Name:______________________

Signature:______________________
J. Number:______________________

MA 125 Calculus 1
Exam 1

1. Attempt all 7 questions.
2. Write your name and J. number at the top of this page.
3. Answer the questions in the spaces provided.
4. Show all your work required to obtain your answers.
5. **No calculators are allowed.**
6. This is a closed book test.

Question	Mark
1	/
2	/
3	/
4	/
5	/
6	/
7	/
total	/

Some standard limits:

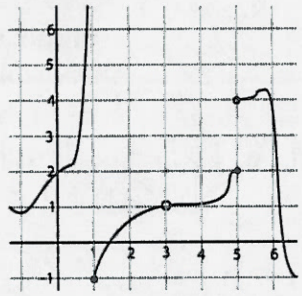
$$\lim_{x\to c} x = c, \qquad \lim_{x\to c} k = k, \qquad \lim_{x\to 0}\frac{\sin(x)}{x} = 1, \qquad \lim_{x\to\infty} k = k, \qquad \lim_{x\to 0}\frac{1}{x} = \infty, \qquad \lim_{x\to\infty}\frac{1}{x} = 0,$$

$$\lim_{x\to 0}\frac{\sin(x)}{x} = 1, \qquad \lim_{x\to 0}\frac{1-\cos(x)}{x} = 0,$$

Some continuous functions:

$$x, \quad ,k, \quad e^x, \quad \sin(x), \quad ,\cos(x), \quad \sqrt[n]{x}$$

1. The graph of a function $f(x)$ is shown in below.



write down the values of the following limits or state that they do not exist. [] 6

$\lim_{x\to 3} f(x) = 1$, $\lim_{x\to 1+} f(x) = -1$, $\lim_{x\to 5} f(x) =$ DNE

2 2 2

2. Define what it means for a function $f(x)$ to be continuous at a point $x = c$. [] 6

Defined on a interval containing c 2

$\lim_{x\to c} f(x) = f(c)$ 4

3. For each of the following functions, state whether or not it is continuous. If you claim the function is discontinuous, write down all of its points of discontinuity. **(You do not need to justify your answer in this question.)** [8]

(a) $f(x) = \frac{\sin(x)}{1+4^x}$ []

Cts

4

(b) $f(x) = \frac{1-2x}{x^2-x-6}$ []

Not cts

discontinuity at $x = -2, 3$ 4

4. Using any of the results and methods (except for numerical estimates) you have seen in the course, determine the following limits. **Justify your answer.**

(a) $\lim_{x\to 2} \frac{x^3-4x}{x-2} = \lim_{x\to 2} \frac{x(x-2)(x+2)}{x-2} = \lim_{x\to 2} x(x+2) = 8$ $\square$ 6

(b) $\lim_{x\to 0} \frac{\sin(x)\cos(x)}{x} = \left(\lim_{x\to 0} \frac{\sin(x)}{x}\right)\left(\lim_{x\to 0} \cos(x)\right) = 1$ $\square$ 6

(c) $\lim_{x\to 0} \frac{\sin(7x)}{3x}$ $\square$ 6

Let $u = 7x \Rightarrow 3x = 3u/7$

$\lim_{x\to 0} \frac{\sin(7x)}{(3x)} = \lim_{u\to 0} \frac{\sin(u)}{3u/7} = \frac{7}{3}\left(\lim_{u\to 0} \frac{\sin(u)}{u}\right) = \frac{7}{3}$

(d) $\lim_{x\to\infty} \frac{2x}{3x^2} = \frac{2}{3} \lim_{x\to\infty} \frac{1}{x} = 0$ ▯ 6

(e) $\lim_{x\to 0} \frac{e^x - e^{2x}}{1-e^x} = \lim_{x\to 0} \frac{e^x(1-e^x)}{1-e^x} = \lim_{x\to 0} e^x = 1$ ▯ 6

(f) $\lim_{x\to 3} \frac{\sqrt{x+1}-2}{x-3} \cdot \frac{\sqrt{x+1}+2}{\sqrt{x+1}+2} = \lim_{x\to 3} \frac{x+1-4}{x-3(\sqrt{x+1}+2)}$ ▯ 6

$= \lim_{x\to 3} \frac{1}{\sqrt{x+1}+2} = \frac{1}{4}$

5. Use the definition of a derivative to find $f'(2)$ where $f(x) = \frac{3}{x}$. **Justify your answer.**

(Recall that $f'(a) = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$.*)* ▯ 10

$f(2) = \frac{3}{2}$

$f(2+h) = \frac{3}{2+h}$

$$f'(2) = \lim_{h \to 0} \frac{\frac{3}{2+h} - \frac{3}{2}}{h} = \lim_{h \to 0} \frac{6 - 6 - 3h}{2h(2+h)} = \lim_{h \to 0} \frac{-3}{2(2+h)} = \frac{-3}{4}$$

(5) (5)

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6. Use the squeeze theorem to evaluate $\lim_{x \to 0_+} \sqrt{x} e^{\cos(\pi/x)}$. **Justify your answer.** ▯ 8

$-1 \leq \cos\left(\frac{\pi}{x}\right) \leq 1$ (1)

e^x is an increasing function so (1)

$e^{-1} \leq e^{\cos\left(\frac{\pi}{x}\right)} \leq e$ (1)

$\sqrt{x} \geq 0$ so (1)

$\frac{\sqrt{x}}{e} \leq \sqrt{x} e^{\cos\left(\frac{\pi}{x}\right)} \leq \sqrt{x} e$ (1)

So

$0 = \lim_{x \to 0_+} \frac{\sqrt{x}}{e} \leq \lim_{x \to 0_+} \sqrt{x} e^{\cos\left(\frac{\pi}{x}\right)} \leq \lim_{x \to 0_+} \sqrt{x} e = 0$ (1)

So by squeeze Thm (1)

$\lim_{x \to 0_+} \sqrt{x} e^{\cos \frac{\pi}{x}} = 0$ (1)