)+\phi(0)\pi^{red}(\overline{G})\big)+\frac{1}{2}\big(\phi(1)\pi^{green}(G)+\phi(0)\pi^{green}(\overline{G})\big)\Big)+\\
&\quad\frac{1}{2}\Big(\frac{1}{2}\big(\phi(x)\pi^{red}(G)+\phi(0)\pi^{red}(\overline{G})\big)+\frac{1}{2}\big(\phi(x)\pi^{green}(G)+\phi(0)\pi^{green}(\overline{G})\big)\Big)=\\
&\quad\frac{1}{2}\Big(\phi(1)\frac{1}{2}+\phi(x)\frac{1}{2}\Big)\Big(\pi^{red}(G)+\pi^{green}(G)\Big)=0\frac{1}{2}\Big(\pi^{red}(G)+\pi^{green}(G)\Big)=0.
\end{aligned}
$$

The definition of μ^I entails that the distributions π^{red} or π^{green} on urn outcomes are equally likely - whether we condition on $\{\pi \mid \pi(H)=1\}$ or on $\{\pi \mid \pi(T)=1\}$. The inequality follows from the strict concavity of ϕ together with $\pi^{red}(G), \pi^{green}(G), \pi^{red}(\overline{G})$, and $\pi^{green}(\overline{G})$ all being positive. I also replaced $\mu^I(\{\pi \mid \pi(H)=1\})$ and $\mu^I(\{\pi \mid \pi(T)=1\})$ by one half. The next equality follows from $\phi(0)=0$. Finally $\phi(1)\frac{1}{2}+\phi(x)\frac{1}{2}=0$ is none other than the value of the bet b. In sum the coin and the urn are not independent according to V^I.

A necessary condition for $\succsim$ to be transparent is that $f_H g \sim x_{f_H} g \sim f_H x_g$ holds for all urn-acts f, g. The necessary condition implies that $f_H g \sim x_{f_H} x_g$ holds for all urn-acts f, g. Since the agent prefers bets with higher payoffs to bets with lower payoffs he prefers $f^*_H g^*$ to all possible choices $f_H g$ in the mechanism if he prefers f^* to all $f \in S_H$ and g^* to all $g \in S_T$. But this is none other than the agent preferring to choose f^* from S_H and g^* from S_T in the random incentive mechanism to any other option. Now compare the condition $f_H g \sim x_{f_H} g \sim f_H x_g$ holds for all urn-acts f, g with the requirement that the coin and the urn are independent. Letting $f = g$ the condition implies $f \sim f_H x_f \sim x_{f_H} f$ for any urn-act f. We could think of this last requirement as the "first half of independence". An additional symmetric "second half of independence" has to hold for the independence of the coin and the urn: $b_E x_b \sim b$ has to hold for any bet on the coin and any urn-event E. So independence symmetrically imposes two comparatively weak conditions of indifference. Conversely transparency requires a condition that is strong and asymmetric. The proof of Theorem 1 revolves around showing that transparency clashes with the "second half of independence".

For the proof of Theorem 1 I fix the set of all random incentive mechanisms that use the randomization device $\mathcal{D}$ and to elicit preferences over $\mathcal{A}$-acts. As a first step of the proof I identify a necessary and sufficient condition for $\succsim$ to be transparent. This - strong and asymmetric - condition is stated precisely in Lemma 1. In Lemma 2 I characterize the set of preferences with a MMEU representation that satisfy the condition identified in Lemma 1. Lemma 3 does the same for smooth representations. Finally Lemma 4 shows that the violation of independence is not specific to the representations U^I and V^I. I fix any preferences that satisfy the characterization of either Lemma 2 or 3 and assume that agent's preferences over $\mathcal{A}$-acts are strictly ambiguity averse. I show that these assumptions preclude the the device $\mathcal{D}$ and $\mathcal{A}$ from being independent.

5 Discussion

Let me take up three suggestions to salvage random incentive mechanisms when preferences are ambiguity averse. Recall that Theorem 1 is due to the clash among transparency, ambiguity aversion, and independence. The first suggestion is to impose a weaker notion of independence. The second is to drop independence altogether. The third is to weaken transparency.

Given that there is no agreed upon notion of stochastic independence for ambiguity averse preferences one might consider replacing the Klibanoff's [9] notion of independence by a weaker one. However, Klibanoff's [9] notion is already very weak. Gilboa and Schmeidler [4], for example, introduced a more restrictive notion of independence in their original article on MMEU representations: any two events that are independent according to that notion are also independent according to Klibanoff's [9]. The converse does not hold. Theorem 1 continues to hold if we replace Klibanoff's independence with any other notion proposed in the literature on ambiguity aversion.

