1. Write down the definition of a relation.

2. List all of the 2^2 relations between $\{1\}$ and $\{1,2\}$

3. List all of the 2^2 relations between $\{1,2\}$ and $\{1\}$

4. List the 2^4 relations on $\{x,y\}$.

5. Define $R = \{(n,m) \in \mathbb{Z}\times\mathbb{Z} \mid n > m \text{ and } 2n - m \geq 0\}$

   Which of the following statements are true and which are false

   (i) $(2,4) \in R$; (ii) $(-2,-4) \in R$; (iii) $1 R (-4)$; (iv) $2 \not R 3$;

   (v) $a R 1 \iff a \in \{n \in \mathbb{Z} \mid n \geq 2\}$

   (vi) $-2 R a \iff a \in \{n \in \mathbb{Z} \mid n < -3\}$

6. For the relation in 5., describe the sets $\{a \mid 0 R a\}$ and $\{a \mid a R 0\}$.

7. Define a relation on $\mathbb{Z}$ by

$$n R m \iff n = am \text{ for some } a \in \mathbb{Z}.$$

   (i) $5 R 10$?; (ii) $9 R 5$?; (iii) $-7 R 21$?; (iv) $36 R -6$?;

   (v) Describe the set $\{x \in \mathbb{Z} \mid 21 R x\}$;

   (vi) Describe the set $\{x \in \mathbb{Z} \mid x R 2\}$

8. What does it mean for a relation to be reflexive?

9. List all of the relations from 4. that are reflexive

10. Is the relation in 6 reflexive?

11. Is the relation in 7 reflexive?

12. Questions 8–11 but replace "reflexive" with symmetric.

13. Questions 8–11 but replace "reflexive" with transitive.

14. Write down the definition of an equivalence relation.

15. For each of the relations in Q4. List those that are equivalence relations.

16. Write down the definition of an equivalence class

17. For all of the equivalence relations in Q 15. describe $[x]$ and $[y]$

1. A relation between A and B is a subset of $A \times B$

2. $\{1\} \times \{1,2\} = \{(1,1),(1,2)\}$ so the possible relations are.

$\emptyset, \{(1,1)\}, \{(1,2)\}, \{(1,1),(1,2)\}$

3. $\emptyset, \{(1,1)\}, \{(2,1)\}, \{(1,1),(2,1)\}$

4. $\{x,y\} \times \{x,y\} = \{(x,x),(x,y),(y,x),(y,y)\}$ so the relations are.

$\emptyset, \{(x,x)\}, \{(x,y)\}, \{(y,x)\}, \{(y,y)\},$
$\{(x,x),(x,y)\}, \{(x,x),(y,x)\}, \{(x,x)(y,y)\}, \{(x,y),(y,x)\}$
$\{(x,y),(y,y)\}, \{(y,x),(y,y)\}, \{(x,x)(x,y)(y,x)\}, \{(x,x)(y,x),(y,y)\}$
$\{(x,x),(x,y),(y,y)\}, \{(x,y),(y,x),(y,y)\}, \{(x,x),(x,y),(y,x),(y,y)\}$

5. (i) F; (ii) T; (iii) T; (iv) T; (v) T; (vi) T

6. $\{a \mid 0Ra\} = \{m \in \mathbb{Z} \mid m < 0\}$, $\{a \mid aR0\} = \mathbb{N}$

7 (i) F; (ii) F; (iii) F; (iv) T; (v) $\{\pm 1, \pm 3, \pm 7, \pm 21\}$; (vi) $\{2k \mid k \in \mathbb{Z}\}$ = even integers.

9. $\{(x,x),(y,y)\}, \{(x,x),(y,y),(y,x)\}$ $\{(x,x),(y,y)(x,y)\}$, $\{(x,x)(y,y),(x,y)(y,x)\}$

10. no

11. yes

12. (ii) $\emptyset, \{(x,x)\}, \{(y,y)\}, \{(x,y),(y,x)\}, \{(x,x),(x,y),(y,x)\},$
$\{(y,y),(x,y),(y,x)\}, \{(x,x),(y,y),(x,y),(y,x)\}.$

(iii) No, (iv) yes.

13 (ii) $\emptyset, \{(x,x)\}, \{(y,y)\}, \{(x,y)\}, \{(y,x)\}, \{(x,x),(y,y)\}, \{(x,x)(x,y)\}$
$\{(x,x),(y,x)\}, \{(y,y),(x,y)\}, \{(y,y),(y,x)\}, \{(x,x),(x,y),(y,y)\}$
$\{(x,x),(y,x),(y,y)\}, \{(x,x),(y,x)(x,y),(y,y)\}$

(iii) Yes (note $n > m \Rightarrow 2n > m \Rightarrow 2n - m > 0$ so second condition is redundant)
(iv) Yes.

17. $\{(x,x),(y,y)\}$ $[x]=\{x\}$, $[y]=\{y\}$

$\{(x,x),(y,y),(x,y),(y,x)\}$ $[x]=\{x,y\}$ $[y]=\{x,y\}$

19. $\{\{1\}\{2\}\{3\}\}$, $\{\{1,2\},\{3\}\}$, $\{\{1,3\}\{2\}\}$ $\{\{2,3\},\{1\}\}$, $\{\{1,2,3\}\}$

21 $\{(1,1)(2,2)(3,3)\}$, $[1]=\{1\}$, $[2]=\{2\}$, $[3]=\{3\}$

$\{(1,1)(2,2)(3,3)(1,2)(2,1)\}$ $[1]=[2]=\{1,2\}$, $[3]=\{3\}$

$\{(1,1)(2,2)(3,3)(1,3)(3,1)\}$ $[1]=[3]=\{1,3\}$ $[2]=\{2\}$

$\{(1,1)(2,2)(3,3)(2,3)(3,2)\}$ $[3]=[2]=\{3,2\}$, $[1]=\{1\}$

$\{(1,1)(2,2)(3,3)(1,2)(2,1)(1,3)(3,1)(2,3)(3,2)\}$ $[1]=[2]=[3]=\{1,2,3\}$

22. $\{\{1,2\},\{3,4,5\}\}$