**Name:**______________________________

**Signature:**______________________________

**J. Number:**______________________________

# MA 126 Calculus II
# Exam 1

1. Attempt all of the questions.
2. Write your name and J. number at the top of this page.
3. Answer the questions in the spaces provided.
4. Show all your work required to obtain your answers.
5. **The quality of your solution will be assessed..**
6. Simplify your answers when it is possible.
7. **No calculators are allowed.**
8. You may use a notecard of maximum size one side of letter paper.

1. Evaluate $\int (3x^2 + 2x)x\,dx$.

$$\int (3x^2+2x)x\,dx = \int 3x^3+2x^2 dx = \frac{3}{4}x^4 + \frac{2}{3}x^3 + C$$ 6

2. Evaluate $\int x(x+1)^9\,dx$.

let $u = x+1 \Rightarrow x = u-1$

$du = dx$ 8

$$\int x(x+1)^9 dx = \int (u-1)u^9 du = \int u^{10} - u^9 du$$

4

$$= \frac{1}{11}u^{11} - \frac{1}{10}u^{10} + C$$ 2

$$= \frac{1}{11}(x+1)^{11} - \frac{1}{10}(x+1)^{10} + C$$ 2

3. Evaluate $\int_1^6 \sqrt{x+3}\,dx$.

let $u = x+3$

$du = dx$

$u(1) = 4 \quad u(6) = 9$

10

$$\int_1^6 \sqrt{x+3}\,dx = \int_4^9 \sqrt{u}\,du = \frac{2}{3}u^{\frac{3}{2}}\Big|_4^9$$ 4 2

$$= \frac{2}{3}\left((\sqrt{9})^3 - (\sqrt{4})^3\right) = \frac{2}{3}(27-8) = \frac{2}{3}(19) = \frac{38}{3}$$

2 2

4. Evaluate $\int \frac{dx}{\sqrt{9-x^2}}$. $= \int \frac{dx}{\sqrt{9\left(1-\left(\frac{x}{3}\right)^2\right)}} = \frac{1}{3}\int \frac{dx}{\sqrt{1-\left(\frac{x}{3}\right)^2}}$ (4)

let $u=\frac{x}{3}$; then $du=\frac{1}{3}dx$, and

$$\frac{1}{3}\int\frac{dx}{\sqrt{1-\left(\frac{x}{3}\right)^2}} = \int \frac{du}{\sqrt{1-u^2}} = \sin^{-1}(u)+C = \sin^{-1}\left(\frac{x}{3}\right)+C$$

(2) (2)

5. Evaluate $\int x\cos(x)\,dx$.

let $u=x$ $\quad v'=\cos x$ (4)

so $u'=1$ $\quad v=\sin(x)$

$$\int x\cos(x)\,dx = \int uv'dx = uv-\int u'v\,dx = x\sin(x)-\int \sin(x)dx = x\sin(x)+\cos(x)+C$$

(4)

6. Evaluate $\int_0^1 x^2e^x\,dx$.

let $u=x^2$, $v'=e^x$ so $u'=2x$, $v=e^x$ (2)

$\int x^2e^x dx = x^2e^x-2\int xe^x dx$ (4)

For $\int xe^x dx$, let $u=x$, $v'=e^x$, so $u'=1$, $v=e^x$ (2)

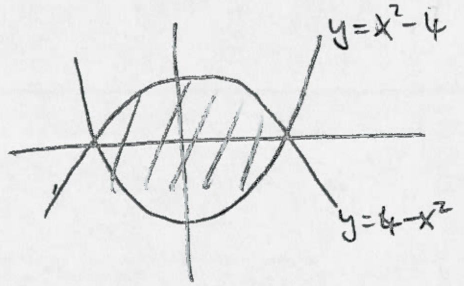
Then $\int xe^x dx = xe^x-\int e^x dx = xe^x-e^x+C$ (4)

So $\int x^2e^x dx = x^2e^x-2xe^x+2e^x+C = (x^2-2x+2)e^x+C$ (1)

The

$\int_0^1 x^2e^x dx = (1-2+2)e^1 - 2e^0 = e-2$ (2)

7. (a) Plot the curves $y = 4 - x^2$ and $y = x^2 - 4$ and determine their points of intersection.




intersect at $(-2,0)$ and $(2,0)$

3

(b) Determine the area between the curves $y = 4 - x^2$ and $y = x^2 - 4$.

$$\text{Area} = \int_{-2}^{2} y_{top} - y_{bot}\,dx = \int_{-2}^{2} (4-x^2)-(x^2-4)\,dx = \int_{-2}^{2} 8-2x^2\,dx$$

4

$$= \left[8x - \tfrac{2}{3}x^3\right]_{-2}^{2} = \left(16 - \tfrac{16}{3}\right) - \left((-16) - \left(-\tfrac{16}{3}\right)\right) = \left(\tfrac{48}{3} - \tfrac{16}{3}\right) - \left(-\tfrac{48}{3} + \tfrac{16}{3}\right) = \tfrac{64}{3}$$

