1 A quick trigonometry review

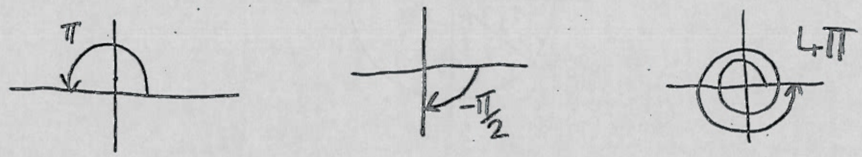
READING	Read Section 1.4 of Rogawski.

Reading

1.1 Angles

- Angles are measure in **radians**.
- Rotating an object by 2π radians will rotate by one complete revolution in the *anti-clockwise* direction.
- π radians $= 180°$

Example 1.



Example 2. Convert 60° into radians.

Solution.

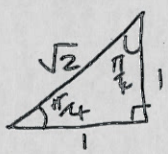
π radians $= 180°$.
So $1° = \frac{\pi}{180}$ radians.
Then $60° = \frac{60\pi}{180}$ radians $= \frac{\pi}{3}$ radians.

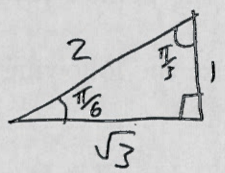
HOMEWORK	Rowgowski Section 1.4: Q 3 & 4.

Homewor

1.2 Two triangles to remember

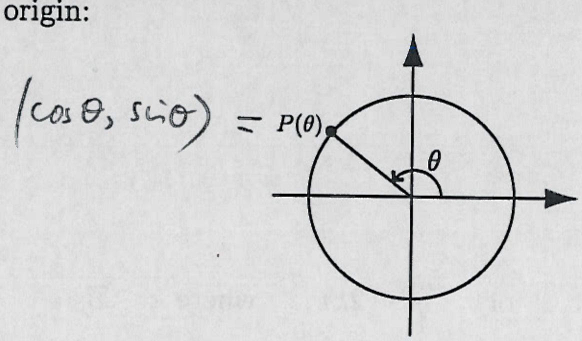
Memorize





1.3 Sine and cosine

Every angle θ defines a point $P(\theta)$ on the unit circle by rotating the point $(0, 1)$ by an angle of θ about the origin:



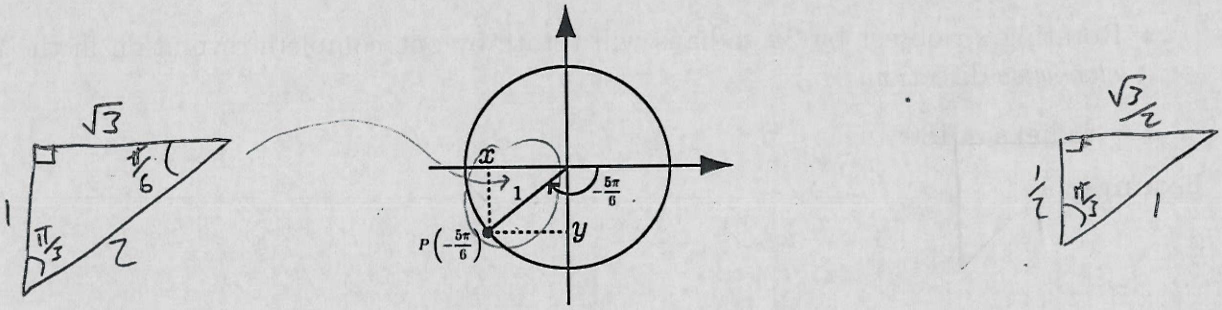
By definition, Memorize

$$P(\theta) = (\cos(\theta), \sin(\theta)).$$

Thus, $\cos(\theta)$ is the x-coordinate of $P(\theta)$; and $\sin(\theta)$ is the y-coordinate of $P(\theta)$.

Example 3. Find the exact values of $\cos\left(-\frac{5\pi}{6}\right)$ and $\sin\left(\frac{7\pi}{6}\right)$.

Solution. We begin by finding the coordinates of $P\left(-\frac{5\pi}{6}\right)$. Consider the following figure:



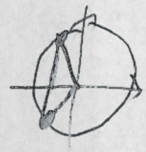
Using our standard triangles, we find that

$$P\left(-\frac{5\pi}{6}\right) = \left(\cos\left(-\frac{5\pi}{6}\right), \sin\left(-\frac{5\pi}{6}\right)\right) = \left(\frac{-\sqrt{3}}{2}, \frac{-1}{2}\right)$$

Finally, since $P\left(-\frac{5\pi}{6}\right) = P\left(\frac{7\pi}{6}\right)$, we have

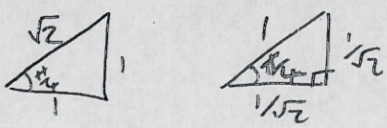
$$\sin\left(\frac{7\pi}{6}\right) = \sin\left(-\frac{5\pi}{6}\right) = -\frac{1}{2}$$

Example 4. Solve the equation $\cos(\theta) = \frac{-\sqrt{2}}{2}$.