The second suggestion is to drop the requirement of independence. To make sure that agents reveal their true preferences in the mechanism we must then instead assume that the randomization device does not induce hedging. There are two problems with this approach. First, hedging is a subjective concept. It is not clear what objective information about the randomization device could convince experimental subjects that they cannot use the randomization device to smooth out ambiguity. With independence, in contrast, we can reasonably hope that an expected utility maximizing agent would consider a randomization device independent if the experimenter publicly states that the device is indeed objectively independent. The second problem is that ambiguity aversion is defined as a preference for hedging. So the brute-force assumption that randomization devices cannot serve to hedge would erase the very phenomenon under study.

The third suggestion is to weaken transparency. Let me once again use the example of the coin and the urn to illustrate. Call an urn-act whose payoff in the event G is at least as high as its the payoff in the event R a G-act. Using a comparable definition for R-acts, observe that any urn-act is either a G-act or an R-act (or both). Replace the assumption of transparency with the assumption that $\succsim$ does not exhibit reversals in any mechanism in which only G-acts can be chosen. In such experiments, the subjects cannot use the randomization device to hedge since for all possible choices the favorable outcome is associated with the same ambiguous event (G). These assumptions only allow us to elicit preferences over G-acts.

But if we assume that the agent treats the events R and G symmetrically we can identify his preferences over all urn-acts. More specifically, the symmetry assumption implies that the agent is indifferent between the G-act f and the R-act g if $f(G) = g(R)$ and $f(R) = g(G)$ hold. The data on G-acts, the symmetry assumption and the assumption of transitivity allow the experimenter to identify the agent's preference over any two urn-acts. This approach indeed works for the agent with the MMEU preferences defined in the Introduction: this agent truthfully reveals his preferences over G-acts. Moreover, the symmetry assumption allows us to correctly derive his preferences over R-acts. While this suggestion works well in the introductory example, it is not directly transferable to other cases. For the suggestion to work, the assumption that a preference does not exhibit reversals in some subset of mechanisms has to be complemented with some assumption that allows the researcher to identify the remainder of the agent's preferences. In the case of the urn and the coin, the assumption that agents would treat G and R symmetrically appears less fraught than the assumption of transparency. In other cases there will be no clear solution.

6 Appendix

The proof of Theorem 1 is by contradiction. Fix a preference $\succsim$ that either has a smooth or a MMEU representation such that $\succsim^{\mathcal{A}}$ is strictly ambiguity averse. The contradiction I obtain is that $\succsim$ cannot be transparent if $\mathcal{A}$ and the randomization device $\mathcal{D}$ are independent. This proof is broken down into a series of steps. Lemma 1 identifies a necessary and sufficient condition for $\succsim$ to be transparent. Lemmas 2 and 3 characterize the set of MMEU and smooth representations that satisfy the condition identified in Lemma 1 given that the preference over $\mathcal{A}$-acts is strictly ambiguity averse. Finally Lemma 4 shows that $\mathcal{D}$ and $\mathcal{A}$ cannot be independent if $\succsim$ has a representation that is characterized by either one of the two preceding Lemmas. But first I need to define some more notation and concepts.

Notation: For any partition $\mathcal{P}$ of the set Ω let $\sigma_{\mathcal{P}}$ denote the algebra generated by $\mathcal{P}$. So an event E is a $\mathcal{P}$-event if and only if $E \in \sigma_{\mathcal{P}}$. An act f is a $\mathcal{P}$-act if and only if f is measurable with respect to the algebra $\sigma_{\mathcal{P}}$. Let $\succsim^{\mathcal{P}}$ denote the restriction of $\succsim$ to the set of $\mathcal{P}$-acts. For any set $C \subset \Delta\Omega$ and any partition $\mathcal{P}$ of Ω define $C_{\mathcal{P}}$ as the set marginal distributions on $\sigma_{\mathcal{P}}$ of all priors in the set C, formally $C_{\mathcal{P}} = \{\pi_{\mathcal{P}} \mid \pi \in C\}$. Let $\Sigma_{\mathcal{A}}$ be the algebra on $\Delta\Omega$ generated by the partition of $\Delta\Omega$ into sets $\{\pi \mid \pi_{\mathcal{A}} = \pi^*_{\mathcal{A}}\}$. For any prior μ over priors $\Delta\Omega$, let $\mu_{\Sigma_{\mathcal{A}}}$ be the marginal distribution with respect to the algebra $\Sigma_{\mathcal{A}}$. For any set S let $co(S)$ be the convex hull of the set S, so $co(S)$ is the smallest convex set that contains S.

