Solutions

Name:_____

Signature:_____

J. Number:_____

MA 125 Calculus 1
Test 6

1. Attempt all of the questions.
2. Write your name and J. number at the top of this page.
3. Answer the questions in the spaces provided.
4. Follow the instructions inside the paper.
5. **Simplify your answers when it is possible.**
6. No graphing calculators are allowed.
7. You may use your table of derivatives.

INSTRUCTIONS: for questions 1 to 9 write your answer in the space provided. You do not need to show your working for these questions. Partial credit is generally not available for these questions. You should use extra paper for your rough working.

1. Evaluate $\int_{-1}^{2} x\, dx$.

ANSWER: $3/2$

---

2. Evaluate $\int 5x^3 - x^{-1} - x^{3/5}\, dx$

ANSWER: $\frac{5}{4}x^4 - \ln|x| - \frac{5}{8}x^{8/5} + C$

---

3. Evaluate $\int_{1}^{-1} 4t^3 - t^2\, dt$

ANSWER: $2/3$

---

4. Evaluate $\int_{0}^{\pi} \cos(\theta)\, d\theta$

ANSWER: 0

---

5. Evaluate $\int 25e^{5x}\, dx$.

ANSWER: $5e^{5x} + C$

---

6. Express $\int_2^9 f(x)\,dx - \int_4^9 f(x)\,dx$ as a single integral.

ANSWER: $\int_2^4 f(x)\,dx$

---

7. State what it means for $F(x)$ to be an antiderivative of $f(x)$.

ANSWER: $F'(x) = f(x)$

---

8. Evaluate the sum $\sum_{i=2}^{5} f(i)$, where $f(x) = i + 2$

ANSWER: 22

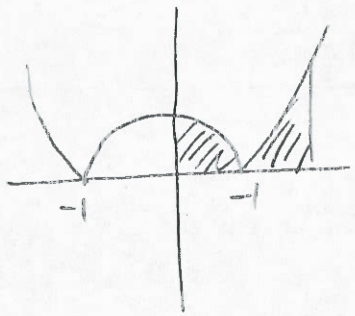
---

9. Using the Fundamental Theorem of Calculus, Part II, evaluate $\frac{d}{dx}\int_1^x \sin(t^2)\,dt$.

ANSWER: $\sin(x^2)$

INSTRUCTIONS: for questions 10 to 11, you must provide full solutions. Your solutions will be assessed.

10. This question has *three* parts. Sketch the graph of $|x^2 - 1|$; shade the area of your graph represented by the integral $\int_0^3 |x^2 - 1|\, dx$; finally, evaluate $\int_0^3 |x^2 - 1|\, dx$.



(2) (2)

$$\int_0^3 |x^2-1|\,dx = \int_0^1 1-x^2\,dx + \int_1^3 x^2-1\,dx \qquad (2)$$

$$= \left(x\Big|_0^1\right) - \left(\tfrac{1}{3}x^3\Big|_0^1\right) + \left(\tfrac{1}{3}x^3\Big|_1^3 - x\Big|_1^3\right) \qquad (2)$$

$$= 1 - \tfrac{1}{3} + \left(9 - \tfrac{1}{3} - (3-1)\right)$$

$$= \tfrac{2}{3} + \tfrac{26}{3} - \tfrac{6}{3} = \tfrac{22}{3} \qquad (2)$$

$7 - \frac{1}{3}$

$21 - 1$

11. Use the Fundamental Theorem of Calculus, Part II, to evaluate $\frac{d}{dx}\int_0^{x^2} \sin^2(t)\,dt$.

let $G(x) = \int_0^{x^2} \sin^2(t)\,dt$

and $A(x) = \int_0^{x} \sin^2(t)\,dt$

Then $G(x) = A(x^2)$

$A'(x) = \sin^2(x)$

So $G'(x) = 2x\,A'(x) = 2x\sin^2(x)$