Q1

$$f(x) = \frac{x^5 - 3x^3 + 4}{7x^2 - 9}$$ is defined as long as $7x^2 - 9 \neq 0$.

$$7x^2 - 9 = 0 \iff x^2 = \pm\sqrt{9/7}$$

So f is defined for all real numbers except $\pm \frac{3}{\sqrt{7}}$

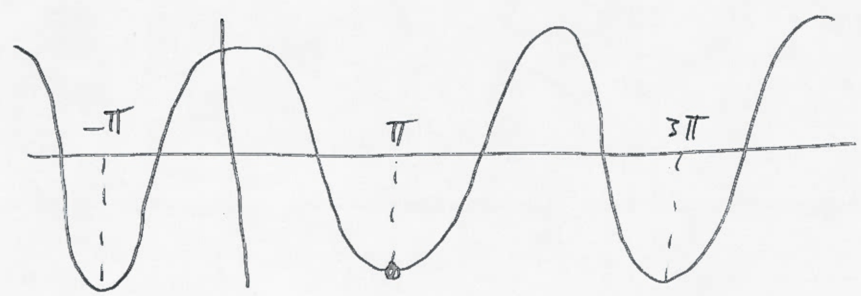
Q2

$$\frac{1}{x-2} - \frac{4}{x^2-4} = \frac{x^2 - 4 - 4(x-2)}{(x-2)(x^2-4)} = \frac{x^2 - 4x + 4}{(x-2)(x-2)(x+2)}$$

$$= \frac{\cancel{(x-2)}\cancel{(x-2)}}{\cancel{(x-2)}\cancel{(x-2)}(x+2)} = \frac{1}{(x+2)}$$

Q3

Consider the graph of cos



So $\pi + 2k\pi$ where k is an integer

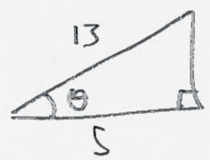
This simplifies to

$$(2k+1)\pi \quad , k \text{ an integer}$$

(4) $0 \leq \theta \leq \pi/2$ so we can use right angled triangles

$$\cos(\theta) = \frac{adj}{hyp} = \frac{5}{13}$$

So we have



By Pythagoras' Theorem $opp = \sqrt{13^2 - 5^2} = 12$

$$\tan(\theta) = \frac{opp}{adj} = \frac{12}{5}$$

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(5) $y = \frac{x+3}{x-1} \Rightarrow y(x-1) = x+3 \Rightarrow yx - y = x+3$

$\Rightarrow yx - x = y+3 \Rightarrow x = \frac{y+3}{y-1} \Rightarrow f^{-1}(x) = \frac{x+3}{x-1}$

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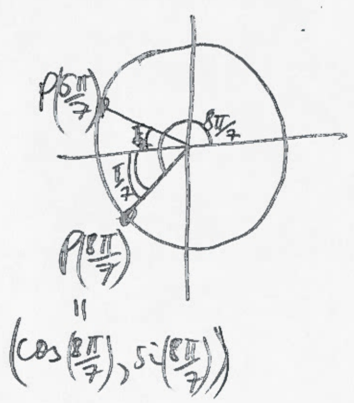
(6) $\cos^{-1}(\cos(\frac{8\pi}{7}))$ is the angle, θ, in $[0,\pi]$ s.t. $\cos(\theta) = \cos(\frac{8\pi}{7})$



From the figure $P(\frac{8\pi}{7})$ and $P(\frac{6\pi}{7})$ have the same x-coord so

$$\cos^{-1}(\cos(\frac{8\pi}{7})) = \frac{6\pi}{7}$$

(7) $9^{-t} = 3^{t-1} \iff 3^{-2t} = 3^{t-1} \iff -2t = t-1 \iff t = \frac{1}{3}$

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(8) $5\ln(x^{\frac{1}{2}}) - \ln(x) = \ln(x^{5/2}) - \ln(x) = \ln\left(\frac{x^{\frac{5}{2}}}{x}\right) = \ln(x^{\frac{3}{2}})$

$\ln(x^{\frac{3}{2}}) = 0 \iff e^{\ln(x^{\frac{3}{2}})} = e^0 \iff x^{3/2} = 1 \iff x = 1.$

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(9) Note $t^2 - 2t + 1 = (t-1)^2$

so putting $t = x^2$ we see

$$x^4 - 2x^2 + 1 = (x^2-1)^2$$

This is zero iff $x^2 = 1$ iff $x = \pm 1$

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(10) putting $x = \sin(\theta)$ in the equation in Q (9) we see

$$\sin(\theta) = \pm 1$$

so $\theta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$