Fix an event E and suppose $\succsim$ either has a MMEU or a smooth representation. In the MMEU-case the event E is **ambiguous** if $\{\pi(E) \mid \pi \in C\}$ is not a singleton. If $\succsim$ has a smooth representation, then E is ambiguous if ϕ is strictly concave and $\{\pi(E) \mid \pi \in supp(\mu)\}$ is neither a singleton nor a subset of $\{0, 1\}$. The preference $\succsim^{\mathcal{A}}$ is strictly ambiguity averse if and only if there exists an ambiguous $\mathcal{A}$-event. If no $\mathcal{A}$-event is ambiguous, then $\succsim^{\mathcal{A}}$ has an expected utility representation.

The randomization device $\mathcal{D}$ is said to be **isolated** from the set of events $\mathcal{A}$ if $f \sim x_{D_i} f$ holds for $x \in \mathbb{R}$, act f and $D_i \in \mathcal{D}$ if and only if $x \sim f[i]$. So if $\mathcal{D}$ is isolated from $\mathcal{A}$ then knowing D_i does not make $f[i]$ any more or less attractive than $x_{f[i]}$, no matter what happens for all the other outcomes of the randomization device.

Any act f can be represented as a list of $\mathcal{A}$-acts $(f[1], \cdots, f[n])$ with the understanding that $f(A \cap D_i) = f[i](A)$ holds for all $A \in \mathcal{A}$ and all i. When $\mathcal{D}$ is isolated from $\mathcal{A}$ we have that $(f[1], \cdots, f[n]) \sim (x_{f[1]}, \cdots, f[n])$. Inductively applying isolation we obtain that $f = (f[1], \cdots, f[n]) \sim (x_{f[1]}, \cdots, f[n]) \sim (x_{f[1]}, x_{f[2]}, \cdots, f[n]) \sim \cdots \sim (x_{f[1]}, \cdots x_{f[n]})$. This observation is used in the proofs of Lemmas 1 and 2:

Fact 1 If $\mathcal{D}$ is isolated from $\mathcal{A}$ then $f \sim (x_{f[1]}, \cdots x_{f[n]})$ holds for any act f.

Lemma 1 *Assume that* $\succsim$ *is monotonic in the sense that* $f \succsim f'$ *holds when* $f(\omega) \succsim f'(\omega)$ *holds for all* ω*. Then* $\succsim$ *is transparent if and only if* $\mathcal{D}$ *is isolated from* $\mathcal{A}$*.*

Proof The statement can be formalized as $(I) \Leftrightarrow (II)$ with

$$(I): \quad (f^* \succsim f \ \forall f \in S) \Leftrightarrow (f^*[i] \succsim f[i] \ \forall \ f[i] \in S_i, i) \quad \forall (S, f^*)$$
$$(II): \quad (x_{D_i} f \sim f) \Leftrightarrow (x \sim f[i]) \quad \forall (x, i, f).$$

To see $(I) \Rightarrow (II)$, fix a triple (x, i^*, f^*) and assume that (II) does not hold. To simplify notation let $g :\ = f^*[i^*]$ and $D_{i^*} :\ = D$.

First assume that $f^\circ :\ = x_D f^* \sim f^*$ and $x \not\sim g$ hold. Define S through $S_{i^*} = \{x, g\}$ and $S_i = \{f^*[i]\}$ for all $i \neq i^*$. So $f^\circ \sim f^* \succsim f$ holds for all $f \in S$, however $x \not\sim g$ implies that one of the two acts $f^\circ[i^*] = x$ and $f^*[i^*] = g$ must be strictly preferred. So (I) is violated.

Next assume that $f^\circ :\ = x_{g_D} f^* \not\sim f^*$ holds. Define S through $S_{i^*} = \{x_g, g\}$ and $S_i = \{f^*[i]\}$ for all $i \neq i^*$. Observe that $f^\circ[i] \sim f^*[i] \succsim f[i]$ holds for all $f[i] \in S_i$ and all i (including i^*), however $f^\circ \not\sim f^*$ implies that one of these two acts must be preferred to the other. So - once again - (I) is violated.

To see $(II) \Rightarrow (I)$ fix a tuple (S, f^*) and assume that (II) holds.

First assume that $f^* \succsim f$ holds for all $f \in S$ while $f^*[i^*] \prec g$ holds for some i^* and $g \in S_{i^*}$. Let $D_{i^*} :\ = D$ and define x such that $f^* \sim x_D f^*$ holds. By (II) x is uniquely defined through $x \sim f^*[i^*]$. This, $f^*[i^*] \prec g$, and monotonicity imply $x_D f^* \prec x_{g_D} f^*$. Applying (II) once again we obtain $x_{g_D} f^* \sim g_D f^*$ and therefore $g_D f^* \succ f^*$, which stands in contradiction with $f^* \succsim f$ for all $f \in S$. We can conclude that $(f^* \succsim f \ \forall f \in S)$ implies $(f^*[i] \succsim f[i] \ \forall \ f[i] \in S_i, i)$ if (II) holds.

