Lecture 11 — 10 August 2019 (11:00 - 12:30)

Plan

Insights from simple limits :

1) noninteracting limit ($U = 0$)
2) atomic limit ($t = 0$)
3) strong coupling limit ($U \gg t$)

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1) noninteracting limit ($U = 0$)

[for details, see P. Fazekas
https://www.worldscientific.com/worldscibooks/10.1142/2945]

a) dispersion relation

⇝ see blackboard

b) thermodynamic quantities

i) partition function Z
ii) occupation n
iii) entropy S

⇝ see blackboard

c) dynamical quantities ⇝ Green's function

i) physical meaning of the Green's function
ii) derivation of the Green's function :

- equation of motion method
- functional integral method

$$G_K(i\omega_n) = \frac{1}{i\omega_n - \xi_K}$$

⇝ see blackboard

[For further details, see A.M. Tremblay
http://www.physique.usherbrooke.ca/tremblay/cours/phy-892/N-corps.pdf]

iii) local density of states

⇝ see blackboard

d) question: correlations for $U=0$?

② atomic limit ($t=0$)

[For details, see P. Fazekas
https://www.worldscientific.com/worldscibooks/10.1142/2945]

a) disconnected sites ⟶ single site model

b) thermodynamic quantities

i) partition function Z

ii) occupation n and charge compressibility κ

iii) entropy S

⇝ see blackboard

c) dynamical quantities ⇝ Green's function

i) derivation with the equation of motion method

$$G(i\omega_n) = \frac{1-\langle n_{-\sigma}\rangle}{i\omega_n+\mu} + \frac{\langle n_{-\sigma}\rangle}{i\omega_n+\mu-U}$$

⇝ see blackboard

[For further details, see A.M. Tremblay
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ii) local density of states

⇝ see blackboard

③ Strong coupling limit ($U \gg t$)

a) idea : low energy effective Hamiltonian

$$H_{\text{Hubbard model}} \longrightarrow H_{t\text{-}J \text{ model}} \xrightarrow{(n=1)} H_{\text{Heisenberg model}}$$

To obtain the t-J model, we can use two equivalent methods :

i) degenerate perturbation theory

ii) canonical transformation

b) canonical transformation method

i) concept : 2 steps :

$$H \longrightarrow \tilde{H} = e^{iS} H e^{-iS}$$

$$\tilde{H}^{(2)} \longrightarrow H_{\text{eff}}$$

⇝ see blackboard (and the following 2 references)

ii) derivation

see • Macdonald et al., PRL 1988

https://journals.aps.org/prb/abstract/10.1103/PhysRevB.37.9753

• Delannay et al., PRB 2005

https://journals.aps.org/prb/abstract/10.1103/PhysRevB.72.115114

iii) physics in the 2 site terms and 3 site terms

see P. Fazekas

https://www.worldscientific.com/worldscibooks/10.1142/2945