A1

(i) $f(1)=2$

$f(1+h)=2(1+h)^2=2+4h+2h^2$

$f'(1)=\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}=\lim_{h\to 0}\frac{2+4h+2h^2-4}{h}=\lim_{h\to 0}(4+2h)=4$

(ii) $f(1)=-2$

$f(1+h)=(1+h)^2-3(1+h)=1+2h+h^2-3-3h=h^2-h-2$

$f'(1)=\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}=\lim_{h\to 0}\frac{h^2-h-2-(-2)}{h}=\lim_{h\to 0}h-1=-1$

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A2

(i) The equation of the tangent line is $y=f'(2)x+c$

First find $f'(2)$

$f(2)=8$

$f(2+h)=(2+h)^3=8+6h^2+12h+h^3$

$f'(2)=\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}=\lim_{h\to 0}h^2+6h+12=12$

So $y=12x+c$

to find c, the line passes through $(2,f(2))=(2,8)$

So $8=24+c$

so $c=-16$

Then $y=12x-16$

A2(iii)

The equation of the tangent line is $y = f'(\pi)x + C$

first find $f'(\pi)$:

$f(\pi) = \sin(\pi) = 0$

$f(\pi+h) = \sin(\pi+h) = \sin(\pi)\cos(h) + \cos(\pi)\sin(h) = -\sin(h)$

so

$$f'(\pi) = \lim_{h\to 0} \frac{f(\pi+h) - f(\pi)}{h} = \lim_{h\to 0} \frac{-\sin(h)}{h} = -\left(\lim_{h\to 0} \frac{\sin(h)}{h}\right) = -1$$

so $y = -x + C$

To find C, the line passes through $(\pi, f(\pi)) = (\pi, 0)$

so $0 = -\pi + C$

so $C = \pi$

so $y = -x + \pi$

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A3:

$y = f'(2)x + C = -3x + C$

the tangent line passes through $(2, f(2)) = (2, 7)$

so $7 = -6 + C$

so $C = 13$

giving $y = -3x + 13$

B1

(i) $f(x) = 2x^2 - x$

$f(x+h) = 2(x+h)^2 - (x+h) = 2x^2 + 4xh + 2h^2 - x - h$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{2x^2 + 4xh + 2h^2 - x - h - 2x^2 + x}{h} = \lim_{h\to 0} 4x + 2h - 1 = 4x - 1$$

(ii) $f(x) = \frac{1}{x}$

$f(x+h) = \frac{1}{x+h}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \left(\frac{1}{h}\left(\frac{1}{x+h} - \frac{1}{x}\right)\right) = \lim_{h\to 0} \left(\frac{x - x - h}{hx(x+h)}\right) = \lim_{h\to 0} \frac{-1}{x(x+h)} = \frac{-1}{x^2}$$

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B2) (i) $3x^2$, (ii) $-3x^{-4} = \frac{-3}{x^4}$ (iii) $3x^2 + 10x^4 - x^{-2}$ (iv) $8x + 3x^{-2} - 25\sin(x)$

(v) (note $\sqrt{t} = t^{\frac{1}{2}}$), $\frac{1}{2}t^{-\frac{1}{2}} + \cos(t)$ (vi) $2e^x + \sec(x)\tan(x) + 4x$

(vii) $0$ ($e^2$ is a constant)

(viii) $\frac{dg}{dt} = -\frac{3}{2}t^{\frac{1}{2}} - 5t^{-\frac{9}{4}}$, $\left.\frac{dg}{dt}\right|_{t=1} = -\frac{3}{2} - 5 = \frac{-13}{2}$

(ix) $-\frac{1}{3}s^{-\frac{4}{3}} + \frac{1}{3}s^{-\frac{2}{3}}$

(x) $f'(x) = 6x + 4$, $f'(-1) = -6 + 4 = -2$

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C1) (i) $\frac{d}{dx}(x+x^2)\cdot\sin(x) = \left(\frac{d}{dx}x + x^2\right)\sin(x) + (x+x^2)\left(\frac{d}{dx}\sin(x)\right)$

$= (1+2x)\sin(x) + (x+x^2)\cos(x)$

or

Let $f(x) = (x+x^2)\sin(x)$, $g(x) = (x+x^2)$, $h(x) = \sin(x)$

Then $f(x) = g(x)\cdot h(x)$.

$g'(x) = 1 + 2x$

$h'(x) = \cos(x)$

$f'(x) = (g\cdot h)'(x) = g'(x)\cdot h(x) + g(x)\cdot h'(x) = (1+2x)\sin(x) + (x+x^2)\cos(x)$

C1

(ii) $\frac{d}{dx} e^x(2x+x^{-4}) = \left(\frac{d}{dx}e^x\right)(2x+x^{-4}) + e^x\left(\frac{d}{dx}(2x+x^{-4})\right) = e^x(2x+x^{-4}) + e^x(2-4x^{-5})$

$= e^x(2x+2+x^{-4}-4x^{-5})$

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(iv) $\frac{d}{dx}\left(e^x(x-2x^{-1})\sin(x)\right) = \left(\frac{d}{dx}e^x\right)\left((x-2x^{-1})\sin(x)\right) + e^x\left(\frac{d}{dx}(x-2x^{-1})\sin(x)\right)$

$\frac{d}{dx}e^x = e^x$

$\frac{d}{dx}(x-2x^{-1})\sin(x) = \left(\frac{d}{dx}(x-2x^{-1})\right)\sin x + (x-2x^{-1})\left(\frac{d}{dx}\sin x\right)$

$= (1+2x^{-2})\sin(x) + (x-2x^{-1})\cos(x)$

so $\frac{d}{dx}\left(e^x(x-2x^{-1})\sin(x)\right) = e^x(x-2x^{-1})\sin(x) + e^x\left[(1+2x^{-2})\sin(x) + (x-2x^{-1})\cos(x)\right]$

$= e^x(x+1-2x^{-1}+2x^{-2})\sin(x) + e^x(x-2x^{-1})\cos(x)$

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(vii) $\frac{d}{dx}\frac{x^2\sin(x)}{e^x-x} = \frac{\left(\frac{d}{dx}x^2\sin(x)\right)(e^x-x) - x^2\sin(x)\left(\frac{d}{dx}(e^x-x)\right)}{(e^x-x)^2}$

$\frac{d}{dx}x^2\sin(x) = \left(\frac{d}{dx}x^2\right)\sin x + x^2\left(\frac{d}{dx}\sin x\right) = 2x\sin(x) + x^2\cos(x)$

$\frac{d}{dx}(e^x-x) = e^x-1$

so $\frac{d}{dx}\frac{x^2\sin x}{e^x-x} = \frac{(2x\sin(x)+x^2\cos(x))(e^x-x) - x^2\sin(x)(e^x-1)}{(e^x-x)^2} = \ldots$

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(ix) note $z^{-2}e^{-z} = \frac{z^{-2}}{e^z}$

so $\frac{d}{dz}z^{-2}e^{-z} = \frac{d}{dz}\frac{z^{-2}}{e^z} = \frac{\left(\frac{d}{dz}z^{-2}\right)e^z - z^{-2}\left(\frac{d}{dz}e^z\right)}{e^{2z}} = \frac{-2z^{-3}e^z - z^{-2}e^z}{e^{2z}}$

$= \frac{-2-z}{z^3e^z}$