1. A bullet is fired vertically from an initial height of $s_0 = 0$ with initial velocity v_0. Suppose that the bullet follows Galileo's equation of motion $s(t) = s_0 + v_0 t - \frac{1}{2} g t^2$.

(a) Show that the bullet will attain its maximum height at time $t = \frac{v_0}{g}$. [5]

Attains max height when $V(t) = 0$ (2)

<u>either</u> $V(t) = S'(t) = V_0 - gt$, so $V(t) = 0 \Leftrightarrow V_0 - gt = 0 \Leftrightarrow t = \frac{V_0}{g}$ (1) for $S'(t)$

<u>or</u> $V(t) = S'(t) = V_0 - gt$, and $V\left(\frac{V_0}{g}\right) = V_0 - g\frac{V_0}{g} = 0$. (2) correct of sol

(b) Using you answer to part (a), or otherwise, determine the initial velocity v_0 required for the bullet to reach a maximum height of $2km$. [10]

Max height at time $t = \frac{V_0}{g}$

Max height is then

$$S\left(\frac{V_0}{g}\right) = V_0\left(\frac{V_0}{g}\right) - \frac{1}{2}g\left(\frac{V_0}{g}\right)^2$$

$$= \frac{V_0^2}{g} - \frac{V_0^2}{2g} = \frac{V_0^2}{2g}$$

Then

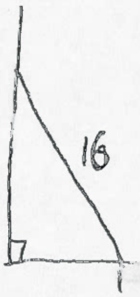
$$S\left(\frac{V_0}{g}\right) = 2000m$$

$$\Leftrightarrow \quad \frac{V_0^2}{2g} = 2000m$$

$$\Leftrightarrow \quad V_0 = +\sqrt{4000g} \ m/s$$

2. A $16ft$ ladder leans against a wall. The bottom of the ladder is $5ft$ from the wall at time $t = 0$ and slides away from the wall at a rate of $3ft/s$.

(a) Determine the distance of the bottom of the ladder from the wall at time $t = 1$ and the height of the top of the ladder from the ground at time $t = 1$. [5]



Distance of ladder from wall at $t=1$ is

$$5 + 3\times 1 = 8ft$$

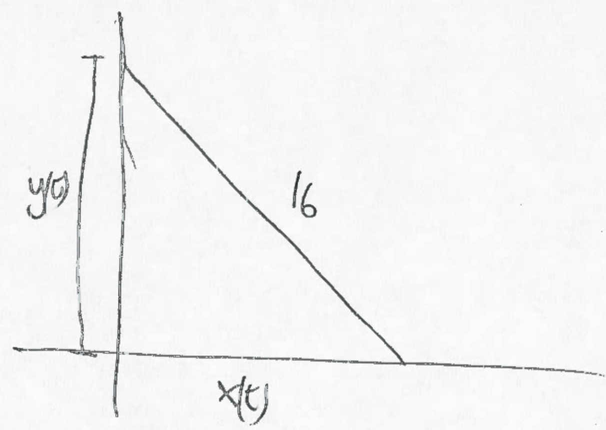
Using pythagoras height of ladder at $t=1$ is

$$\sqrt{16^2 - 8^2} = \sqrt{192}\ ft \approx 13.86ft$$

(b) Find the velocity of the top ladder at time $t = 1$. [10]

$x(t)$ = distance between wall and bottom of ladder at time t

$y(t)$ = height of top of ladder at time t.



$$\frac{dx}{dt} = 3.$$

we need to find $\left.\frac{dy}{dt}\right|_{t=1}$

$$x(t)^2 + y(t)^2 = 16^2$$

so

$$2x(t)\frac{dx}{dt} + 2y(t)\frac{dy}{dt} = 0$$

So $$\frac{dy}{dt} = -\frac{x(t)}{y(t)}\frac{dx}{dt}$$

From part (a), when $t=1$, $x(t) = 8$, $y(t) = \sqrt{192}$

so $$\left.\frac{dy}{dt}\right|_{t=1} = \frac{-8\times 3}{\sqrt{192}} = \frac{-24}{\sqrt{192}}\ ft/s \approx -1.7\ ft/s$$