Name:______________________________

Signature:______________________________

J. Number:______________________________

MA 125 Calculus 1
Exam 2

1. Attempt all of the questions.
2. Write your name and J. number at the top of this page.
3. Answer the questions in the spaces provided.
4. Show all your work required to obtain your answers.
5. **Simplify your answers when it is possible.**
6. No graphing calculators are allowed.
7. You may use your table of derivatives.

Question	Mark
1	/
2	/
3	/
4	/
5	/
6	/
7	/
total	/

1. If $f(x) = (x^2+9)(2-e^x)$, calculate $f'(x)$. Simplify your answer by collecting together the e^x terms $\square$ 5

$$f'(x) = 2x(2-e^x) + (x^2+9)(-e^x)$$

$$= 4x - 2xe^x - x^2e^x - 9e^x$$

$$= 4x - (x^2+2x-9)e^x$$

2. If $g(x) = \frac{x}{x+1}$, calculate $g''(1)$. Simplify your answer $\square$ 10

$$g'(x) = \frac{(x+1)-x}{(x+1)^2} = \frac{1}{(x+1)^2}$$ 5

$$g''(x) = \frac{d}{dx}(x+1)^{-2} = \frac{-2}{(x+1)^3}$$ 5

$$g''(1) = \frac{-2}{8} = -\frac{1}{4}$$

3. If $y = \sqrt{\sin(x)\cos(x)}$, calculate $\frac{dy}{dx}$. ▯ 10

let $u = \sin(x)\cos(x)$

$y = \sqrt{u}$

$$\frac{du}{dx} = \cos^2(x) - \sin^2(x)$$

$$\frac{dy}{du} = \frac{1}{2}u^{-\frac{1}{2}} = \frac{1}{2\sqrt{u}}$$

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = \frac{\cos^2(x) - \sin^2(x)}{2\sqrt{\sin(x)\cos(x)}}$$

(d) If $f(x) = 5^{x^2-2x+9}$, calculate $\frac{df}{dx}\Big|_{x=1}$. ▯ 5

let $u = x^2 - 2x + 9$

$f = 5^u$

$$\frac{df}{dx} = \frac{df}{du}\frac{du}{dx} = \ln(5)5^u(2x-2)$$

$$= 2\ln(5)(x-1)\,5^{x^2-2x+9}$$

$$\cancel{\frac{df}{dx}\Big|_{x=1} = 0}$$

calculate $f'\left(\frac{\pi}{4}\right)$

4. Let $f(x) = \csc(x) - \cot(x)$. ~~Find the equation of the tangent line to $f(x)$ at $x = \pi/4$.~~ []

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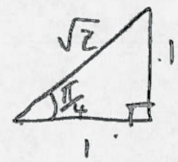
$$f'(x) = -\csc(x)\cot(x) + \csc^2(x)$$ 4

$$\csc\left(\frac{\pi}{4}\right) = \frac{1}{\sin\left(\frac{\pi}{4}\right)} = \sqrt{2}$$



$$\cot\left(\frac{\pi}{4}\right) = \frac{1}{\tan\left(\frac{\pi}{4}\right)} = 1$$

$$f'\left(\frac{\pi}{4}\right) = -\sqrt{2} + 2 = 2 - \sqrt{2}$$ 3

5. A slingshot is used to shoot a pebble into the air vertically from ground level with an initial velocity of $20\,m/s$. Find the pebbles maximum height. (You may ignore air resistance and assume Galileo's equation $s(t) = -\frac{1}{2}gt^2 + v_0t + s_0$ holds.)

$$S(t) = -\frac{1}{2}gt^2 + 20t + 0$$

max height when $V(t) = 0$

$$V(t) = S'(t) = -gt + 20$$

$$S'(t) = 0 \iff -gt + 20 = 0 \iff t = \frac{20}{g}$$

So max height at time $t = \frac{20}{g}$

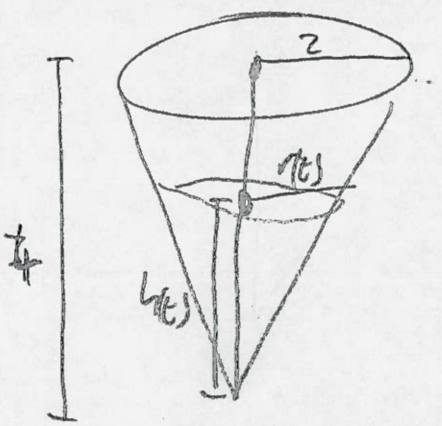
So max height is

$$S\left(\frac{20}{g}\right) = -\frac{1}{2}g\left(\frac{20}{g}\right)^2 + 20\left(\frac{20}{g}\right)$$

$$= -\frac{200}{g} + \frac{400}{g}$$

$$= \frac{200}{g}$$

6. A conical tank has height $4m$ and radius $2m$ at the top. Water flows in at a rate of $3m^3/min$. How fast is the water level rising when it is $2m$. (Recall that the volume of a cone is given by $V = \frac{1}{3}\pi r^2 h$.) □



$V(t)$ = vol of water at time t.

$h(t)$ = water level at time t.

$r(t)$ = radius of top of water at time t

$$\frac{dV}{dt} = 3$$

Want to find $\left.\frac{dh}{dt}\right|_{h(t)=2}$

$$\frac{h(t)}{r(t)} = \frac{4}{2} \Rightarrow r(t) = \frac{1}{2}h(t)$$

$$V(t) = \frac{\pi}{3}(r(t))^2 h(t) = \frac{\pi}{3}\left(\frac{1}{2}h(t)\right)^2 h(t) = \frac{\pi}{12} h(t)^3$$

$$\frac{dV}{dt} = \frac{\pi}{12} 3 (h(t))^2 \frac{dh}{dt} = \frac{\pi}{4}(h(t))^2 \frac{dh}{dt}$$

evaluating at $h(t)=2$

$$3 = \frac{\pi}{4} 4 \left.\frac{dh}{dt}\right|_{h(t)=2} \Rightarrow \left.\frac{dh}{dt}\right|_{h(t)=2} = \frac{3}{\pi}\ m/s$$

Let $f(x) = e^x \cos(x)$. Find the equation of the tangent line to $f(x)$ at $x=0$

The tangent line is given by

$$y = mx + b$$

$$m = f'(0)$$

$$f'(x) = e^x \cos(x) - e^x \sin(x)$$

so

$$m = f'(0) = 1$$

So $y = x + b$

The line passes through $(0, f(0)) = (0, 1)$

So

$$1 = 0 + b$$

So $b = 1$

So the line is

$$y = x + 1$$