4 2

8. Use the reduction formula

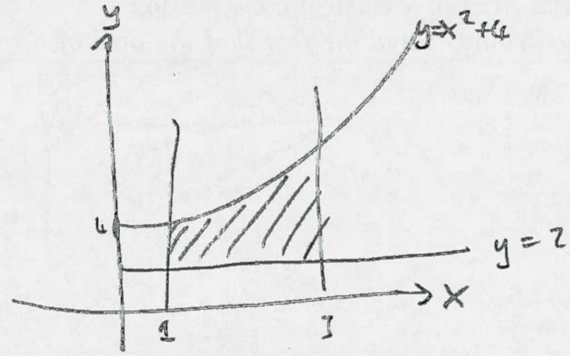
$$\int \sin^n(x) = \frac{-1}{n}\sin^{n-1}(x)\cos(x) + \frac{n-1}{n}\int \sin^{n-2}(x)\,dx$$

to calculate $\int \sin^4(x)\,dx$. $= -\frac{1}{4}\sin^3(x)\cos(x) + \frac{3}{4}\int \sin^2(x)\,dx$ 3

$$= -\tfrac{1}{4}\sin^3(x)\cos(x) + \tfrac{3}{4}\left(-\tfrac{1}{2}\sin(x)\cos(x) + \tfrac{1}{2}\int dx\right)$$ 3

$$= -\tfrac{1}{4}\sin^3(x)\cos(x) - \tfrac{3}{8}\sin(x)\cos(x) + \tfrac{3}{8}x + C$$ 2

9. (a) Sketch the region in the $xy$-plane enclose by the curves $y = x^2 + 4$, $y = 2$, $x = 1$, and $x = 3$.




4

(b) Find the volume of the solid obtained by rotation this region about the $x$-axis.

$$\text{Vol} = \pi \int_1^3 (x^2+4)^2 - 2^2 \, dx \qquad 4$$

$$= \pi \int_1^3 x^4 + 8x^2 + 12 \, dx$$

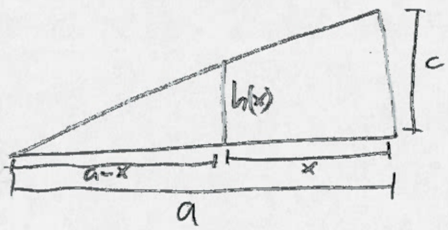
$$= \pi \left[ \tfrac{1}{5}x^5 + \tfrac{8}{3}x^3 + 12x \right]_1^3 \qquad 4$$

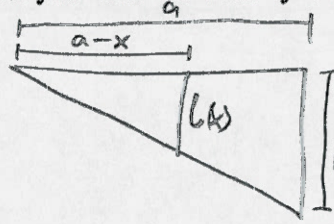
$$= \pi \left( \left( \frac{3^5}{5} + \frac{8 \cdot 3^3}{3} + 12 \cdot 3 \right) - \left( \tfrac{1}{5} + \tfrac{8}{3} + 12 \right) \right) \qquad 2$$

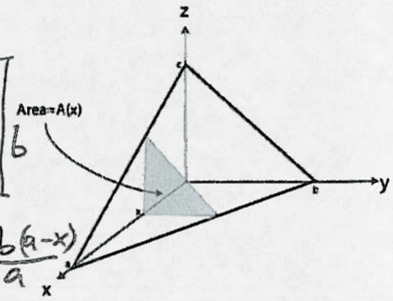
$$= \frac{2126}{15} \pi \qquad 1$$

10. Consider the wedge determined by the coordinates $(0,0,0)$, $(a,0,0)$, $(0,b,0)$, $(0,0,c)$.

(a) Determine the area $A(x)$ of a vertical cross section.
*(Hint: use similar triangles and the fact that the area of a triangle is $\frac{1}{2}$base$\times$height.)*




$$\frac{c}{a} = \frac{h(x)}{a-x} \Rightarrow h(x) = \frac{c(a-x)}{a} \qquad \frac{b}{a} = \frac{l(x)}{a-x} \Rightarrow l(x) = \frac{b(a-x)}{a}$$

$$A(x) = \frac{1}{2} l(x)\, h(x) = \frac{1}{2}\frac{cb(a-x)^2}{a^2} = \frac{cb}{2a^2}(a-x)^2$$

(4)

(b) Determine the volume of the solid.

$$\text{Vol} = \int_0^a A(x)\,dx = \frac{cb}{2a^2}\int_0^a (a-x)^2\,dx = \frac{cb}{2a^2}\int_0^a a^2-2ax+x^2\,dx$$

(2)

$$= \frac{cb}{2a^2}\left[a^2x - ax^2 + \frac{x^3}{3}\right]_0^a = \frac{cb}{2a^2}\left(a^3 - a^3 + \frac{a^3}{3}\right) = \frac{abc}{6}$$

(2) (2)