Now assume that $f^*[i] \succsim f[i]$ holds for all $f[i] \in S_i$ and all i. So we have that $x_{f^*[i]} \succsim x_{f[i]}$ holds for all $f[i] \in S_i$ and all i. Monotonicity and Fact 1 (which follows from (II)) imply that $f^* \sim (x_{f^*[1]}, \cdots, x_{f^*[n]}) \succsim (x_{f[1]}, \cdots, x_{f[n]}) \sim f$ holds for all $f \in S$. In sum we obtain that $(f^*[i] \succsim f[i]\ \forall\ f[i] \in S_i, i)$ implies $(f^* \succsim f\ \forall f \in S)$ when (II) holds.

□

Lemma 2 *Assume that* $\succsim$ *has a MMEU representation* $U(f) = \min_{\pi \in C} \sum_\Omega f(\omega)\pi(\omega)$ *and let* $C^* := co\big(\{\pi \mid \pi_\mathcal{D} \in C_\mathcal{D} \text{ and } \pi_\mathcal{A}(\cdot \mid D_i) \in C_\mathcal{A} \text{ for all } i = 1, \cdots, n\}\big)$. *Then* $\mathcal{D}$ *is isolated from* $\mathcal{A}$ *if and only if* $C^* = C$ *as well as* $\pi(D_i) > 0$ *for all* i *and* $\pi \in C$.

Proof Let $C = C^*$ as well as $\pi(D_i) > 0$ for all i and $\pi \in C$. Fix any act f and i^*, define $D_{i^*} := D$ and $f[i^*] := g$, so $f = g_D f$ holds and $U(f)$ can be calculated as

$$\min_{\pi \in C} \Big(\pi(D) \sum_{A \in \mathcal{A}} g(A)\pi(A \mid D) + \sum_{i \neq i^*} \pi(D_i) \sum_{A \in \mathcal{A}} f[i](A)\pi(A \mid D_i)\Big) =$$

$$\pi^*_\mathcal{D}(D) \min_{\pi_\mathcal{A} \in C_\mathcal{A}} \sum_{A \in \mathcal{A}} g(A)\pi_\mathcal{A}(A) + \sum_{i \neq i^*} \pi^*_\mathcal{D}(D_i) \min_{\pi^i_\mathcal{A} \in C_\mathcal{A}} \sum_{A \in \mathcal{A}} f[i](A)\pi^i_\mathcal{A}(A) =$$

$$\pi^*_\mathcal{D}(D)x_g + \sum_{i \neq i^*} \pi^*_\mathcal{D}(D_i) \min_{\pi^i_\mathcal{A} \in C_\mathcal{A}} \sum_{A \in \mathcal{A}} f[i](A)\pi^i_\mathcal{A}(A) = U\big(x_{g_D} f\big).$$

The first equality follows from $C = C^*$ and the definition $\pi^*_\mathcal{D} \in C_\mathcal{D}$ as the marginal on $\mathcal{D}$ at which the sum is minimized, the second follows from the definition of the certainty equivalent of g. Given that $\pi^*_\mathcal{D}(D) > 0$ this second equality and thereby $U(f) = U\big(x_D f\big)$ only holds for $x = x_g$. We can conclude that $\mathcal{D}$ is isolated from $\mathcal{A}$ if $C^* = C$ as well as $\pi(D_i) > 0$ for all i and $\pi \in C$.

To see the necessity of the conditions for isolation, suppose first of all that there exist a $D \in \mathcal{D}$ and a $\pi^* \in C$ such that $\pi^*(D) = 0$. Then we have $U(1_D 0) = 0 = U(0) = U(0_D 0)$ even though $1 \not\sim 0$. So isolation is violated.[7]

Next suppose that $C^* \neq C$. First suppose there exists a $\pi^* \in C^* \setminus C$. Since C and C^* are both convex the separating hyperplane theorem implies the existence of an act f^* such that $\min_{\pi \in C^*} \sum_\Omega f^*(\omega)\pi(\omega) < \min_{\pi \in C} \sum_\Omega f^*(\omega)\pi(\omega)$. The arguments in the preceding part of the proof imply that $\min_{\pi \in C^*} \sum_\Omega f(\omega)\pi(\omega) = \min_{\pi_\mathcal{D} \in C_\mathcal{D}} \sum_{i=1}^n \pi_\mathcal{D}(D_i)x_{f[i]} =$

[7]This argument also applies to the ambiguity neutral case: $\succsim$ is not transparent if the agent has an expected utility representation with a prior π such that $\pi(D) = 0$ holds for some $D \in \mathcal{D}$.

