let $u = x^2$

so $du = 2x\,dx$

Then

$$\int x^3 \cos(x^2)\,dx = \frac{1}{2}\int u \cos(u)\,du$$

Now use integration by parts

Set

$w = u \qquad v' = \cos(u)$

$w' = 1 \qquad v = \sin(u)$

Then

$$\frac{1}{2}\int u\cos(u)\,du = \frac{1}{2}\left(u\sin(u) - \int \sin(u)\,du\right)$$

$$= \frac{1}{2}u\sin(u) + \frac{1}{2}\cos(u) + C$$

$$= \frac{1}{2}x^2\sin(x^2) + \frac{1}{2}\cos(x^2) + C$$

a) Use partial fractions

$$\frac{1}{(x+2)(2x+3)} = \frac{A}{x+2} + \frac{B}{2x+3}$$

$\Rightarrow \quad 1 = A(2x+3) + B(x+2)$

$\Rightarrow \quad 1 = (2A+B)x + (3A+2B)$

$\Rightarrow \quad B = -2A \quad \text{and} \quad 3A+2B = 1$

$\Rightarrow \quad A = -1 \quad \text{and} \quad B = 2$

Then we have

$$\int \frac{dx}{(x+2)(2x+3)} = 2\int \frac{dx}{2x+3} - \int \frac{dx}{x+2} = \ln|2x+3| - \ln|x+2| + C$$

$$= \ln\left|\frac{2x+3}{x+2}\right| + C$$

b)

$$\int_1^\infty \frac{dx}{(x+2)(2x+3)} = \lim_{R\to\infty} \int_1^R \frac{dx}{(x+2)(2x+3)} = \lim_{R\to\infty}\left(\ln\left|\frac{2x+3}{x+2}\right| - \ln\left(\frac{5}{3}\right)\right)$$

$$= \ln\left|\lim_{R\to\infty} \frac{2x+3}{x+2}\right| - \ln\left(\frac{5}{3}\right) = \ln(2) - \ln\left(\frac{5}{3}\right) = \ln\left(\frac{6}{5}\right)$$

We have $\cos^2(\theta) = 1 - \sin^2(\theta)$

Thus

$$\int \sin^5(\theta)\cos^3(\theta)\, d\theta = \int \sin^5(\theta)\left(1-\sin^2(\theta)\right)\cos\theta \, d\theta \qquad (*)$$

Now use substitution,

let $u = \sin(\theta)$

so $du = \cos(\theta)\, d\theta$

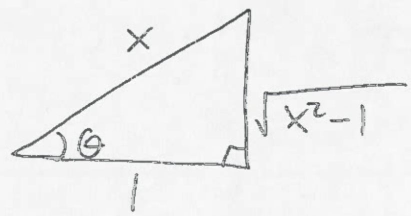
so

$$(*) = \int u^5(1-u^2)\, du = \int u^5 - u^7 \, du$$

$$= \frac{1}{6}u^6 - \frac{1}{8}u^8 + C$$

$$= \frac{1}{6}\sin^6(\theta) - \frac{1}{8}\sin^8(\theta) + C$$

Use trig. subrtitution



let

$$x = \sec(\theta)$$

so $dx = \sec(\theta)\tan(\theta)\,d\theta$

Then

$$\int \frac{dx}{x(x^2-1)^{3/2}} = \int \frac{\sec(\theta)\tan(\theta)}{\sec(\theta)\left(\sec^2(\theta)-1\right)^{3/2}}\,d\theta$$

$$= \int \frac{\sec(\theta)\tan(\theta)}{\sec(\theta)\tan^3(\theta)}\,d\theta = \int \cot^2(\theta)\,d\theta$$

$$= -\cot(\theta) - \theta + C$$

we have

$$\theta = \sec^{-1}(x)$$

and

$$\cot(\theta) = \frac{1}{\sqrt{x^2-1}}$$

so

$$\int \frac{dx}{x(x^2-1)^{3/2}} = -\sec^{-1}(x) - \frac{1}{\sqrt{x^2-1}} + C$$