Name:____________________________

Signature:____________________________

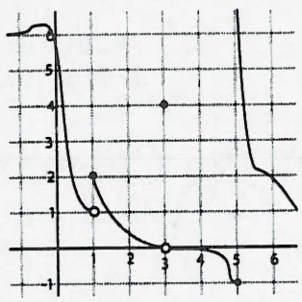
J. Number:____________________________

MA 125 Calculus 1
Test 2

1. Attempt all 7 questions.
2. Write your name and J. number at the top of this page.
3. Answer the questions in the spaces provided.
4. Show all your work required to obtain your answers.
5. **No calculators are allowed.**
6. You may use a notecard.

Question	Mark
1	/
2	/
3	/
4	/
5	/
6	/
76	/
total	/

1) The function $f(x)$ shown in the graph below has three points of discontinuity.



(a) Write down the one-sided limits (or state that they do not exist) of the function $f(x)$ at each of the three points of discontinuity. [6]

$\lim_{x \to 1_-} f(x) = 1$ (1) $\lim_{x \to 3_+} f(x) = 0$ (1) $\lim_{x \to 5_+} f(x) = \infty$ or DNE (1)

$\lim_{x \to 1_+} f(x) = 2$ (1) $\lim_{x \to 3_-} f(x) = 0$ (1) $\lim_{x \to 5_-} f(x) = -1$ (1)

(b) State whether the function $f(x)$ is right-continuous, left-continuous, both or neither at the points $x = 1$, $x = 2$, $x = 3$ and $x = 5$. [4]

$x=1$: right-cts (1)
$x=2$: both (1)
$x=3$: neither (1)
$x=5$: left-cts (1)

2) The table below shown some values of $f(x) = \frac{\sin(x)}{x^3 - x}$

x	f(x)	x	f(x)	x	f(x)
-0.987	-32.72679750	-1.000012	35061.56118	-1.012	34.70178074
-1.112	3.408594464	-0.98987	-41.89579171	-0.12	-1.012177077
-0.012	-1.000120017	-0.0012	-1.000001200	-0.96	-10.88424171
-.999987	-32364.62089	0.12	-1.012177077	0.0012	-1.000001200

Write down the following limits: [4]

$$\lim_{x \to -1_-} f(x) = +\infty \qquad \lim_{x \to -1_+} f(x) = -\infty$$

(2) (2)

3) (a) Using the facts that $\cos(x)$, e^x and x are continuous functions, and the laws of continuity to show that $\frac{\cos(\pi e^{(x^2)})}{x}$ is continuous at $x = 2$. **Clearly state which of the laws of continuity you are using throught your solution.** ~~[2]~~ 6

1 x cts $\Rightarrow$ x^2 cts by prod. law

1 e^x and x^2 cts $\Rightarrow$ e^{x^2} cts by comp law

1 e^{x^2} cts $\Rightarrow$ πe^{x^2} cts by const. mult. law

1 $\cos(x)$ and πe^{x^2} cts $\Rightarrow$ $\cos(\pi e^{x^2})$ cts by comp law

2 $\cos(\pi e^{x^2})$ and x cts, also $x \neq 0$ at $x=2$ so

$\frac{\cos(\pi e^{x^2})}{x}$ cts at $x=2$ by quotient law

4) (a) State what it means for a function $f(x)$ to be continuous at a point $x = c$. [2]

f(x) is defined on an interval containing c 1

$\lim_{x \to c} f(x) = f(c)$ 1

(b) For what values of of b and c will the function $f(x) = \begin{cases} 5 & \text{for } x = 2 \\ 2x + b & \text{for } x < 2 \\ x^2 + 2b + c & \text{for } x > 2 \end{cases}$ be continuous at $x = 2$. ~~[3]~~ $\boxed{5}$

f(x) is cts at x=2 iff $\lim_{x \to 2_-} f(x) = \lim_{x \to 2_+} f(x) = f(2)$ 1

$f(2) = 5$

$\lim_{x \to 2_-} f(x) = \lim_{x \to 2_-} (2x+b) = 4+b$ 1

$\lim_{x \to 2_+} f(x) = \lim_{x \to 2_+} (x^2+2b+c) = 4+2b+c$ 1

So cts at x=2

$\Leftrightarrow$ $4+b=5$ and $4+2b+c=5$

$\Leftrightarrow$ $b=1$ and $4+2+c=5$ 2

$\Leftrightarrow$ $b=1$ and $c=-1$

5) Using any of the results and methods (except for 'numerical' methods) you have seen in the course, calculate the following limits. Justify your answers.

