Ambiguity Aversion in Models of Political Economy

by

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Four papers that use ambiguity aversion in political economy are reviewed. The first two (Bade, 2011a and 2011b) find that two important puzzles (equilibrium existence with multidimensional issue spaces and platform convergence) of the Downs–Hotelling model of electoral competition can be addressed by modeling parties as ambiguity-averse. The second two papers pertain to the Condorcet model of information aggregation. Ghirardato and Katz (2006) explains selective abstention through ambiguity aversion. Ellis (2012) shows that this abstention motive can be strong enough to prevent any information aggregation. (JEL: D70, D81)

1 *Introduction*

The Downs–Hotelling model of platform positioning and the Condorcet model of information aggregation through elections are the two most important pillars of the formal theory of political economy. In Condorcet models there are two candidates L and R running in an election. There are two states of the world, l and r, and voters have identical preferences: they prefer candidate L in state l and candidate R in state r. However, the state of the world is not known, and voters differ in their information about the state of the world. Depending on their signal, voters may have varying a posteriori preferences over the two candidates. Condorcet models are used to investigate whether and how elections serve to aggregate the information dispersed among all voters.

The Downs–Hotelling model was set up to study the strategic choice of platforms by parties. In these models, voter preferences are defined over some political spectrum, and two parties strategically choose their platforms in this spectrum. In the standard version of the Downs–Hotelling model the political spectrum is unidimensional (say from left to right), voter preferences are single-peaked, and the goal of parties is to maximize their vote shares or winning probabilities. This model always

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has an equilibrium: in any equilibrium both parties will announce a median of the distribution of the voters' ideal points as their platform.

Ambiguity aversion has been introduced into both models. In the Condorcet model the voters are the strategic agents and they face some uncertainty about the state of the world. In the standard model voters behave as expected-utility maximizers in the face of this uncertainty. Both Ghirardato and Katz (2006) and Ellis (2012) replace expected-utility maximization with the assumption that voters are ambiguity-averse. Parties are the strategic agents in the Downs–Hotelling model. In the standard model they precisely know the preferences of all voters. In Bade (2011a) and Bade (2011b), I modify the Downs–Hotelling model and assume that parties are uncertain about voter preferences and ambiguity-averse.

The papers on Condorcet voting find that ambiguity-averse voters have a higher propensity to abstain than their expected-utility-maximizing counterparts. This might go so far that all voters abstain in an equilibrium of a Condorcet game. The papers on the Downs–Hotelling model address two puzzles in political economy: first, that party platforms converge in any equilibrium of a classical Downs–Hotelling model, and second, that Downs–Hotelling models that allow for a multidimensional political spectrum typically have no equilibrium. With ambiguity-averse parties the conditions for the existence of equilibrium in Downs–Hotelling models with multidimensional issues are much less strict than when parties are expected-utility maximizers. Moreover, such models might have equilibria with divergent platforms.

2 *Ambiguity Aversion*

First, let me introduce the concept of ambiguity aversion. An event is considered ambiguous if the agent is not sure what the probability of the event is. With risky events, in contrast, the probabilities are known. The classic example to delineate these two forms of uncertainty is that of a bet on the color of a ball drawn from an urn, which contains 10 yellow and green balls. If the agent knows that exactly half the balls are green (urn 1), he knows his probability of winning to be one-half. If the agent does not know anything beyond the fact that there are 10 green and yellow balls (urn 2), his chances of winning are ambiguous.

The phenomenon of ambiguity aversion arises if an agent's choices are biased away from situations where the payoff is ambiguous. Let the bet X_k be defined by agent 1 winning 1 if a ball of color X is drawn from urn k and getting 0 otherwise, where $X = G, Y$ for G a green and Y a yellow ball, and $l = 1, 2$ for urn 1 being the urn with known odds and urn 2 being the uncertain urn as described above. According to the Ellsberg paradox, agents typically exhibit the following preferences: $G_1 \sim Y_1 \succ G_2 \sim Y_2$, meaning that agents strictly prefer betting on either color in the urn with known odds to betting on either color in the other urn. This choice behavior stands in contrast with expected-utility maximization. If the agent were an expected-utility maximizer, he would have to consider the event that a green ball is drawn from urn 2 to be either more or less probable than $1/2$. In the former case the agent would have the preference $G_2 \succsim G_1$; in the latter he would have the preference $Y_2 \succsim Y_1$.

In decision theory the phenomenon of ambiguity aversion is usually captured by David Schmeidler's (1989) axiom of ambiguity aversion. This axiom is satisfied if an agent who is indifferent between two ambiguous acts prefers any objective randomization of these two acts to the original two acts. To illustrate this axiom, consider G_2 and Y_2 as the two indifferent ambiguous acts. Also consider a third act where the draw from urn 2 is preceded by a toss of an objective coin, which determines whether green or yellow is the winning color. The probability of winning for this act is 1/2, regardless of the color composition of the urn. Consequently, the third act corresponds to G_1 and Y_1, which means that the preferences $Y_1 \sim G_1 \succ G_2 \sim Y_2$ exhibit ambiguity aversion as defined by Schmeidler.

The Ellsberg paradox has inspired a large decision-theoretic literature on models of preference representation that can capture ambiguity-averse behavior. Only a few of these models are used frequently in applied models. In political economy only max-min expected utilities by Gilboa and Schmeidler (1989) and by Bewley's theory of Knightian uncertainty Bewley (2002) have been used as models of ambiguity-averse preferences. Let me therefore lay out these two models. Both concern preferences over Anscombe–Aumann acts $f : \Omega \to \Delta(X)$, where Ω is some state space, X a set of outcomes, and $\Delta(X)$ the set of all objective lotteries on X. For simplicity I assume Ω and X to be finite. So acts map states to objective lotteries.