$U\big((x_{f[1]},\cdots,x_{f[n]})\big)$ holds for all acts f. In sum we obtain $(x_{f^*[1]},x_{f^*[2]},\cdots,x_{f^*[n]}) \prec (f^*)$ which stands in contradiction with Fact 1 which requires these two acts to be indifferent when $\mathcal{D}$ is isolated from $\mathcal{A}$. The case that there exists a $\pi^* \in C \setminus C^*$ is covered by the same arguments mutatis mutandis. □

Lemma 3 *Assume that* $\succsim$ *has a smooth representation* $V(f) = \int_{\Delta\Omega} \phi\big(\sum_{\Omega} f(\omega)\pi(\omega)\big)d\mu(\pi)$ *and that the preference* $\succsim^{\mathcal{A}}$ *over* $\mathcal{A}$*-acts is strictly ambiguity averse. Then* $\mathcal{D}$ *is isolated of* $\mathcal{A}$ *if and only if* $\pi(D)\pi(\overline{D}) = 0$, $\mu(\{\pi \mid \pi(D) = 1\}) > 0$ *and* $\mu_{\Sigma_{\mathcal{A}}} = \mu_{\Sigma_{\mathcal{A}}}(\cdot \mid \pi(D) = 1)$ *hold for all* $D \in \mathcal{D}$ *and all* $\pi \in supp(\mu)$.

Proof First assume that $\pi(D)\pi(\overline{D}) = 0$, $\mu(\{\pi \mid \pi(D) = 1\}) > 0$ and $\mu_{\Sigma_{\mathcal{A}}} = \mu_{\Sigma_{\mathcal{A}}}(\cdot \mid \pi(D) = 1)$ hold for all $D \in \mathcal{D}$ and all $\pi \in supp(\mu)$. Fix any act f and any $D \in \mathcal{D}$. Define g as the $\mathcal{A}$-act for which $g_D f = f$ holds. Since $\pi(D)\pi(\overline{D}) = 0$ holds for all $\pi \in supp(\mu)$ we can represent $V(f)$ as $\int_{\Delta\Omega,\pi(D)=1} \phi\Big(\sum_{\Omega} f(\omega)\pi(\omega)\Big)d\mu(\pi) + \int_{\Delta\Omega,\pi(D)=0} \phi\Big(\sum_{\Omega} f(\omega)\pi(\omega)\Big)d\mu(\pi)$. Rewrite the first term of the sum as follows:

$$\int_{\Delta\Omega,\pi(D)=1} \phi\Big(\sum_{\Omega} f(\omega)\pi(\omega)\Big)d\mu(\pi) = \int_{\Delta\Omega,\pi(D)=1} \phi\Big(\sum_{A\in\mathcal{A}} g(A)\pi(A)\Big)d\mu(\pi) =$$

$$\mu(\{\pi \mid \pi(D) = 1\}) \int_{\Delta\Omega} \phi\Big(\sum_{A\in\mathcal{A}} g(A)\pi(A)\Big)d\mu(\pi \mid \pi(D) = 1) =$$

$$\mu(\{\pi \mid \pi(D) = 1\}) \int_{\Delta\Omega} \phi\Big(\sum_{A\in\mathcal{A}} g(A)\pi(A)\Big)d\mu(\pi) =$$

$$\mu(\{\pi \mid \pi(D) = 1\})\phi\big(x_g\big) = \int_{\Delta\Omega,\pi(D)=1} \phi\big(x_g\big)d\mu(\pi).$$

The first and second equality follow from the restriction to probability measures π with $\pi(D) = 1$ and the definition of the conditional probability $\mu(\cdot \mid \pi(D) = 1)$. The third equality holds since the marginal $\mu_{\Sigma_{\mathcal{A}}}$ is equal to the conditional marginal $\mu_{\Sigma_{\mathcal{A}}}(\cdot \mid \pi(D) = 1)$. The fourth equality uses the definition of the certainty equivalent of g. In sum we have $V\big(f\big) = V\big(g_D f\big) = V\big(x_{g_D} f\big)$. Since $\mu(\{\pi \mid \pi(D) = 1\})$ is positive the equality $V\big(f\big) = V\big(x_D f\big)$ holds if and only if $x = x_g$. We can conclude that $\mathcal{D}$ is isolated from $\mathcal{A}$ if $\pi(D)\pi(\overline{D}) = 0$, $\mu(\{\pi \mid \pi(D) = 1\}) > 0$ and $\mu_{\Sigma_{\mathcal{A}}} = \mu_{\Sigma_{\mathcal{A}}}(\cdot \mid \pi(D) = 1)$ hold for all $D \in \mathcal{D}$ and all $\pi \in supp(\mu)$.

