1. Evaluate $\int_0^{\sqrt{\ln(2)}} xe^{x^2}\, dx$.

let $u = x^2$

$$\frac{du}{dx} = 2x \Rightarrow dx = \frac{1}{2x} du$$

$u(0) = 0 \qquad u(\sqrt{\ln(2)}) = \ln(2)$

So

$$\int_0^{\sqrt{\ln(2)}} x e^{x^2} dx = \frac{1}{2}\int_0^{\ln(2)} e^u du = \frac{1}{2}\left[e^u\right]_0^{\ln(2)}$$

$$= \frac{1}{2}(2-1) = \frac{1}{2}$$

2. Evaluate $\int 64x^2\sqrt{4x-1}\,dx$.

let $u = 4x - 1 \Rightarrow x = \frac{1}{4}(u+1)$

$\frac{du}{dx} = 4 \Rightarrow dx = \frac{1}{4}du$

$$\int 64x^2\sqrt{4x-1}\,dx = \int \frac{64}{64}(u+1)^2\sqrt{u}\,du$$

$$= \int (u^2+2u+1)u^{\frac{1}{2}}\,du$$

$$= \int u^{\frac{5}{2}} + 2u^{\frac{3}{2}} + u^{\frac{1}{2}}\,du$$

$$= \frac{5}{7}u^{7/5} + \frac{4}{5}u^{\frac{5}{2}} + \frac{2}{3}u^{\frac{3}{2}} + C$$

$$= \frac{5}{7}(4x-1)^{7/5} + \frac{4}{5}(4x-1)^{\frac{5}{2}} + \frac{2}{3}(4x+1)^{\frac{3}{2}} + C$$

3. Evaluate $\int \frac{dt}{\sqrt{16-t^2}}$.

$$\sqrt{16-t^2} = 4\sqrt{1-\left(\tfrac{t}{4}\right)^2}$$

let $u = \frac{t}{4}$

$$\frac{du}{dt} = \frac{1}{4} \Rightarrow dt = 4du$$

so

$$\int \frac{dt}{\sqrt{16-t^2}} = \frac{1}{4}\int \frac{dt}{\sqrt{1-\left(\frac{t}{4}\right)^2}} = \int \frac{du}{\sqrt{1-u^2}}$$

$$= \sin^{-1}(u) + C$$

$$= \sin^{-1}\left(\frac{t}{4}\right) + C$$

4. Evaluate $\int e^{\sqrt{x}}\,dx$.

let $u = \sqrt{x} = x^{\frac{1}{2}}$

$$\frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}} \Rightarrow dx = 2\sqrt{x}\,du = 2u\,du$$

So

$$\int e^{\sqrt{x}}\,dx = \int 2ue^{u}\,du$$

Now integrate by parts using $\int vw'\,du = vw - \int v'w\,du$

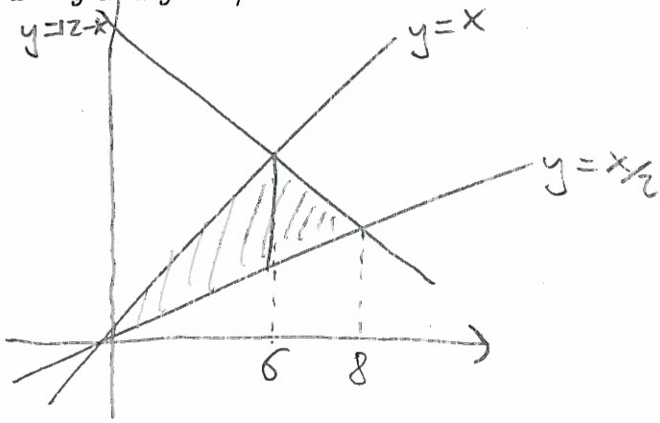
$v = 2u \qquad w' = e^{u}$

$v' = 2 \qquad w = e^{u}$

so
$$\begin{aligned}\int 2ue^{u}\,du &= 2ue^{u} - \int 2e^{u}\,du \\ &= 2ue^{u} - 2e^{u} + C \\ &= 2\sqrt{x}\,e^{\sqrt{x}} - 2e^{\sqrt{x}} + C \\ &= 2e^{\sqrt{x}}(\sqrt{x} - 1) + C\end{aligned}$$

5. (a) Plot the curves $y = 12 - x$ and $x = y$ and $y = x/2$ on the same xy-axis.

(b) Using integration, determine the area bounded by the curves $y = 12 - x$ and $x = y$ and $y = x/2$.



$x = 12 - x \Rightarrow x = 6$

$\frac{x}{2} = 12 - x \Rightarrow \frac{3}{2}x = 12 \Rightarrow x = \frac{24}{3} = 8$

$$\text{Area} = \int_0^6 x - \frac{x}{2}\,dx + \int_6^8 (12-x) - \frac{x}{2}\,dx$$

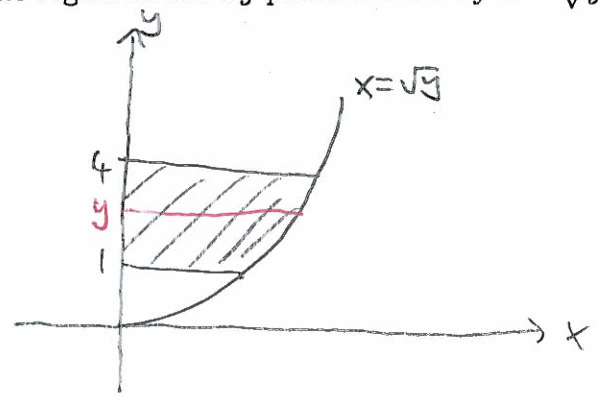
$$= \int_0^6 \frac{x}{2}\,dx + \int_6^8 \left(12 - \frac{3x}{2}\right)dx$$

$$= \left[\frac{x^2}{4}\right]_0^6 + \left[12x - \frac{3x^2}{4}\right]_6^8$$

$$= 9 + ((96 - 48) - (72 - 27))$$

$$= 12$$

7. (a) Sketch the region in the xy-plane enclose by $x = \sqrt{y}$, $x = 0$, $y = 1$, and $y = 4$. [5]



(b) Find an integral that determines volume of the solid obtained by rotation this region about the y-axis. ~~(You do not need to evaluate this integral).~~ [10]

$$A(y) = \pi(\sqrt{y})^2$$

$$\text{Vol} = \pi\int_1^4 (\sqrt{y})^2\,dy = \pi\int_1^4 y\,dy = \frac{\pi y^2}{2}\Big|_1^4$$

$$= \frac{16\pi}{2} - \frac{\pi}{2} = \frac{15\pi}{2}$$