As an illustration reconsider the bets defined on urns above. We could frame the state space as the number of green balls in the urn: $\Omega = \{0, 1, \cdots, 10\}$. The outcome space X consists of 1 (winning) and 0 (losing): $X = \{0, 1\}$. Since the outcome space is binary, $\Delta(X)$ can be represented by the interval [0, 1] of winning probabilities. So the bet G_2 can formally be described as the act $G_2 : \Omega \to [0, 1]$, where $G_2(\omega) = \omega/10$ is the probability of winning at the state ω in which there are ω green balls in urn 2. If the agent were an expected-utility maximizer, he would assign a probability distribution π over the states in Ω to calculate his utility of an act f as $U(f) = \int_\omega u(f(\omega))d\pi(\omega)$. The model of Gilboa and Schmeidler differs from the subjective expected-utility model in that the single prior π of the subjective expected-utility model is replaced by a convex and compact set of priors C. Preferences $\succsim^{\mathrm{GS}}$ have a max-min expected-utility representation if there exists a convex and compact set of priors C and a Bernoulli utility functional $u : \Delta(X)$ such that

$$f \succsim^{\mathrm{GS}} g \Leftrightarrow \min_{\pi \in C} \int_\omega u(f(\omega))d\pi(\omega) \geq \min_{\pi \in C} \int_\omega u(g(\omega))d\pi(\omega).$$

To see that such preferences $\succsim^{\mathrm{GS}}$ can accommodate the Ellsberg paradox, let C be the convex hull of the two priors π' and π'' on Ω with $\pi'(5) = \pi''(5) = 2/3$, $\pi'(4) = 1/3$, and $\pi''(6) = 1/3$. So the cumulative probability of green being drawn from urn 2 is below 1/2 if one uses π' to evaluate the urn. Conversely, when using π'' for the same task, the probability of green being drawn is above 1/2. The pessimistic max-min expected-utility maximizer will use π' to evaluate his winning chances when betting on green and will use π'' when evaluating his chances in the complementary case of betting on yellow. In either case he will appear to believe that his winning chances in the urn with unknown odds are below $1/2$, in line with the Ellsberg paradox.

Truman Bewley's (2002) incomplete-preferences theory of Knightian uncertainty provides a different approach towards the Ellsberg paradox. An agent prefers one act f to another act g if f yields a higher expected utility than g for an entire set of priors C on Ω. Formally, preferences $\succsim^{\mathrm{B}}$ satisfy Bewley's model if there exists a convex and compact set of priors C and a Bernoulli utility functional $u : \Delta(X)$ such that

$$f \succsim^{\mathrm{B}} g \Leftrightarrow \int_{\omega} u(f(\omega))d\pi(\omega) \geq \int_{\omega} u(g(\omega))d\pi(\omega) \quad \text{holds for all } \pi \in C\,.$$

Preferences $\succsim^{\mathrm{B}}$ and $\succsim^{\mathrm{GS}}$ are very similar. They both require the evaluation of the expected utility of an act for an entire set of priors C. Holding C and u fixed, $f \succsim^{\mathrm{B}} g$ implies $f \succsim^{\mathrm{GS}} g$. However, the converse does not hold. To see this consider the act G_2 and the belief set constructed above. It was already argued that $1/2 \succ^{\mathrm{GS}} G_2$ holds. However, $1/2 \succ^{\mathrm{B}} G_2$ does not hold. For π' the agent obtains a lower expected utility from G_2 than from $1/2$, while for π'' $1/2$ yields a higher expected utility than G_2. So the preferences $\succsim^{\mathrm{B}}$ are incomplete: G_2 and $1/2$ are unranked.

Given this incompleteness, the preferences $\succsim^{\mathrm{B}}$ cannot be used to explain or predict the full range of behavior of an agent. The most common assumption to fill the gap is to suppose status quo bias. For a given status quo, an agent chooses a different alternative only if that alternative is preferred according to $\succsim^{\mathrm{B}}$. If the alternative is unranked, the agent remains at his status quo. The Ellsberg paradox can be explained using Bewley-type preferences if we assume that agents view the lottery with 50% winning chances as the natural fallback option or status quo.

In addition to Ellsberg's intuitively convincing examples, ambiguity aversion is also empirically well documented. Camerer and Weber (1992) provide an early summary of the experimental evidence. More and more applied models use ambiguity aversion as a tool: ambiguity-averse agents have made an appearance in models of general equilibrium, contracts, and finance, in mechanism design, and in political economy.

3 *Ambiguity Aversion in Models of Political Economy*

Situations are typically considered more ambiguous rather than risky if agents have less experience with a situation. Coin throws are the paragon of risky situations: they have been performed often and are well understood. Similarly, roulette wheels, urns with known ball compositions, and even weather events and bus arrivals in one's home town can safely be considered risky situations. Conversely, situations that agents encounter only rarely or even just once are more likely to be considered ambiguous by agents. So political economy would appear to be a prime arena for the application of ambiguity aversion.

Elections typically concern some new issues. New wars are fought, new technologies need to be regulated, values shift. As an illustration consider the voters in a Condorcet-type model of an election that pitches a proponent of nuclear energy against an opponent. The proponent is a good candidate in the state of the world in which nuclear power plants will not blow up. In the alternative state the opponent is the better candidate. Now, of course, our knowledge of nuclear power plants differs greatly from our knowledge of dice. The probabilities of catastrophic incidents at power plants are notoriously hard to gauge, as is the probable extent of the damage. For obvious reasons we cannot run a set of controlled experiments on this subject before taking action. In short, we are facing a situation of subjective uncertainty over the state of the world, and agents will likely consider a set of probability distributions over nuclear-power-plant accidents.

