SOLUTIONS

Name:____________________________

Signature:____________________________
J. Number:____________________________

MA 125 Calculus 1
Test 4

1. Attempt all of the questions.
2. Write your name and J. number at the top of this page.
3. Answer the questions in the spaces provided.
4. Show all your work required to obtain your answers.
5. **Simplify your answers when it is possible.**
6. No graphing calculators are allowed.
7. You may use your table of derivatives.

Question	Mark
1	/6
2	/9
3	/10
4	/19
total	/

1. Let $f(x) = (2x - x^2)e^x$. This function has a critical point at $x = -\sqrt{2}$. Use the **second derivative test** to determine if $f(x)$ has a local minimum or a local maximum at $x = -\sqrt{2}$. If the second derivative test is inconclusive, state that this is the case. [6]

$f'(x) = (2-2x)e^x + (2x-x^2)e^x = (2-x^2)e^x$ 2

$f''(x) = -2x\, e^x + (2-x^2)e^x$

$= (-x^2-2x+2)e^x$ 2

$f''(-\sqrt{2}) > 0$ so local min 2

3. The concentration $C(t)$ (in mg/cm^3) of a drug in a patient's blood-stream after t hours is given by

$$C(t) = \frac{t}{60(t^2+4t+4)}.$$

Find the maximum concentration in the first 4 hours after the patient receives the drug, and the time at which it occurs. $C(t)$ is $C(t)$ in $[0,4]$ [10]

$$C'(t) = \frac{1}{60}\left[\frac{t^2+4t+4 - t(2t+4)}{(t^2+4t+4)^2}\right] = \frac{1}{60}\frac{-t^2+4}{(t+2)^4} = \frac{1}{60}\frac{(2-t)(2+t)}{(t+2)^4}$$

$$= \frac{1}{60}\frac{(2-t)}{(t+2)^3}$$ 3

$C'(t)$ is not defined at $t=-2$, but since $C(t)$ is not defined at $t=-2$, $t=2$ is the only critical point. 2

We need to find max in the interval $[0,4]$ 1

$t=2$ is in this interval 1

$C(0) = 0$

$C(2) = \frac{2}{60(16)} = \frac{1}{480}$ 2

$C(4) = \frac{4}{60(36)} = \frac{1}{540}$

So max concentration is $\frac{1}{480}$ mg/cm³ and it occurs after 2 hours.

1

2. **(a)** Find all of the local maxima of $f(x) = x^3 - 27x - 20$. [7]

$f'(x) = 3x^2 - 27$ (1)

$f'(x)$ is defined everywhere (1)

$f'(x) = 0 \Leftrightarrow x = \pm 3$ (1)

(1)	(1)	(1)
$(-\infty, -3)$	$f'(-10) > 0$	$\nearrow$
$(-3, 3)$	$f'(0) < 0$	$\searrow$
$(3, \infty)$	$f'(10) > 0$	$\nearrow$

So local maxi at $x = -3$ (1)

min $x = 3$

(b) Does the function $f(x) = x^3 - 27x - 20$ have an absolute maximum? Briefly justify your answer. [2]

NO

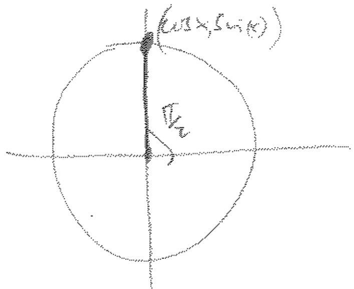
$\lim_{x \to \infty} f(x) = \infty$ so function is unbounded.

4. Let $f(x) = \cos(x) + x$, with domain $[0, 2\pi]$.
(a) Determine on which intervals $f(x)$ is increasing and on which $f(x)$ is decreasing. [7]

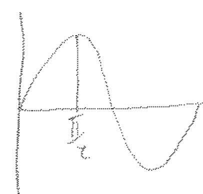
$f'(x) = -\sin x + 1$ 1

$f'(x)$ is defined everywhere 1

$f'(x) = 0 \Leftrightarrow -\sin(x) + 1 = 0 \Leftrightarrow \sin(x) = 1$ 1



or



gives $x = \frac{\pi}{2}$

So $\frac{\pi}{2}$ is the only critical point 1

interval	test point	behaviour
$(0, \frac{\pi}{2})$	$f'(\frac{\pi}{4}) > 0$	↑
$(\frac{\pi}{2}, 2\pi)$	$f'(\pi) > 0$	↑

1 2

(b) Determine on which intervals $f(x)$ is concave up and on which it is concave down. [8]

$f''(x) = -\cos(x)$ 1

$f''(x)$ is defined on $[0, 2\pi]$ 1

$f''(x) = 0 \iff -\cos(x) = 0$

$\iff x = \frac{\pi}{2}$ or $\frac{3\pi}{2}$ 2

so critical points are $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$

interval	test value	concavity
$(0, \frac{\pi}{2})$	$f''(\frac{\pi}{4}) < 0$	down
$(\frac{\pi}{2}, \frac{3\pi}{2})$	$f''(\pi) > 0$	up
$(\frac{3\pi}{2}, 2\pi)$	$f''(\frac{7\pi}{4}) < 0$	down

1 3

(c) Sketch the graph of $f(x)$, **marking the points of inflection and the critical points** on your graph. [4]

$f(x) = \cos(x) + x$

$f(0) = 1$