Name:____________________________

Signature:____________________________

J. Number:____________________________

MA 125 Calculus 1
Exam 1

1. Attempt all 6 questions.
2. Write your name and J. number at the top of this page.
3. Answer the questions in the spaces provided.
4. Show all your work required to obtain your answers.
5. **No calculators are allowed.**
6. This is a closed book test.

Question	Mark
1	/
2	/
3	/
4	/
5	/
6	/
total	/

Some standard limits:

$$\lim_{x\to c} x = c, \qquad \lim_{x\to c} k = k, \qquad \lim_{x\to 0} \frac{\sin(x)}{x} = 1, \qquad \lim_{x\to\infty} k = k, \qquad \lim_{x\to\infty} \frac{1}{x} = 0,$$

Some continuous functions:

$$x, \quad ,k, \quad e^x, \quad \sin(x), \quad ,\cos(x), \quad \sqrt[n]{x}$$

For finding derivatives:

$(f+g)' = f' + g'$, $(cf)' = cf'$, $(fg)' = f'g + g'f$, $\left(\frac{f}{g}\right)' = \frac{f'g - g'f}{g^2}$

$\frac{d}{dx}x^n = nx^{n-1}$, $\frac{d}{dx}\sin(x) = \cos(x)$, $\frac{d}{dx}\cos(x) = -\sin(x)$, $\frac{d}{dx}e^x = e^x$

1. Find all of the solutions in $[0, 2\pi]$ to the equation

$$\sin(x)\cos(x) + \sin(x) = 0.$$

(Hint: start by factorizing.) ▯ 6

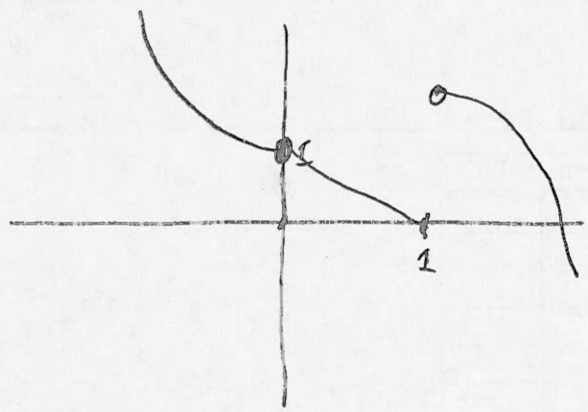
$\sin(x)\cos(x) + \sin(x) = 0 \iff \sin(x)(\cos(x)+1) = 0$

$\iff \sin(x) = 0$ or $\cos(x) = -1$ ③ ③

$\iff x = 0, \pi, 2\pi$

2. Let $f(x) = \begin{cases} x^2+1 & \text{if } x \le 0 \\ 1-x & \text{if } 0 > x \ge 1 \\ 4-x^3 \text{ if } x > 1 & \end{cases}$

(a) Plot the graph of $f(x)$. ▯ 4



(b) Write down the points of discontinuity. ▯ 2

$x = 1$

(c) State whether $f(x)$ is right-continuous or left-continuous at these points. ▯ 2

left-cts at $x=1$

3. Using any of the results and methods (except for numerical estimates) you have seen in the course, determine the following limits. **Justify your answer.**

(a) $\lim_{x\to 3} \frac{1-x}{1+x}$. $= \frac{1-3}{1+3} = \frac{-2}{4} = \frac{-1}{2}$ ▯ 4

(b) $\lim_{x\to 1} \frac{x^3-x}{x-1}$. $= \lim_{x\to 1} \frac{x(x^2-1)}{x-1} = \lim_{x\to 1} \frac{x(x-1)(x+1)}{x-1}$ ▯ 6

$= \lim_{x\to 1} x(x+1) = 2$

(c) $\lim_{x\to 1} \left(\frac{1}{x-1} - \frac{2}{x^2-1}\right)$. $= \lim_{x\to 1} \left(\frac{1}{x-1} - \frac{2}{(x-1)(x+1)}\right) = \lim_{x\to 1} \frac{x+1-2}{(x-1)(x+1)}$ ▯ 6

$= \lim_{x\to 1} \frac{x-1}{(x-1)(x+1)} = \lim_{x\to 1} \frac{1}{x+1} = \frac{1}{2}$

(d) $\lim_{x\to 3} \frac{\sqrt{x+1}-2}{x-3}$. $= \lim_{x\to 3} \frac{(\sqrt{x+1}-2)(\sqrt{x+1}+2)}{(x-3)(\sqrt{x+1}+2)}$ ▯ 6

$= \lim_{x\to 3} \frac{x+1-4}{(x-3)(\sqrt{x+1}-2)} = \lim_{x\to 3} \frac{(x-3)}{(x-3)(\sqrt{x+1}+2)} = \lim_{x\to 3} \left(\frac{1}{\sqrt{x+1}+2}\right) = \frac{1}{4}$

(e) $\lim_{t\to 9} \frac{\sqrt{t}-3}{t-9}$. $= \lim_{t\to 9} \frac{\sqrt{t}-3}{(\sqrt{t}-3)(\sqrt{t}+3)} = \lim_{t\to 9} \frac{1}{\sqrt{t}+3} = \frac{1}{6}$ ▯ 6