Now consider the strategic choices of two parties that compete in an election that has at least two issues, one being nuclear energy, the other immigration. Consider an amendment of the Downs–Hotelling model that allows for voters to have preferences over these two issues. To this end represent each issue by an interval on the real line, I_n and I_i. To evaluate their choices, parties need to be able to ascribe probabilities of winning the election for any combination of platform profiles in the issue spaces $I_n \times I_i$. So they not only need to have a firm idea of the distribution of voters' ideal points over the issue space $I_n \times I_i$, they also need to form probabilistic judgments on any trade-offs that agents are willing to make between the two issues. Parties need to come up with educated guesses on how many voters they will lose (or gain) for any combination of immigration law with an energy policy. Yet again the complexity of these probability judgments far exceeds the complexity of forming preferences over dice – or even over urns with unknown odds. Matters are further aggravated by the fact that parties often have little time to gather information on voter preferences, which change continuously (think of an election shortly after the catastrophe of Fukushima).

All in all, political economy stands out as a prime candidate for the application of ambiguity aversion. Still, as far as I am aware, only a few papers have followed this route. I review Ghirardato and Katz (2006), Ellis (2012), Bade (2011a), and Bade (2011b) in the following two sections.

4 *Downs–Hotelling Models of Political Competition*

4.1 *The Classic Model*

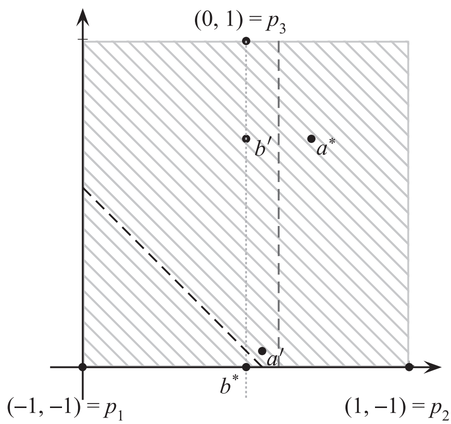
I used ambiguity aversion as a modeling tool in Downs–Hotelling environments in two papers: "Electoral Competition with Uncertainty Averse Parties" and "Divergent Platforms." In these two papers I address two major puzzles of the Downs–Hotelling model. The first is that the counterfactual assumption of a unidimensional issue space is essential to the existence of equilibrium. The second is the counterfactual prediction that parties announce identical platforms in equilibrium.

To fix ideas, let me formally define and discuss (a simple version of) the Downs–Hotelling model. There is a unidimensional political spectrum: $[-1, 1]$. Voters have single-peaked preferences over this spectrum. Preferences $\succsim$ are single-peaked if there exists an ideal point $p \in [-1, 1]$ such that either $a < b < p$ or $p > b > a$ implies $a \prec b$. Note that any platforms to the two sides of the ideal point (say a, b with $a < p < b$) are not ranked by this requirement. The distribution π of voter ideal points has a unique median. There are two parties A and B. These parties strategically choose their platforms a and b in $[-1, 1]$ to maximize their vote shares. Voters mechanically vote for the party that offers their preferred platform. Indifferent voters vote for either party with probability $1/2$. In equilibrium both parties announce the median of the distribution π as their platform.

This equilibrium prediction is counterfactual: we do not observe elections with indistinguishable parties. Moreover the prediction is hard to integrate into broader models of costly politics: why would anyone vote, campaign, or even pay attention to elections if the implemented platform were independent of the winner? This is one of the two puzzles I address by introducing ambiguity aversion into a Downs–Hotelling model of electoral competition.

The second puzzle I address is that the existence of equilibrium in the classic Downs–Hotelling model depends on the assumption of a unidimensional political spectrum. To get an intuition for this consider *model 1*, which has a two-dimensional political spectrum $[-1, 1]^2$ with just three voters: $i = 1, 2, 3$, as presented in Figure 1. Assume that each voter's preferences are represented by $U_i(p) = -d(p, p_i)$, where $d(p, p_i)$ is the Euclidean distance between platforms p and p_i, which in turn is voter i's ideal point. Let $p_1 = (-1, -1)$, $p_2 = (1, -1)$, and $p_3 = (0, 1)$. Suppose (a^*, b^*) were an equilibrium of the game. Then $a^*_1 = 0$ would have to hold, meaning that with respect to issue 1, party A would have to offer the median preferred

Figure 1
There is No Equilibrium in Model 1



platform. To see this, suppose we had $a_1^* > 0$. Then party B could obtain 2 votes by offering the platform $b' = (0, a_2^*)$, which voters 1 and 3 strictly prefer to a^* (all voters with an ideal point to the left of the gray dashed line prefer a^* to b'). So (a^*, b^*) cannot be an equilibrium, since party A would be strictly better off by simply offering the same platform as party B. By the same logic we can rule out that $a_1^* < 0$ or that $b_1^* \neq 0$. In short, we must have that $(a^*, b^*) = ((0, a_2^*), (0, b_2^*))$. Now suppose that either $a_2^* > -1$ or $b_2^* > -1$ held; then the other party would gather the support of voters 1 and 2 on offering $(0, -1)$ as its platform. So we must have $a^* = b^* = (0, -1)$. Now, finally, to see that this cannot be an equilibrium either, fix party B's platform at $b^* = (0, -1)$ and observe that party A would win the votes of 1 and 3 if it changed its platform to $a' = (\epsilon, -1 + \epsilon)$ for a small ϵ. In Figure 1 all voters with ideal points above the black dashed line would vote for a' over b^*.

