

## Programme

academic year 2018-2019

I am listing here the titles of sections and subsections of my lecture notes. Notation:

■ **Examinable topics.**

▲ **Examinable topics with the indicated limitations.**

□ **Introductory topics and useful reminders for the understanding of other topics of the course. They are not examinable in their own, but it is assumed that they have to be known for the part necessary to the understanding of other topics of the course.**

◆ **These topics were covered, or covered incompletely, in class, but will not be examined in this course.**

Please have a look also at the [Moodle page](#) of the course, where you can find all details of the course (notes, materials where my notes are taken, supplementary material such as codes and videos), tutorials, homeworks, problems in class, and further readings.

### I. THE ORIGINS OF QUANTUM MECHANICS: A BRIEF REVIEW

#### 1. Introduction

#### 2. Waves and particles: old and new phenomena

- ◆ The blackbody radiation
- ◆ The photoelectric effect
- ◆ The Compton effect
- ◆ The Bohr atom

#### 3. De Broglie's hypothesis

- Einstein-De Broglie equations
- De Broglie's waves and particle velocities. Free wave packet
- Quantisation of atomic energy levels
- Experimental confirmation of the wave-like nature of particles

### II. THE WAVE FUNCTION AND THE SCHRÖDINGER EQUATION

#### 1. The Schrödinger equation

- The Schrödinger equation

#### 2. Born's statistical interpretation of the wave function

- Probabilities of the results of a measurement of the position of a particle
- Normalisation condition
- Superposition principle
- Norm preservation
- Probability current
- Expectation values of position and momentum
- Expectation values of other dynamical variables
- Digression: the operator concept. Commutators

#### 3. Statistical interpretation and particle-wave duality

- The double slit experiment with tennis balls
- The double slit experiment with water waves
- The double slit experiment with quantum particles

#### 4. Heisenberg uncertainty relations

- Position-momentum uncertainty relations

#### 5. The measurement procedure

- Outcome and probabilities
- Repeatability of a measurement
- Possible results of a measurement. Reduction of the wave packet
- Probabilities
- Individual versus multiple measurements
- 6. **Particle in a time-independent scalar potential**
  - Time-independent the Schrödinger equation
  - Stationary states

### III. ONE DIMENSIONAL SYSTEMS

1. **The infinite square potential well**
  - Eigenvalues, eigenfunctions, and their properties
2. **The free particle**
  - Free wave packet
  - ◆ Parseval-Plancherel's theorem
  - Form of a free wave packet at a given time
  - Time evolution of a free wave packet
  - One-dimensional gaussian wave packet (results only)
3. **General properties of the solutions**
  - Bound states and scattering states
4. **The finite square potential well**
  - Eigenvalues and eigenfunctions
5. **Scattering from potential well**
  - Reflection and transmission coefficients
6. **The square potential barrier**
  - The “tunnel” effect
7. **The potential step**
  - Eigenvalues and eigenfunctions. Reflection and transmission coefficients
8. **The harmonic oscillator**
  - Analytic method: eigenvalues, eigenfunctions, and their properties
  - Comparison classical mechanics-quantum mechanics
  - Expectation values of position and momentum

### IV. THE MATHEMATICAL TOOLS OF QUANTUM MECHANICS

- One-particle wave-function space (discrete basis only)
- State space
- Representations on state space (discrete basis only)
- Eigenvalue equation. Observables (results and definitions only)

### V. THE GENERAL PRINCIPLES OF QUANTUM MECHANICS

1. **Introduction**
  - Introduction
2. **General principles**
  - General principles
3. **Physical interpretation of the principles concerning observables and their measurement**
  - Mean value of an observable in a given state
  - Compatibility of observables
  - Uncertainty relations

4. **Physical implications of the Schrödinger equation**
  - General properties of the Schrödinger equation
  - The case of conservative systems. Ehrenfest theorem
5. **The superposition principle and physical predictions**
  - Probability amplitudes and interference effects
6. **A familiar example: polarisation of light**
  - Polarisation states of a photon
  - Measurement of photon polarisation. Successive measurements
7. **Example: the one-dimensional harmonic oscillator**
  - Algebraic solution

## VI. THREE DIMENSIONAL QUANTUM SYSTEMS

1. **Systems with rectangular symmetry**
  - Eigenvalues and eigenfunctions
  - Example: free particle in three dimensions
  - Example: three-dimensional harmonic oscillator
2. **Systems with central symmetry**
  - Definitions. Separation of variables
  - ▲ The angular equation (results only)
  - ▲ The radial equation (results only)

## VII. PARTICLE IN A CENTRAL POTENTIAL: THE HYDROGEN ATOM

1. **General remarks on the hydrogen atom**
  - The hydrogen atom
2. **The Schrödinger equation for a two-body problem**
  - ◆ The Schrödinger equation for a two-body problem
3. **The radial wave function**
  - Introduction
  - Change of variables
  - Asymptotic behaviour
  - Solutions in the form of power series in  $\rho$
  - Eigenvalues and eigenfunctions

## VIII. ORBITAL ANGULAR MOMENTUM

1. **Introduction**
  - Definitions and properties
2. **The observables  $L^2$  and  $L_z$** 
  - Eigenvalue equation of  $L^2$  and  $L_z$
  - $L_+$  and  $L_-$  operators
  - ▲ Standard basis of angular momentum (results only)
  - ▲ Matrix elements of the angular momentum operators (results only)
3. **Eigenvalues and eigenfunctions of  $L^2$  and  $L_z$** 
  - Introduction. Values of  $l$  and  $m$ .

## IX. THE SPIN

1. **Introduction**
  - The concept of spin
2. **Effects of a magnetic field on the electron states**

- Zeeman effect
- 3. **Goudsmit and Uhlenbeck hypothesis**
  - The hypothesis of electron spin
- 4. **The internal dynamics: quantum formalism**
  - Quantum formalism
- 5. **Quantum formalism for spin 1/2. Pauli matrices**
  - Quantum formalism for spin 1/2
  - Pauli matrices
- 6. **Spin measurements: Stern and Gerlach experiment**
  - Classical description
  - Quantum description
- 7. **Addition of two spins 1/2 particles**
  - Notation and state space
  - Total spin  $S$ . Commutation relations
  - The basis change to be performed
  - The eigenvalues of  $S_z$  and their degrees of degeneracy
  - Diagonalisation of  $S^2$
  - Eigenvalues and eigenvectors of  $S^2$
  - Results: spin singlets and spin triplets

## X. TIME-INDEPENDENT PERTURBATION THEORY

- 1. **Introduction**
  - Introduction
- 2. **Description of the method**
  - Description of the method
- 3. **Perturbation of a non-degenerate level**
  - First-order energy corrections
  - First-order eigenvector corrections

## XI. SYSTEMS OF IDENTICAL PARTICLES

- 1. **Introduction**
  - Definition
- 2. **Two-particle systems**
  - Two-particle systems
- 3. **Bosons and fermions**
  - Consequences of identity
  - Connection between spin and statistics
  - The Pauli exclusion “principle”
  - The symmetrisation principle
- 4. **Atoms**
  - ◆ The Aufbau principle and the periodic table