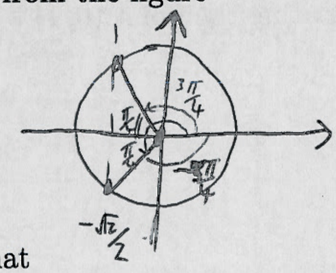


Solution. $\cos(\theta)$ is the x-coordinate of the point $P(\theta)$. So we need to find all of the values θ such that $P(\theta) = (-\sqrt{2}/2, y)$.

The appearance of $\sqrt{2}$ suggests we use the following standard triangle:



Then from the figure



we see that

$$\theta = \frac{3\pi}{4} + 2k\pi, \quad \text{or} \quad \frac{5\pi}{4} + 2k\pi, \quad \text{where } k \in \mathbb{Z}.$$

The following identity is very useful.

Theorem 1.

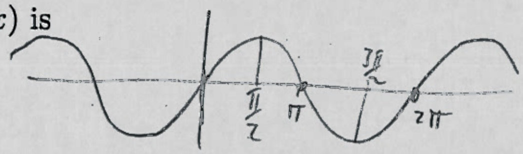
$$\cos^2(\theta) + \sin^2(\theta) = 1$$

HOMEWORK	Rowgowski Section 1.4: Q 7, 9, 13 & 30.

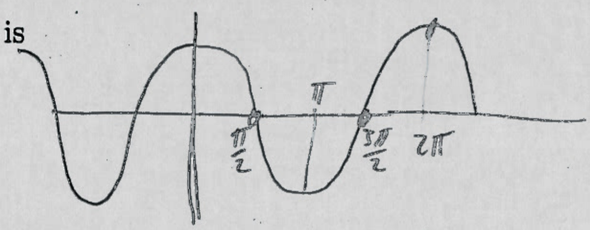
Homewor

1.4 The graphs of sine and cosine

The graph of $y = \sin(x)$ is

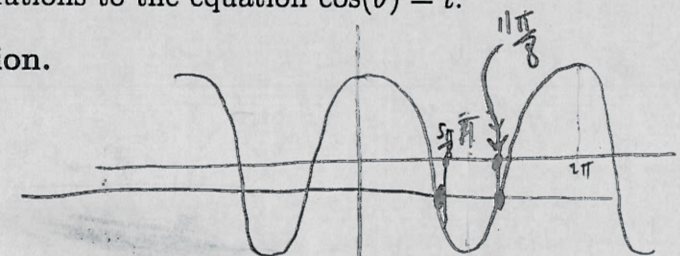


The graph of $y = \cos(x)$ is

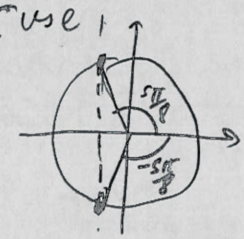


Example 5. Let $t := \cos(5\pi/8)$. Use the graph of the cosine function to write down all of the solutions to the equation $\cos(\theta) = t$.

Solution.



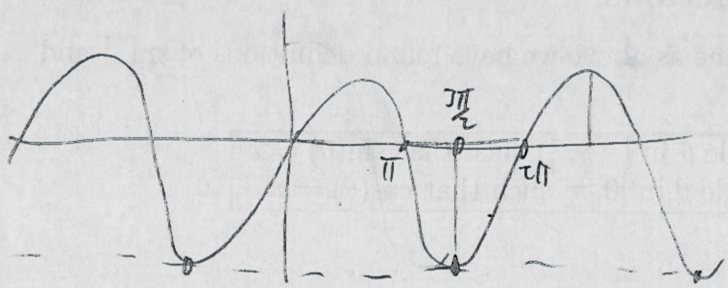
or use



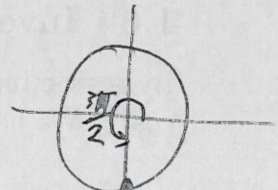
$\frac{5\pi}{8} + 2k\pi$ or $\frac{11\pi}{8} + 2k\pi$

Example 6. Use the graph of sine to write down all of the solutions to $\sin(x) + 1 = 0$, where $2\pi \leq x \leq 4\pi$.

