

AZ iii)

$$f'(x) = \ln(x) + \frac{x}{x} = \ln(x) + 1$$

$$f'(x) = 0 \Leftrightarrow \ln(x) + 1 = 0 \Leftrightarrow \ln(x) = -1 = \ln\left(\frac{1}{e}\right)$$

$x = \frac{1}{e}$ is the only critical point.

interval	sign	behaviour
$(0, \frac{1}{e})$	$f'(x) < 0$	decreasing
$(\frac{1}{e}, \infty)$	$f'(x) > 0$	increasing

AZ iv)

$$f'(x) = 1 - \sin(x)$$

$$f'(x) = 0 \Leftrightarrow 1 - \sin(x) = 0 \Leftrightarrow 1 = \sin(x) \Leftrightarrow x = \frac{\pi}{2} + 2k\pi, \quad k \text{ an integer}$$

on the interval $(0, 2\pi)$ we have

interval	sign	behaviour
$(0, \frac{\pi}{2})$	$f'(\frac{\pi}{4}) > 0$	inc
$(\frac{\pi}{2}, 2\pi)$	$f'(\pi) > 0$	inc

Since $\sin(x)$ is cyclic, $f(x)$ is always increasing

AZ ii)

$$f'(x) = \frac{1-x^2}{(x^2+1)^2}$$

critical points are $x = \pm 1$

interval	sign	behaviour
$(-\infty, -1)$	-	dec
$(-1, 1)$	+	inc
$(1, \infty)$	-	dec.

BII (i)

$$y' = 4x - 4$$

$$y' = 0 \Leftrightarrow x = 1$$

critical point is $x = 1$

$$f(1) = 0$$

$$f(0) = 2$$

$$f(3) = 8$$

max is $f(3) = 8$

min is $f(1) = 0$

BII (iv)

$$f'(x) = 1 - \frac{4}{(x+1)^2} = \frac{(x-1)(x+3)}{(x+1)^2}$$

$$f'(x) = 0 \Leftrightarrow x = 1 \text{ or } x = -3$$

critical points are $1, -3, -1$

only $x = 1$ is in the interval

$$f(1) = -1 \quad f(0) = 0 \quad f(3) = 0$$

max is $f(0) = f(3) = 0$

min is $f(1) = -1$

BIV $f(x) = \sec^2 x - 2$

$$f'(x) = 0 \Leftrightarrow \sec(x) = \pm\sqrt{2} \Leftrightarrow \frac{1}{\cos(x)} = \pm\sqrt{2} \Leftrightarrow \cos(x) = \pm\frac{1}{\sqrt{2}} \\ \Leftrightarrow x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

only $\frac{\pi}{4}$ is in $[0, 1]$

$$f(0) = 0$$

$$f\left(\frac{\pi}{4}\right) \approx -0.57$$

$$f(1) \approx -0.44$$

so min is $f\left(\frac{\pi}{4}\right) = 1 - \frac{\pi}{2}$, max is $f(0) = 0$