4. (a) Write down the definition of the derivative $f'(x)$ of a function $f(x)$ ▯ 2

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

(b) Use the definition of a derivative to find $g'(2)$ where $g(x) = \frac{1}{x}$. ▯ 8

$$f(2+h) = \frac{1}{2+h}$$

$$f'(2) = \lim_{h\to 0} \frac{\frac{1}{2+h} - \frac{1}{2}}{h} = \lim_{h\to 0} \frac{\left(\frac{2-2+h}{4+2h}\right)}{h}$$

$$= \lim_{h\to 0} \frac{h}{h(4+2h)} = \lim_{h\to 0} \frac{1}{4+2h} = \frac{1}{4}$$

5. (a) State the Intermediate Value Theorem (IVT). ▯ 3

~~let~~ $f(x)$ is cts on $[a,b]$
if $f(a) \neq f(b)$, then for all M between $f(a)$ and $f(b)$
There is a c in $[a,b]$ such that $f(c) = M$.

(b) Show that $\cos(x) - x$ has a ~~solution~~ root in the interval $[0, 1]$. ▯ 6

x is cts $\Rightarrow$ $-x$ is cts by const. mult. law
$\cos(x)$ is cts
so $\cos(x) - x$ is cts by sum law

$\cos(0) - 0 = 1$
$\cos(1) - 1 < 0$

So by IVT there is some c in $[0,1]$ such that
$\cos(c) - c = 0$

1. Find all of the solutions in $[0, 2\pi]$ to the equation

$$\cos(x)\sin(x) + \cos(x) = 0.$$

(Hint: start by factorizing.)

2. Let $f(x) = \begin{cases} x^2 & \text{if } x \le 1 \\ 2 - x & \text{if } x > 1 \end{cases}$

   (a) Plot the graph of $f(x)$.

   (b) Write down the points of discontinuity

   (c) State whether $f(x)$ is right-continuous or left-continuous at these points.

3. (a) State what it means for a function $f(x)$ to be continuous at a point c.

   (b) State what it means for a function to have a jump continuity at $x = c$.

   (c) State what it means for a function to have a removable continuity at $x = c$.

   (d) For what value of c is the function $f(x) = \begin{cases} 2x + 9 & \text{if } x \le 3 \\ -4x + c & \text{if } x > 3 \end{cases}$ continuous at x=3? Justify your answer.

4. (a) State the Intermediate Value Theorem (IVT).

   (b) Find an interval of length 1/4 in $[0, 1]$ containing a root of $x^5 - 5x + 1$.

5. Using any of the results and methods (except for numerical estimates) you have seen in the course, determine the following limits. **Justify your answer.**

   (a) $\lim\limits_{x \to 4}(3 + x^{1/2})$.

   (b) $\lim\limits_{x \to -1} \frac{3x^2+4x+1}{x+1}$.

   (c) $\lim\limits_{h \to 0} \frac{\frac{1}{3+h} - \frac{1}{3}}{h}$.

   (d) $\lim\limits_{x \to 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{x}$.

   (e) $\lim\limits_{t \to -2} \frac{2t+4}{12-3t^2}$.

6. (a) Write down the definition of the derivative $f'(x)$ of a function $f(x)$

   (b) Let $f(x) = (x + 1)^2$. Use the definition of a derivative to prove that $f'(x) = 2(x + 1)$.

   (c) Use the definition of a derivative to find $g'(-1)$ where $g(t) = \frac{2}{1-t}$.

7. Calculate the following derivatives.

   Recall: $(f + g)' = f' + g'$, $(cf)' = cf'$, $(fg)' = f'g + g'f$, $\left(\frac{f}{g}\right)' = \frac{f'g - g'f}{g^2}$
   $\frac{d}{dx}x^n = nx^{n-1}$, $\frac{d}{dx}\sin(x) = \cos(x)$, $\frac{d}{dx}\cos(x) = -\sin(x)$, $\frac{d}{dx}e^x = e^x$

   (a) $\frac{d}{dz}7z^{-3} - 7x^2 + 10x + 9$

   (b) $\frac{d}{dx}3e^x - 2\cos(x)$

6. Calculate the following derivatives. *Use the standard derivatives and the laws of derivatives listed on the front page.*

(a) $\frac{d}{dx}x^{3/2} = \frac{3}{2}x^{\frac{1}{2}} = \frac{3}{2}\sqrt{x}$ $\square$ 4

(b) $\frac{d}{dx}2x^3 - 10x^{-1} + \cos(x) = 6x^2 + 10x^{-2} - \sin(x)$ $\square$ 4

(c) $\frac{d}{dx}x^2e^x = 2xe^x + x^2e^x = (2x+x^2)e^x$ $\square$ 4

(d) $\frac{d}{dx}\frac{x}{x+1}. = \frac{(1)(x+1) - x(1)}{(x+1)^2} = \frac{1}{(x+1)^2}$ $\square$ 4

7. What is the equation of the tangent line at $x = 3$ assuming that $f(3) = 5$ and $f'(3) = 2$. $\square$ 6

$y = mx + b$

$m = f'(3) = 2$

so $y = 2x + b$

Line passes through $(3,5)$ so $5 = 6 + b \Rightarrow b = -1$

so $y = 2x - 1$