Solution.



or use



$\frac{3\pi}{2} + 2k\pi$

$\frac{3\pi}{2} + 2k\pi$

1.5 Other trigonometric functions

Recall

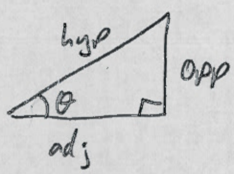
Memorize

$$\tan(\theta) := \frac{\sin(\theta)}{\cos(\theta)}, \qquad \cot(\theta) := \frac{1}{\tan(\theta)}, \qquad \sec(\theta) := \frac{1}{\cos(\theta)}, \qquad \csc(\theta) := \frac{1}{\sin(\theta)}$$

Example 7. If $\cot(\theta) = 4$ and $0 \leq \theta \leq \frac{\pi}{2}$, find $\sin(\theta)$.

Solution.

Since $0 \leq \theta \leq \pi/2$ we can use right-angled triangles



$$\sin(\theta) = \frac{o}{h}$$

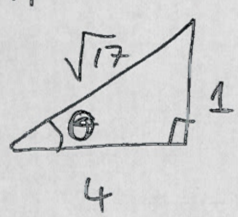
$$\cos(\theta) = \frac{a}{h}$$

$$\tan(\theta) = \frac{o}{a}$$

so $\cot(\theta) = \dfrac{\text{adj}}{\text{opp}}$

Then $\dfrac{\text{adj}}{\text{opp}} = 4$ so take, eg, adj $= 4$ opp $= 1$ hyp $= \sqrt{4^2+1^2} = \sqrt{17}$

giving



so $\sin(\theta) = \dfrac{1}{\sqrt{17}}$

HOMEWORK	Rowgowski Section 1.4: Q 19, 21, 24, 25 & 27

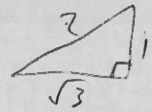
Homework

1.6 Inverse trigonometric functions

By restricting the domain of sine and cosine as above we have found definitions of $\sin^{-1}$ and $\cos^{-1}$. We have:

Memorize

$\sin^{-1}(x)$ is the unique angle θ in $[-\frac{\pi}{2}, \frac{\pi}{2}]$ such that $\sin(\theta) = x$
$\cos^{-1}(x)$ is the unique angle θ in $[0, \pi]$ such that $\cos(\theta) = x$



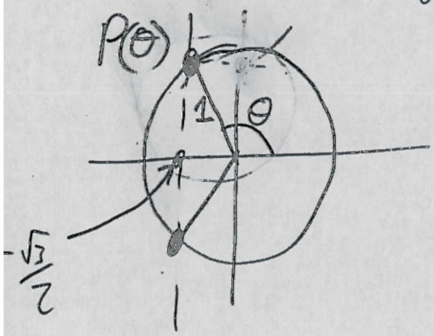
We can similarly define inverses of all of the other trig. functions. These definitions can be found in Section 1.5 of the textbook.

Example 8. Calculate $\cos^{-1}(-\sqrt{3}/2)$.

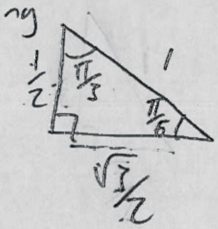
Solution.

$\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ is the unique angle θ in $[0, \pi]$ with $\cos(\theta) = \frac{-\sqrt{3}}{2}$

$\cos(\theta)$ is the x-coord of $P(\theta)$ so we have



Using

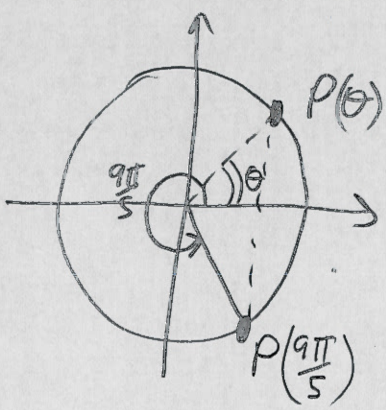


we see $\theta = \frac{5\pi}{6}$

Example 9. Calculate $\cos^{-1}(\cos(9\pi/5))$.

Solution. $\cos^{-1}\left(\cos\left(\frac{9\pi}{5}\right)\right)$ is the unique angle θ in $[0, \pi]$ with $\cos(\theta) = \cos\left(\frac{9\pi}{5}\right)$

$\cos(\theta)$ is the x-coord of $P(\theta) = P\left(\frac{9\pi}{5}\right)$



Using symmetry we see $\theta = \frac{\pi}{5}$

So $\cos^{-1}\left(\cos\left(\frac{9\pi}{5}\right)\right) = \frac{\pi}{5}$

HOMEWORK	Rowgowski Section 1.5: Q 23, 25, 27, 29, 31, 33, 35, 37

Homeworl