Now, it has to be said that model 1 does not represent an oddity. Quite to the contrary, the nonexistence of equilibrium is generic in the case of political competition à la Downs–Hotelling with a multidimensional issue space. Plott (1967), Davis, DeGroot, and Hinich (1972), Grandmont (1978), and McKelvey (1979) all state some extremely stringent conditions that have to be met for equilibria to exist in their respective multidimensional versions of the Downs–Hotelling model. This of course presents a major problem, since the assumption of a unidimensional issue space appears as rather counterfactual.

4.2 *The Existence of Equilibrium*

In Bade (2011a) I propose to solve this riddle by assuming that parties do not know their odds of winning for every possible constellation of opposing platforms. I assume that parties hold a set of theories on the voters' preferences and that their choice behavior follows the max-min expected-utility model by Gilboa and Schmeidler (1989): to evaluate their vote shares, parties take the full set of beliefs on the preferences of the electorate into account. Keeping the opponent's platform fixed, each party chooses the platform that maximizes its worst vote share according to the full set of these beliefs. The parties' theories on voter behavior stay conservatively Downsian in the following two respects: any indifferent voter votes for each party with equal probability, and all voters have single-peaked preferences.

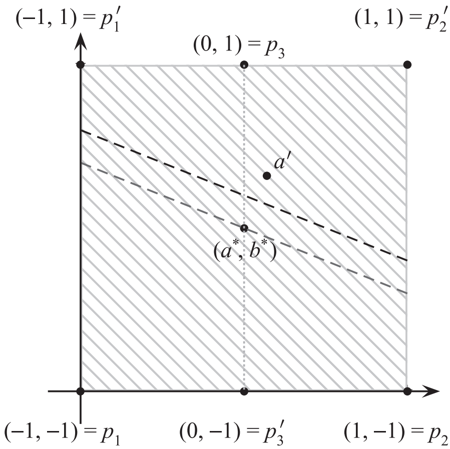
To see why the introduction of ambiguity aversion should favor the existence of equilibrium, observe that the model of Downs–Hotelling competition I have sketched resembles the Ellsberg paradox. For a fixed platform of the opposing party, each party faces the choice between a lottery of winning the election with a chance of 50% or winning the election with ambiguous odds. Either party can obtain the 50% chance of winning by matching the platform of the opposing party. Conversely, if the two parties offer different platforms, the parties' beliefs on voter preferences matter. Here both parties might believe that – under the worst-case scenario – they would obtain less than half the votes.

Let me illustrate this point with *model 2*, a variation on model 1. Just as in model 1, model 2 has 3 voters with Euclidean preferences over the issue space

$[-1, 1]^2$. Differently from model 1, the parties in model 2 do not know the ideal points of the voters. Parties maximize the max-min vote shares, each with the belief set C consisting of two distributions.[1] The first one, π, is identical to the one posited above. The second one, μ, is such that the three voter ideal points are at $p'_1 = (-1, 1)$, $p'_2 = (1, 1)$, and $p'_3 = (0, -1)$.

The platform profile $(a^*, b^*) = ((0, 0), (0, 0))$ is an equilibrium in model 2. Consider the deviation a' for party A as shown in Figure 2. The black dashed line, which is perpendicular to the line between $(0, 0)$ and a' and cuts this line exactly in half, separates the ideal points of all voters who strictly prefer a' to $(0, 0)$ from the voters with the opposite preferences. Any voter whose ideal point is on this black line is indifferent between the two platforms. The deviation to a' is not preferred, as party A only obtains one vote (voter 3's) when using the distribution π to evaluate the electorate. Of course, this is just one among a myriad of possible deviations.

Figure 2
The Equilibrium of Model 2



To see that no deviation in the same direction as a' would win more than one vote according to π, move the platform a' closer and closer to $(0, 0)$. Observe that the line that separates the voters who prefer the deviation from those who prefer $(0, 0)$ moves closer and closer to the gray dashed line. Analogously, there is a straight line through $(0, 0)$, for any deviation, such that all voters who prefer the deviation are strictly contained in one of the half spaces to one side of that line. The proof

[1] Technically this model does not qualify as a max-min expected-utility model following Gilboa and Schmeidler, since they require a convex and compact C. The result remains identical when requiring that agents hold the convex hull of π and μ as their set of beliefs.

that (a^{*}, b^{*}) is an equilibrium is concluded with the observation that if such a half space contains more than one voter according to one of the relevant distributions, it contains only one voter according to the other distribution. So for each deviation the deviating party's minimal vote count is at most one, meaning that the party is better off keeping its platform at $(0, 0)$.

Needless to say, model 2 is an extreme example. For the nitty-gritty details of the much weaker conditions under which equilibria exist in multidimensional Downs–Hotelling games with ambiguity-averse parties, see Bade (2011a). There I do not restrict attention to Euclidean preferences and allow all single-peaked preferences. Parties can be uncertain about the voters' willingness to trade off issues against each other, as well as about the voters' ideal points.

In addition to the result on the existence of equilibrium, there is a result on equilibrium characterization in the paper: in equilibrium both parties announce a median preferred policy for every issue. Consider the two distributions in the above example. The median with respect to issue 1 (the horizontal axis) is 0. The median with respect to the other issue is -1 for π, and 1 for μ (according to μ, all voters have their ideal point on or below the line $(x_1, 1)$, while two-thirds have their ideal point on or above that line, so this line is the median line). Since parties are certain that the median preferred policy with respect to issue 1 is 0, according to my result any equilibrium platform x must have $x_1 = 0$. On the other hand, no restriction is implied for the second issue; equilibrium platforms x might have any $x_2 \in [-1, 1]$.