To see the necessity of the conditions for isolation, suppose first of all that $\mu(\{\pi \mid$

$\pi(D) = 1\}) = 0$ holds for some D. Then we have $V(1_D0) = 0 = V(0) = V(0_D0)$ even though $1 \not\sim 0$. So isolation is violated.[8]

I show next that $\pi(D)\pi(\overline{D}) = 0$ has to hold for all $\pi \in supp(\mu)$ and all $D \in \mathcal{D}$ for $\mathcal{D}$ to be isolated from $\mathcal{A}$ when $\succsim^{\mathcal{A}}$ is strictly ambiguity averse. Since $\succsim^{\mathcal{A}}$ is strictly ambiguity averse there must exist an $\mathcal{A}$-event A such that $\{\pi(A) \mid \pi \in supp(\mu)\}$ is neither a singleton set, nor a subset of $\{0, 1\}$, moreover ϕ must be strictly concave. Normalize ϕ such that $\phi(0) = 0$ and ϕ is strictly concave in some neighborhood around 0. Implicitly define a function $y : \mathbb{R}_0^+ \to \mathbb{R}_0^-$ through $V\big(x_A y(x)\big) = 0$ and let $b[x]\colon = x_A y(x)$, so $y(0) = 0$. To see that y cannot be linear at 0 suppose we had $y(x) = -\rho x$ for all $x \in [0, \overline{x}]$ for some $\rho > 0$ and $\overline{x} > 0$. Fix an $x \in (0, \overline{x}]$ to obtain the following contradiction

$$0 = V(b[\frac{x}{2}]) = \int_{\Delta\Omega} \phi\big((\frac{1}{2}0 + \frac{1}{2}x)(\pi(A) - \rho\pi(\overline{A}))\big) d\mu(\pi) >$$
$$\frac{1}{2}\int_{\Delta\Omega} \phi(0) d\mu(\pi) + \frac{1}{2}\int_{\Delta\Omega} \phi(x(\pi(A) - \rho\pi(\overline{A}))) d\mu(\pi) = 0 + \frac{1}{2}V(b[x]) = 0.$$

The inequality follows since ϕ is strictly concave around 0, and since $\pi(A) - \rho\pi(\overline{A}) \neq 0$ must hold for a set of π that has positive measure according to μ (given that $\{\pi(A) \mid \pi \in supp(\mu)\}$ is not a singleton). The conclusion follows from $\phi(0) = 0$ and the definition of $b[x]$.

If $\mathcal{D}$ is isolated from $\mathcal{A}$ then $b[x]_D 0 \sim 0 \sim 0_D b[x]$ must hold for all $x \geq 0$ and all $D \in \mathcal{D}$, implying

$$0 = V\big(b[x]_D 0\big) + V\big(0_D b[x]\big) =$$
$$\textstyle\int_{\Delta\Omega} \phi\big(x\pi(A \cap D) + y(x)\pi(\overline{A} \cap D)\big) d\mu(\pi) + \int_{\Delta\Omega} \phi\big(x\pi(A \cap \overline{D}) + y(x)\pi(\overline{A} \cap \overline{D})\big) d\mu(\pi) =$$
$$\textstyle\int_{\Delta\Omega} \phi\big(x\pi(A \cap D) + y(x)\pi(\overline{A} \cap D)\big) + \phi\big(x\pi(A \cap \overline{D}) + y(x)\pi(\overline{A} \cap \overline{D})\big) d\mu(\pi) \geq$$
$$\textstyle\int_{\Delta\Omega} \phi\big(x\pi(A \cap D) + y(x)\pi(\overline{A} \cap D) + x\pi(A \cap \overline{D}) + y(x)\pi(\overline{A} \cap \overline{D})\big) d\mu(\pi) =$$
$$\textstyle\int_{\Delta\Omega} \phi\big(x\pi(A) + y(x)\pi(\overline{A})\big) d\mu(\pi) = 0$$

The concavity of ϕ implies the weak inequality, a contradiction is achieved if the inequality holds strictly for some x. The strict concavity of ϕ at 0 together with the assumption that $\phi(0) = 0$ implies that $\phi(\alpha) + \phi(\beta) > \phi(\alpha + \beta)$ holds, if and only if $\alpha \neq 0 \neq \beta$.

