PH2110: Quantum Mechanics
Royal Holloway University of London

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Programme

academic year 2014-2015

I am listing here the titles of sections and subsections of my lecture notes. Notation:
■ Examinable topics
▲ Examinable topics with the indicated limitations
♦ These topics were covered in class, but will not be examined

Take a look also at the Moodle page of the course, where you can find all details of the course (notes, materials where my notes are taken, supplementary material such as codes and videos), tutorials, homeworks, and further readings.

I. THE ORIGINS OF QUANTUM MECHANICS: A BRIEF REVIEW

A. Waves and particles: old and new phenomena

♦ The blackbody radiation

♦ The photoelectric effect

♦ The Compton effect

♦ The Bohr atom

B. De Broglie's hypothesis

■ Einstein-De Broglie equations

■ De Broglie's waves and particle velocities. Free wave packet

■ Quantisation of atomic energy levels

■ Experimental confirmation of the wave-like nature of particles

II. THE WAVE FUNCTION AND THE SCHRÖDINGER EQUATION

A. Invention of the Schrödinger equation

■ The Schrödinger equation

B. Born's statistical interpretation of the wave function

■ Probabilities of the results of a measurement of the position of a particle

■ Normalisation condition

■ Superposition principle

■ Norm preservation

■ Probability current

■ Expectation values of position and momentum

■ Expectation values of other dynamical variables

■ Digression: the operator concept. Commutator

C. Statistical interpretation and particle-wave duality

■ The double slit experiment with tennis balls

■ The double slit experiment with water waves

■ The double slit experiment with quantum particles

D. Heisenberg uncertainty relations

■ Position-momentum uncertainty relations

E. **The measurement procedure**

- ■ Outcome and probabilities
- ■ Repeatability of a measurement
- ■ Possible results of a measurement. Reduction of the wave packet
- ■ Probabilities
- ■ Individual versus multiple measurements

F. **Particle in a time-independent scalar potential**

- ■ Time-independent the Schrödinger equation
- ■ Stationary states

III. ONE DIMENSIONAL SYSTEMS

A. **The infinite square potential well**

- ■ Eigenvalues, eigenfunctions, and their properties

B. **The free particle**

- ■ Free wave packet
- ♦ Parseval-Plancherel's theorem
- ■ Form of a free wave packet at a given time
- ■ Time evolution of a free wave packet
- ▲ One-dimensional gaussian wave packet (results only)

C. **General properties of the solutions**

- ■ Bound states and scattering states

D. **The finite square potential well**

- ■ Eigenvalues and eigenfunctions

E. **Scattering from potential well**

- ■ Reflection and transmission coefficients

F. **The square potential barrier**

- ■ The "tunnel" effect

G. **The potential step**

- ■ Eigenvalues and eigenfunctions. Reflection and transmission coefficients

H. **The harmonic oscillator**

- ■ Analytic method: eigenvalues, eigenfunctions, and their properties
- ■ Comparison classical mechanics-quantum mechanics
- ■ Expectation values of position and momentum

IV. THE GENERAL PRINCIPLES OF QUANTUM MECHANICS

A. **The mathematical tools of quantum mechanics**

- ▲ One-particle wave-function space (discrete basis only)
- ■ State space: Dirac notation
- ▲ Representations on state space (discrete basis only)
- ▲ Eigenvalues and eigenvectors. Observables (results and definitions only)

B. **The general principles of quantum mechanics**

- ■ Introduction

- ■ General principles
- ■ Physical interpretation of the principles concerning observables and their measurement
- ■ Physical implications of the Schrödinger equation. Ehrenfest theorem
- ■ The superposition principle and physical predictions

C. **A familiar example: polarisation of light**

- ■ Polarisation states of a photon
- ■ Measurement of photon polarisation. Successive measurements

D. **Example: the one-dimensional harmonic oscillator**

- ■ Algebraic solution

V. THREE DIMENSIONAL SYSTEMS

A. **Systems with rectangular symmetry**

- ■ Eigenvalues
- ■ Example: free particle in three dimensions
- ■ Example: three-dimensional harmonic oscillator

B. **Systems with central symmetry**

- ■ Separation of variables
- ▲ The angular equation (results only)
- ▲ The radial equation (results only)
- ♦ Example: the infinite spherical well

VI. PARTICLE IN A CENTRAL POTENTIAL: THE HYDROGEN ATOM

A. **General remarks on the hydrogen atom**

- ■ The hydrogen atom

B. **The Schrödinger equation for a two-body problem**

- ♦ The Schrödinger equation for a two-body problem

C. **The radial wave function**

- ■ Change of variables
- ■ Asymptotic behaviour
- ■ Solutions in the form of power series in ρ
- ■ Eigenvalues and eigenfunctions

VII. ORBITAL ANGULAR MOMENTUM

A. **Theory of orbital angular momentum**

- ■ Introduction
- ■ The observables L^2 and L_z
- ■ Eigenvalues and eigenfunctions of L^2 and L_z

VIII. THE SPIN

A.**Generalities**

- ■ The concept of spin

B. **Effects of a magnetic field on the electron states**

■ Zeeman effect

C. **Goudsmit and Uhlenbeck hypothesis**

■ The hypothesis of electron spin

D. **The internal dynamics: quantum formalism**

■ Quantum formalism

E. **Quantum formalism for spin 1/2. Pauli matrices**

■ Quantum formalism for spin 1/2

■ Pauli matrices

F. **Spin measurements: Stern and Gerlach experiment**

■ Classical description

■ Quantum description

G. **Addition of two spins 1/2 particles**

■ State space

■ Total spin S. Commutation relations

■ The basis change to be performed

■ The eigenvalues of S_z and their degrees of degeneracy

■ Diagonalisation of S^2

■ Eigenvalues and eigenvectors of S^2

■ Results: spin singlets and spin triplets

IX. TIME-INDEPENDENT PERTURBATION THEORY

A. **Introduction**

■ Introduction

B. **Description of the method**

■ Description of the method

C. **Perturbation of a non-degenerate level**

■ First-order energy corrections

■ First-order eigenvector corrections

■ Second-order energy corrections

X. SYSTEMS OF IDENTICAL PARTICLES

A. **Identical particles**

■ Definition

B. **Two-particle systems**

■ Two-particle systems

C. **Bosons and fermions. The Pauli principle**

■ The symmetrisation principle

■ The Pauli "principle"

D. **Exchange "forces"**

■ Consequence of identity: exchange "forces"

E. **Atoms**

♦ The Aufbau principle and the periodic table