This generalization of characterization of equilibrium outcomes as preferred policies by median voters is important insofar as it gives some justification to a modeling technique that has been widely used in applied models: Political-economy theories often regard one issue in isolation, say taxes or health care, and then argue by the Downs–Hotelling model that in a democratic society the median preferred policy should obtain. However, the classic Downs–Hotelling model gives no justification for regarding issues in isolation. My paper delineates assumptions under which it is theoretically sound to apply the Downs–Hotelling model issue by issue.

4.3 *Policy Convergence*

The model discussed in the preceding section shares one counterfactual prediction with the standard model: in equilibrium, parties announce identical platforms. Reconsider model 2, and observe that only when announcing the platform of the opponent does a party have firm control over the worst possible expected vote share: in equilibrium each party gets half of the vote share for either of the two distributions. In equilibrium the parties experience no uncertainty. On the other hand, for any deviation, uncertainty aversion does matter: the prospects typically look good for some distribution, but there is always also a pessimistic outlook according to which the deviation lowers the vote share for the deviator. This description does not just hold for the example. Offering the platform of the opponent will generally serve as a hedging device. If both parties offer the same platform, the ambiguous information on voter preferences does not matter. In that case each party gets half the vote share – no matter which distribution over the preferences one assumes. Conversely, if parties offer different platforms, the minimal vote share of each party might be below one-half.

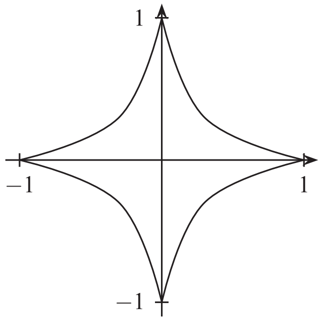
Observe that this rationale for platform convergence is peculiar to the model with ambiguity aversion. In the standard model, platform convergence is driven by the unidimensionality of the issue space and the single-peakedness of preferences. Single-peaked preferences over a unidimensional issue space are necessarily convex, meaning that any voter who prefers platform a to platform b must also prefer platform $\lambda a + (1 - \lambda)b$ to b for any $\lambda \in (0, 1)$. This in turn implies that party A never loses any votes by changing its platform from a to $\lambda a + (1 - \lambda)b$ with b being party B's platform and λ being a number in $(0, 1)$, as any voter who is already voting for party A when it runs on platform a must continue to vote for this party when it changes its platform to $\lambda a + (1 - \lambda)b$, assuming the other party's platform remains fixed at b.

So moving one's platform closer to the other party's platform never decreases one's vote share. If any voters have their ideal point strictly between the two parties' platforms, at least one party benefits from moving closer to the other party's platform. To see this, suppose that there is some voter with ideal point $x \in (a, b)$. Suppose, w.l.o.g., that this voter votes with a probability of at least 50% for party A. Then party B can strictly improve its vote share by offering $(1/2)a + (1/2)x$ as its platform. As argued above, party B will not lose any votes due to this move. In addition it will gain the vote of the voter with ideal point at x, since $a < (1/2)a + (1/2)x < x$. So in any equilibrium (a^*, b^*), platforms must converge.

So to have a Downs–Hotelling model of divergent platforms, both these rationales for platform convergence need to be undermined. To this end I set up a Downs–Hotelling model with a multidimensional issue space and party preferences that follow Bewley's model of Knightian uncertainty in Bade (2011b). With a multidimensional issue space, single-peakedness does not imply convexity of preferences. To see this, consider a two-dimensional issue space $[-1, 1]^2$ and an agent with preferences $u(x) = -\sqrt{|x_1 - p_1|} - \sqrt{|x_2 - p_2|}$. This agent has an ideal point of (p_1, p_2), and his preferences are single-peaked: his utility decreases when he goes from a platform x to a platform y which is further from his ideal point in either direction (formally $|p_1 - y_1| = |p_1 - x_1| + |x_1 - y_1|$ as well as $|p_2 - y_2| = |p_2 - x_2| + |x_2 - y_2|$). According to this criterion a voter with ideal point $p = (0, 0)$ must prefer platform $(0.1, 0.1)$ to platform $(0.2, 0.3)$. To see that the voter's preferences are not convex, consider a voter with ideal point $(0, 0)$ together with the platforms $(1, 0)$, $(0, 1)$ and the intermediate platform $(1/2, 1/2)$. So $u((0, 1)) = u((1, 0)) = -1$ and $u((1/2, 1/2)) = -\sqrt{1/2} - \sqrt{1/2} < -1$. The voter's indifference curve through $(1, 0)$ is illustrated by the black line in Figure 3. In short, the standard rationale for convergence does not apply to the present model.

Now let me argue that the hedging benefit of offering the other party's platform is no longer a decisive advantage. According to the Bewley model of Knightian uncertainty, party A prefers platform a to platform a' – keeping party B's platform fixed at b – if party A obtains a (weakly) higher vote share for platform a' according

Figure 3
Nonconvex Preferences



to *all* priors on the electorate. Parties do not focus on the prior on the electorate that is associated with the minimal vote share. So offering the platform of the opponent, which is associated with a high minimal vote share of $1/2$, has no particular appeal within this framework. To see this, reconsider model 2. Fixing party B's platform at (0, 0), party A best responds by offering platform (0, 1) if it has Bewley preferences with the belief set $\{\pi, \mu\}$ given in model 2. For this deviation party A obtains two votes according to distribution π and one vote according to distribution ρ. The platform (0, 1) is a best reply, since there is no other platform for which the party can improve its vote share according to one distribution without hurting its vote share according to the other.