Therefore the above inequality holds strictly if and only if $x\pi(A \cap D) + y(x)\pi(\overline{A} \cap D)) \neq 0$ and $x\pi(A \cap \overline{D}) + y(x)\pi(\overline{A} \cap \overline{D})) \neq 0$ holds for a set of priors that has positive measure

[8] I used the same argument to rule out $\pi(D) = 0$ for any $\pi \in C$ in the proof of Lemma 2.

according to μ. To avoid such a contradiction any $\pi \in supp(\mu)$ which does not assign probability 1 or 0 to D must satisfy

$$y(x) \in \Big\{ -x\frac{\pi(A \cap D)}{\pi(\overline{A} \cap D)}, -x\frac{\pi(A \cap \overline{D})}{\pi(\overline{A} \cap \overline{D})}\Big\}.$$

If this condition held for some $\pi^* \in supp(\mu)$ we would obtain $y(x) = -\rho x$ for ρ either $\frac{\pi^*(A \cap D)}{\pi^*(\overline{A} \cap D)}$ or $\frac{\pi^*(A \cap \overline{D})}{\pi^*(\overline{A} \cap \overline{D})}$ given that y is continuous and $y(0) = 0$. This contradicts the observation that y cannot be linear in any small neighborhood around 0. We can conclude that either one of the first two cases must hold. In sum, we have that $\pi(D)\pi(\overline{D}) = 0$ holds for all $\pi \in supp(\mu)$ and all $D \in \mathcal{D}$.

If $\mu_{\Sigma_\mathcal{A}} \neq \mu_{\Sigma_\mathcal{A}}(\cdot \mid \pi(D) = 1)$ holds for some $D \in \mathcal{D}$ then there exists an $\mathcal{A}$-act f such that

$$\int_{\Delta\Omega} \phi\Big(\sum_{A \in \mathcal{A}} f(A) d\pi(\omega)\Big) d\mu(\pi) \neq \int_{\Delta\Omega} \phi\Big(\sum_{A \in \mathcal{A}} f(A) d\pi(\omega)\Big) d\mu(\pi \mid \pi(D) = 1).$$

For this act f we have $V\big(f_D x_f\big) = \int_{\Delta\Omega} \phi\Big(\sum_{\mathcal{A}} (f_D x_f)(A)\pi(A)\Big) d\mu(\pi) =$

$$\mu(\{\pi \mid \pi(D) = 1\}) \int_{\Delta\Omega} \phi\Big(\sum_{A \in \mathcal{A}} f(A) d\pi(\omega)\Big) d\mu(\pi \mid \pi(D) = 1) + \mu(\{\pi \mid \pi(D) = 0\})\phi(x_f) \neq$$
$$\mu(\{\pi \mid \pi(D) = 1\})\phi(x_f) + \mu(\{\pi \mid \pi(D) = 0\})\phi(x_f) = V(f).$$

The inequality is implied by $\mu(\{\pi \mid \pi(D) = 1\}) > 0$. In sum $\mu_{\Sigma_\mathcal{A}} = \mu_{\Sigma_\mathcal{A}}(\cdot \mid \pi(D) = 1)$ must hold for all $D \in \mathcal{D}$ for $\mathcal{D}$ to be isolated from $\mathcal{A}$. □

Lemma 4 *Assume that $\succsim$ either has a MMEU or a smooth representation, that $\mathcal{D}$ is isolated from $\mathcal{A}$ and that $\succsim^\mathcal{A}$ is strictly ambiguity averse. Then $\mathcal{D}$ and $\mathcal{A}$ cannot be independent.*

Proof Since $\succsim^\mathcal{A}$ is strictly ambiguity averse there exists an ambiguous $\mathcal{A}$-event A. Fix a $D \in \mathcal{D}$ and define a value x and a bet b such that $b = 1_D x \sim 0$. In the following paragraphs I show that $b_A 0 \not\sim 0$ holds for either representation contradicting the independence of $\mathcal{D}$ and $\mathcal{A}$ which requires $b_A 0 \sim 0$ to hold.

If the preference has the MMEU representation $U(f) = \min_{\pi \in C} \sum_\Omega f(\omega)\pi(\omega)$, then $x = -\min_{\pi \in C}\Big(\pi(D)/\pi(\overline{D})\Big) := \pi^*(D)/\pi^*(\overline{D})$ holds; according to Lemma 2 no prior in C assigns zero probability to any outcome of the randomization device, so x is well defined and non-zero. We can calculate $U\big(b_A 0\big)$ as

$$
\begin{aligned}
\min_{\pi \in C} \Big(\pi(A \cap D) + x\pi(A \cap \overline{D}) + 0\pi(\overline{A})\Big) &= \\
\min_{\pi^1 \in C_{\mathcal{D}}, \pi^2, \pi^3 \in C_{\mathcal{A}}} \Big(\pi^2(A)\pi^1(D) + x\pi^3(A)\pi^1(\overline{D})\Big) &= \\
\min_{\pi^2 \in C_{\mathcal{A}}} \pi^2(A)\pi^*(D) + x \max_{\pi^3 \in C_{\mathcal{A}}} \pi^3(A)\pi^*(\overline{D}) &= \\
\pi^*(D)(\min_{\pi^2 \in C_{\mathcal{A}}} \pi^2(A) - \max_{\pi^3 \in C_{\mathcal{A}}} \pi^3(A)) &< 0
\end{aligned}
$$