(a) $\lim_{x\to 2} \frac{x^3-4x}{x-2}$ $= \lim_{x\to 2} \frac{x(x-2)(x+2)}{x-2}$ [5]

$= \lim_{x\to 2} x(x+2) = 8$

(b) $\lim_{t\to 4} \frac{2t+4}{12-3t^2}$ $= \frac{8+4}{12-3\times(4^2)} = \frac{12}{-36} = -1/3$ [5]

(c) $\lim_{\theta\to\pi} \frac{\cot(\theta)}{\csc(\theta)}$ $= \lim_{\theta\to\pi} \frac{\left(\frac{\cos\theta}{\sin\theta}\right)}{\frac{1}{\sin(\theta)}} = \lim_{\theta\to\pi} \cos(\theta) = \cos(\pi) = -1$ [5]

(d) $\lim_{h\to 2} \frac{\sqrt{2+h}-2}{h} = \frac{\sqrt{2+2}-2}{2} = \frac{0}{2} = 0$ [5]

(e) $\lim_{x\to\infty} \frac{2x^2-3}{-x^2+x} = \lim_{x\to\infty} \left(\frac{2x^2-3}{-x^2+x} \frac{\frac{1}{x^2}}{\frac{1}{x^2}} \right) = \lim_{x\to\infty} \frac{\frac{2x^2}{x^2} - \frac{3}{x^2}}{\frac{-x^2}{x^2} + \frac{x}{x^2}}$ [5]

$= \lim_{x\to\infty} \left(\frac{2 - \frac{3}{x^2}}{-1 + \frac{1}{x}} \right) = \frac{2 - 3\left(\lim_{x\to\infty} \frac{1}{x}\right)^2}{-1 + \lim_{x\to\infty} \frac{1}{x}} = \frac{2-3(0)^2}{-1+0} = -2$

(f) $\lim_{h\to 0} \frac{3^{2x}-1}{3^x-1} = \frac{3^{2x}-1}{3^x-1}$ [5]

Since the function does not depend upon h.

6) (a) State the Intermediate Value Theorem (IVT). [3]

Let f(x) be cts on the interval $[a, b]$
such that $f(a) \neq f(b)$
Then for every M between $f(a)$ and $f(b)$
there exists a c in $[a,b]$ such that $f(c) = M$

(b) Use the IVT to show that the equation $e^{-x^2} = x$ has a solution. [3]

setup ② $e^{-x^2} = x$ has a solution $\Leftrightarrow$ $e^{-x^2} - x = 0$ has a solution
Let $f(x) = e^{-x^2} - x$. We need to show $f(c) = 0$ for some c.

① $f(x)$ is cts

① $f(0) = e^0 - 0 = 1$
$f(1) = e^{-1} - 1 < 0$

① so by IVT $f(c) = 0$ for some c in $(0,1)$

7) For each of the following statements, state whether the statement is **always true** or **always false** or **sometimes true**.

Note that you will get 3 points for a correct answer and -3 points for an incorrect answer.

(a) $\lim_{x\to 3} f(x) = f(3)$ [3]

Sometimes true (f(3) need not be defined.)

(b) If $\lim_{x\to 0} \frac{f(x)}{x} = 1$, then $f(0) = 0$ [3]

sometimes true eg consider $f(x) = x$

or $f(x) = \frac{x^2}{x}$

(c) If $\lim_{x\to -7} f(x) = 8$, then $\lim_{x\to -7} \frac{1}{f(x)} = \frac{1}{8}$ [3]

Always true by quotient law

(d) If $\lim_{x\to 5+} f(x) = 4$ and $\lim_{x\to 5-} f(x) = 8$ then $\lim_{x\to 5} f(x) = 6$ [3]

Always false

(e) If f is a continuous function such that $f(-1) = 1$ and $f(1) = 1$ then the IVT tells us that the graph of $f(x)$ does not cross the x-axis in the interval $(-1, 1)$. [3]

Always false In this situation the IVT tells us nothing

(f) If $\lim_{x\to c}(f(x) + g(x)) = L$, then $\lim_{x\to c} f(x) = L - \lim_{x\to c} g(x)$. [3]

Sometimes true.

eg $f(x) = \frac{1}{x}$, $g(x) = \frac{-1}{x}$ then limits are not defined

or $f(x) = 1$ and $g(x) = 1$