The gist of the paper Bade (2011b) consists in arguing that both features – the multidimensionality of the issue space as well as the Bewley-type preferences – are necessary for the existence of divergent equilibria. I provide an example of a modified Downs–Hotelling game that indeed does have a divergent equilibrium. The necessity of both features might seem surprising, given the discussion of model 2. I claimed that party A had no incentive to exactly match party B's platform in model 2, even though all voters' preferences were assumed to be Euclidean, and therefore convex. The answer lies in what it means for platforms to diverge: some voters must with positive probability have their ideal point between the two platforms. If one were to call any pair of different platforms divergent, even the classical Downs–Hotelling model would have divergent equilibria (consider the case of a distribution of voter ideal points with a nonunique median). Now one might debate what "in between two platforms" means in the context of a multidimensional issue space. One might equally debate what "some voters must with positive probability" means in a world of ambiguity aversion. In Bade (2011b) I provide more and less liberal definitions of these terms and show that my results hold even for the strictest cases. With respect to model 2 one should observe that the platforms (0, 0) and (0, 1) can hardly be considered divergent by the criterion that some voter should have his ideal point with positive probability between these two platforms.

The models in Bade (2011a) and Bade (2011b) differ only in one respect: in Bade (2011a) I follow Gilboa and Schmeidler's (1989) max-min expected utilities to model preferences; in the other paper I follow Bewley's model of Knightian uncertainty.[2] Given that these two models of preference representation are so similar, the difference between the two models is very small indeed. But the difference in results is major: divergent equilibria only obtain with Bewley-type preferences. In Bade (2011a), with max-min preferences, I could provide a very straightforward characterization result that allows us to apply the classic Downs–Hotelling prediction issue by issue: in equilibrium, parties will announce a median preferred policy with respect to every issue. Unfortunately, such a characterization result has proved elusive with respect to the Bewley model. The important question of comparative statics remains completely open. It remains unknown how the set of equilibria changes as a function of the belief set on voter preferences.

5 *Condorcet Models of Information Aggregation*

5.1 *The Classical Model*

In the classical model of Condorcet voting, there are two candidates L and R and two equiprobable states of the world, λ and ρ. All voters have identical preferences: they prefer candidate L in state λ and candidate R in state ρ. They all assign utility values of 1 and 0 to the choice of the correct and the wrong candidate, respectively. Information on the state of the world is spread throughout the population: each voter privately holds a signal $s \in \{l, r\}$ on the state of the world such that s is l with probability $p > 1/2$ in state λ and conversely r is b with the identical probability p in state ρ.

If all voters vote according to their signal, the probability that the electorate chooses the correct candidate approaches one as the size of the electorate grows large. This result dates back to Condorcet. In the light of modern game theory another question arises: would it be optimal for one voter to vote according to his own signal, given that all other voters do so? Or in other words, is sincere voting an equilibrium? In the very symmetric setup sketched above, the answer is yes. Given that all voters vote according to their own signal, voting according to one's own signal is a strict best reply. There are boundless variations of the Condorcet model in the literature. The size of the electorate might be known or unknown, one state of the world might be more probable than the other, signal probabilities might vary, different agents might get signals of different strength, signals might be acquired endogenously, or there might be some pre-electoral sharing of information, to name some of the most obvious variations.

[2] Since the only example of the model with max-min expected utilities assumes that voter preferences are Euclidean, the attentive reader might presume that there is yet another difference between the two, regarding the convexity of the preferences. This is not the case. In Bade (2011a) I do not assume convex preferences.

Since abstention is the main focus of the papers on ambiguity aversion in Condorcet models, let me mention the – by now – classic variation of the Condorcet model used to explain abstention: Feddersen and Pesendorfer's (1996) *swing voter's curse*. In this model, one state is a priori more likely than the other, and voters receive signals of varying strength and behave fully strategically. There are equilibria in which the less informed voters abstain from voting – even if there is no cost of voting and even if each of these voters has a strict preference for one candidate over the other when regarding only his own information. To figure out his optimal strategy a voter considers only the case in which his vote matters to the electoral outcome, namely, when he is pivotal. Now, if a voter with low-quality information is pivotal, this is a sign that more voters with high-quality information received a signal favoring the candidate less likely a priori. So the voter with low-quality information should abstain. This rationale can lead to sizable abstention, which does not diminish as the electorate grows large. However, this form of abstention does not hurt information aggregation in the limit. The probability that the correct candidate is selected approaches one as the electorate grows large.

5.2 *Abstention*

To my knowledge, Ghirardato and Katz (2006) were the first to introduce ambiguity aversion into any model of political economy. They assume that the voters in a Condorcet model are ambiguity-averse and show that an ambiguity-averse voter might strictly prefer abstaining to voting. The reasoning is simple: an ambiguity-averse voter might understand so little about an election that he would rather have the random draw that occurs in the event of a tie than commit himself to either one of the candidates. For a formal example assume that from the point of view of some voter only two states matter: in state λ this voter obtains utility 1 if candidate L is voted into office, and utility 0 otherwise; in the alternative state ρ, the inverse picture holds – the voter assigns utility 1 to candidate R and utility 0 to candidate L. Now assume that the voters' preferences can be represented by the Gilboa–Schmeidler model and that – conditioning on being pivotal and on his signal – the voter holds a set of priors $[\underline{\pi}, \overline{\pi}]$ that the state is ρ.

If $\underline{\pi} < 1/2 < \overline{\pi}$, then abstaining is strictly better for the voter under consideration. To see this, observe that the vote of the agent under consideration only matters if he is pivotal – so the set of priors that conditions on the pivotal event (and the agent's signal) is the relevant set of priors. Now if the agent votes for the left candidate, he obtains a utility of $\min_{\pi\in[\underline{\pi},\overline{\pi}]} 1 \times (1-\pi) = 1-\overline{\pi} < 1/2$. Similarly, the agent's utility of voting for the right candidate is $\min_{\pi\in[\underline{\pi},\overline{\pi}]} 1 \times \pi = 1-\underline{\pi} < 1/2$. On the other hand, abstaining corresponds to randomizing over the two candidates, which yields a utility of $\min_{\pi\in[\underline{\pi},\overline{\pi}]}((1/2)\pi + (1/2)(1-\pi)) = 1/2$. So abstention allows the voter to hedge between the two candidates.