The first equality follows from Lemma 2 which shows that C must be defined as $co(\{\pi \mid \pi_{\mathcal{D}} \in C_{\mathcal{D}}$ and $\pi_{\mathcal{A}}(\cdot \mid D_i) \in C_{\mathcal{A}}$ for all $i = 1, \cdots, n\})$ for $\mathcal{D}$ to be isolated from $\mathcal{A}$. The second equality recognizes the fact that a difference is minimized through minimising the minuend and maximizing the subtrahend. The third uses the definition of x. The inequality holds since A is ambiguous, meaning that $\{\pi(A) \mid \pi \in C\}$ is not a singleton set.

Now assume that $\succsim$ has the smooth representation $V(f) = \int_{\Delta\Omega} \phi(\sum_{\Omega} f(\omega)\pi(\omega))d\mu(\pi)$ with $\phi(0) = 0$ and ϕ strictly concave in some open interval around 0. Define $\lambda \colon = \mu(\{\pi \mid \pi(D) = 1\})$. Since $b \sim 0$, we have $V(b) = \lambda\phi(1) + (1-\lambda)\phi(x) = 0$.

$$
\begin{aligned}
V(b_A 0) = \int_{\Delta\Omega} \phi\Big((\pi(A \cap D) + x\pi(A \cap \overline{D}) + 0\pi(\overline{A} \cap D) + 0\pi(\overline{A} \cap \overline{D})))\pi(\omega)\Big)d\mu(\pi) &= \\
\lambda \int_{\Delta\Omega} \phi\Big(\pi(A) + 0\pi(\overline{A})\Big)d\mu(\pi) + (1-\lambda)\int_{\Delta\Omega} \phi\Big(x\pi(A) + 0\pi(\overline{A}))\Big)d\mu(\pi) &> \\
\lambda \int_{\Delta\Omega} \Big(\phi(1)\pi(A) + \phi(0)\pi(\overline{A}))\Big)d\mu(\pi) + (1-\lambda)\int_{\Delta\Omega} \Big(\phi(x)\pi(A) + \phi(0)\pi(\overline{A})\Big)d\mu(\pi) &= \\
\int_{\Delta\Omega} \Big((\lambda\phi(1) + (1-\lambda)\phi(x))\pi(A)\Big)d\mu(\pi) = \int_{\Delta\Omega} \Big(0\pi(A)\Big)d\mu(\pi) &= 0.
\end{aligned}
$$

The first equality follows from the definition of the act $b_A 0$. The second equality is implied by Lemma 3 which shows that $\pi(D)\pi(\overline{D}) = 0$ and $\mu_{\Sigma_{\mathcal{A}}} = \mu_{\Sigma_{\mathcal{A}}}(\cdot \mid \pi(D) = 1)$ must hold for all $D \in \mathcal{D}$ and all $\pi \in supp(\mu)$ for $\mathcal{D}$ to be isolated from $\mathcal{A}$ when $\succsim^{\mathcal{A}}$ is strictly ambiguity averse. The inequality follows from the assumption that ϕ is strictly concave around 0 and $\mu(\{\pi \mid 0 < \pi(A) < 1\}) > 0$ as implied by A being ambiguous. The next equality follows from $\phi(0) = 0$, finally $\lambda\phi(1) + (1-\lambda)\phi(x) = 0$ yields the conclusion. $\square$

To prove Theorem 1 all preceding Lemmas need to be combined.

Proof Assume that the $\succsim$ has a MMEU or a smooth representation. Given that any such preference is monotonic, Lemma 1 applies; $\succsim$ is transparent if and only if $\mathcal{D}$ is isolated from $\mathcal{A}$. Lemma 4 shows that $\mathcal{D}$ and $\mathcal{A}$ cannot be independent if $\mathcal{D}$ is isolated from $\mathcal{A}$ and if $\succsim^{\mathcal{A}}$ is strictly ambiguity averse. A contradiction is achieved, meaning that $\succsim^{\mathcal{A}}$ must be ambiguity neutral. □

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