The paper is neatly motivated by the empirical observation that voters who arguably face no cost of voting might still abstain: Ghirardato and Katz discuss the case of multiple elections on one ballot. In such elections voters often selectively abstain from the less publicized races. Ambiguity aversion as an explanation comes in handy, since we have empirical evidence that comparisons across decisions dramatically affect agents' perception of ambiguity (Fox and Tversky, 1995). Thus, in the light of the more familiar race, the less familiar ones will appear more ambiguous to the voters. So the comparative-ignorance hypothesis straightforwardly explains the empirical phenomenon of selective abstention.

Given its ability to explain multiple-election behavior, Ghirardato and Katz's model outperforms costly-voting models as an explanation of abstention. However, costly-voting models are not the only and maybe not even the most prominent alternative explanation of abstention in the literature. Feddersen and Pesendorfer's (1996) *swing voter's curse* is the contemporary workhorse model on abstention. Since this is a model that succeeds in explaining mass abstention within the framework of classical expected-utility theory, Ghirardato and Katz (2006) had to distinguish their rationale for abstention from Feddersen and Pesendorfer's (1996) rationale. They do so by analyzing the decision to vote or abstain in a model of voting that drops the common-interest assumption. Note that in my sketch of their results, I always spoke of the preference of the voter under consideration, and made no reference to the question whether this preference is a shared one or not.

Let the states λ and ρ continue to be the relevant states of nature for the voter under consideration. This voter might be a shop owner who either will need to hire an employee (state λ) or will be able to continue to run his store alone (state ρ). In state λ this shop owner prefers politician L, who is more lenient on immigration. Now, the preferences of other voters need not at all be aligned. Another voter might for example want to lower the immigration standard to accommodate more refugees in times of dire catastrophe. By allowing for such diverse preferences Ghirardato and Katz (2006) shut down the channel that drives Feddersen and Pesendorfer's (1996) swing-voter result. In Ghirardato and Katz's article agents do not need to agree on the informational content of the pivot-event. In short, even when the votes of the other citizens have no bearing on the voter's (conditional) preferences over candidates, voters who perceive the state as more ambiguous are more likely to abstain.

5.3 Information Aggregation

Andrew Ellis (2012) takes the insight developed in Ghirardato and Katz (2006) one step further. The paper of Ghirardato and Katz (2006) in a sense remains a "partial equilibrium" model that considers abstention of some voters while taking the behavior of other voters for granted. Ellis (2012) instead asks whether the standard result that information is aggregated efficiently in large elections continues to hold when voters are ambiguity-averse. In contrast to Ghirardato and Katz (2006), Ellis's model remains more closely tied to the standard Condorcetian setup: all voters receive utility 1 if candidate L is elected in state λ or if candidate R is elected in state ρ, and otherwise receive utility 0.

Ellis observes that even with informative signals there are sequences of equilibria in Condorcetian games with ambiguity aversion such that the probability of the correct candidate being selected in either state remains $1/2$ no matter what the size of the electorate. Consider a voter who is so ambiguity-averse that he would rather throw a fair coin to decide between L and R than to choose either one with certainty if this voter had to decide who gets elected based on his own signal. Now consider an electorate that is entirely made up of such voters. Complete abstention is an equilibrium in this model. Given that all other voters abstain, any voter faces the decision to choose the president on his own. By assumption each of these voters prefers a fair lottery over the two candidates (which will happen when he abstains) to choosing either one. This argument at no point invokes the size of the electorate. Therefore there can be equilibria that fail to aggregate information, regardless of the size of the electorate.

More formally, consider an electorate consisting of n ambiguity-averse voters whose preferences can be represented following Gilboa and Schmeidler. Assume that voters can observe two different signals l and r. The signals provide information about the underlying state of nature in that, having observed signal x, a voter updates his set of priors to $[\underline{\pi}_x, \overline{\pi}_x]$, where $\underline{\pi}_l < \underline{\pi}_r$ and $\overline{\pi}_l < \overline{\pi}_r$.

Consider a game in which voters can choose only L, R, or any mixture thereof. Games of this sort have a noninformative equilibrium in which all voters vote for either candidate with probability $1/2$ if $\underline{\pi}_r < 1/2 < \overline{\pi}_l$ (and hence $1/2 \in (\underline{\pi}_l, \overline{\pi}_l)$ and $1/2 \in (\underline{\pi}_r, \overline{\pi}_r)$). Consider the choice of any voter, given that all other voters randomize. Since all other voters randomize, the pivotal event has no informative content: all the information that the voter has is contained in his own signal. But the signals and updating were defined so that even if the voter obtained l as his signal, he would rather randomize between the two candidates than vote for L, since $\min_{\pi \in [\underline{\pi}_x, \overline{\pi}_x]}(1-\pi) < \min_{\pi \in [\underline{\pi}_x, \overline{\pi}_x]}((1/2)\pi + (1/2)(1-\pi)) = 1/2$. In short, the voter is afraid to commit to vote for candidate L, since he believes that malevolent nature would then set the probability of the state being λ to $1 - \overline{\pi}_l < 1/2$. Instead he prefers to hedge by either randomizing between the two candidates, or abstaining in the modified model that allows for abstention. The same holds, mutatis mutandis, for a voter who observes signal r.

6 Outlook

I find it surprising that so far there are just four models that allow for ambiguity-averse agents. I fully agree with the following statement by Ghirardato and Katz (2006, p. 383) on modeling agents in the political sphere as ambiguity-averse: “The choice of voting is a natural first step, for it clearly seems to be a decision problem in which low quality of information is the norm rather than the exception.” Ghirardato and Katz started to work on this “natural first step” to fuse political economy and ambiguity aversion in 1997. Still, as of 2012, I am only aware of four papers that have continued this agenda.

Only the two most classic models in political economy have been modified to allow for ambiguity aversion: the Downs–Hotelling model of political competition and the Condorcet model of information aggregation. But, as argued above, even confining ourselves to these two models, many questions on the effect of ambiguity aversion on classical results remain open. While I have shown in Bade (2011b) that ambiguity aversion *can* lead to divergent equilibria in a Downs–Hotelling model, I do not provide any comparative statistics that would help us to predict when such divergent equilibria exist or where we should expect parties to locate their platforms.

While Ellis (2012) has demonstrated that ambiguity aversion might get in the way of information aggregation in large electorates with informative signals, much remains to be learned about the set of all equilibria in models of Condorcetian voting with ambiguity-averse voters. McLennan (1998) shows that any utility-maximizing strategy profile in a game of common interest is an equilibrium. McLennan assumes expected-utility-maximizing agents. Still, if voters are dynamically consistent updaters, the result can be applied to Condorcetian games with ambiguity-averse agents. So assume that the nonstrategic optimization problem of finding a (signal-contingent) plan for every voter to achieve the ex ante optimal outcome in a Condorcet problem has a well-defined solution that aggregates information. Then the corresponding Condorcetian game has an equilibrium that aggregates information.[3] The open and difficult question is to characterize these equilibria. For such a characterization we should assume not only that voters are dynamically consistent, but also that they receive "independently and identically distributed" signals. While it is well known what this requirement means in the context of expected-utility-maximizing agents, the definition and study of the concept of independently and identically distributed random variables is currently at the frontier of the study of ambiguity aversion (Larry Epstein and coauthors have a sequence of papers on the subject).

While the route of twisting some well-understood models to introduce ambiguity aversion has been taken, there seem to be no papers that make use of ambiguity aversion to tackle some of the untamed questions in the field of political economy. Consider models involving more than two parties, which still pose some large challenges. If parties are able to freely choose their platforms to maximize their vote shares, the existence of equilibrium becomes a delicate issue. Moreover the assumption that voters would simply vote for their most preferred party becomes questionable in a context with three or more parties. Strategic voting might lead voters to vote for the lesser evil rather than their most preferred party, as can occur under a winner-take-all rule. Matters get further complicated if there is no such rule, and parties instead need to form coalitions once they are voted into parliament.

One typical problem in such models is that of the nonexistence of equilibrium. Take the example of Downsian competition with multiple parties, where incentives of parties to deviate are pervasive. It is possible that the assumption that parties are ambiguity-averse relaxes some of these maddeningly strong incentives to deviate and thereby would allow some equilibria to exist. Similarly, in a model of parliamentary democracy, one could imagine using ambiguity aversion to narrow the set of potential

[3] I owe this observation to a discussion among Eran Hanany, Peter Klibanoff, and Michael Mandler.

coalition partners for a party. Finally, ambiguity aversion could be used to render sincere voting more attractive in models with more than two candidates: this could arise out of the ambiguous information on the winning chances of the various candidates. So ambiguity aversion could be used to eliminate some equilibria that rely on the coordination of voters.

References

Bade, S. (2011a), "Electoral Competition with Uncertainty Averse Parties," *Games and Economic Behavior*, 72(1), 12–29.

— (2011b), "Divergent Platforms," Working Paper 2011/25, Preprints of the Max Planck Institute for Research on Collective Goods, Bonn.

Bewley, T. F. (2002), "Knightian Utility Theory: Part 1," *Decisions in Economics and Finance*, 25(2), 79–110.

Camerer, C., and M. Weber (1992), "Recent Developments in Modeling Preferences: Uncertainty and Ambiguity," *Journal of Risk and Uncertainty*, 5(4), 325–370.

Davis, O. A., M. H. DeGroot, and M. J. Hinich (1972), "Social Preference Orderings and Majority Rule," *Econometrica*, 40(1), 147–157.

Ellis, A. (2012), "Condorcet Meets Ellsberg," Mimeo, Boston University, Boston (MA).

Feddersen, T. J., and W. Pesendorfer (1996), "The Swing Voter's Curse," *The American Economic Review*, 86(3), 408–424.

Fox, C. R., and A. Tversky (1995), "Ambiguity Aversion and Comparative Ignorance," *The Quarterly Journal of Economics*, 110(3), 585–603.

Ghirardato, P., and J. N. Katz (2006), "Indecision Theory: Weight of Evidence and Voting Behavior," *Journal of Public Economic Theory*, 8(3), 379–399.

Gilboa, I., and D. Schmeidler (1989), "Maxmin Expected Utility with Non-Unique Prior," *Journal of Mathematical Economics*, 18(2), 141–153.

Grandmont, J.-M. (1978), "Intermediate Preferences and the Majority Rule," *Econometrica*, 46(2), 317–330.

McKelvey, R. D. (1979), "General Conditions for Global Intransitivities in Formal Voting Models," *Econometrica*, 47(5), 1085–1112.

McLennan, A. (1998), "Consequences of the Condorcet Jury Theorem for Beneficial Information Aggregation by Rational Agents," *American Political Science Review*, 92(2), 413–418.

Plott, C. R. (1967), "A Notion of Equilibrium and its Possibility under Majority Rule," *The American Economic Review*, 57(4), 787–806.

Schmeidler, D. (1989), "Subjective Probability and Expected Utility without Additivity," *Econometrica*, 57(3), 571–587.

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