A1

(i) $\cos^4(x-1)+3\cos(x-1)$ (ii) $\cos(x^2+3x-1)$ (iii) $\frac{x^2+3x}{x^2+3x-1}$ (iv) $\frac{\cos(x^2+3x-1)}{\cos(x^2+3x-1)-1}$

A2

(i) $f(x)=g(h(x))$ where $g(x)=x^5$, $h(x)=7x-9$

(ii) $f(x)=(p\circ q\circ r)(x)$ where $r(x)=x^2-3$, $q(x)=\cos(x)$, $p(x)=\sin(x)$

(iii) $f(x)=(p\circ q\circ r)(x)$ where $r(x)=5-6x$, $q(x)=e^x$, $p(x)=\tan(x)$

(iv) $f(x)=(t\circ p\circ q\circ r\circ s)(x)$ where $s(x)=x^2$, $r(x)=\cos(x)$, $q(x)=e^x$, $p(x)=\sin(x)$, $t(x)=x^3$

A3

(6) $(f\circ g)(a)=f(g(a))=f\left(\frac{a+1}{a-1}\right)=\frac{\left(\frac{a+1}{a-1}+1\right)}{\left(\frac{a+1}{a-1}-1\right)}=\frac{\left(\frac{a+1+a-1}{a-1}\right)}{\left(\frac{a+1-a+1}{a-1}\right)}=\frac{\left(\frac{2a}{a-1}\right)}{\left(\frac{2}{a-1}\right)}$

$=\left(\frac{2a}{a-1}\right)\left(\frac{a-1}{2}\right)=\frac{2a(a-1)}{2(a-1)}=a$

So. $(f\circ g)(t)=t$

also since $f(t)=g(t)$ $(f\circ g)(t)=(g\circ f(t))=t$.

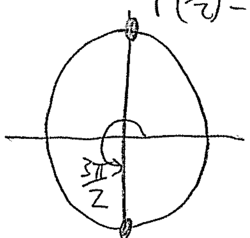
16

A3

(29) Since $\frac{\pi}{3}$ is in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, $\sin^{-1}\left(\sin\left(\frac{\pi}{3}\right)\right) = \frac{\pi}{3}$

(31) $\cos^{-1}\left(\cos\left(\frac{3\pi}{2}\right)\right)$ is the angle θ in $[0, \pi]$ such that $\cos(\theta) = \cos\left(\frac{3\pi}{2}\right)$

$P\left(\frac{\pi}{2}\right) = \left(\cos\left(\frac{\pi}{2}\right), \sin\frac{\pi}{2}\right)$



$P\left(\frac{3\pi}{2}\right) = \left(\cos\left(\frac{3\pi}{2}\right) \sin\left(\frac{3\pi}{2}\right)\right)$

From the figure $\cos\left(\frac{\pi}{2}\right) = \cos\left(\frac{3\pi}{2}\right)$

so $\cos^{-1}\cos\left(\frac{3\pi}{2}\right) = \frac{\pi}{2}$.

B1 (1)(5) see solutions in book

(6). For every $c \neq 1, 2$, $f(c)$ is given by a polynomial, so f is cts at $c \neq 1, 2$

at $x=1$:

$\lim_{x\to 1-} f(x) = \lim_{x\to 1-} (x^2+3) = 4 \neq 9 = f(1)$

$\lim_{x\to 1+} f(x) = \lim_{x\to 1+} (10-x) = 9 = f(1)$ so right cts at 1

at $x=2$

$\lim_{x\to 2-} f(x) = \lim_{x\to 2-} (10-x) = 8 = f(2)$

$\lim_{x\to 2+} f(x) = \lim_{x\to 2+} (6x-x^2) = 8 = f(2)$

so cts at $x=2$

B1

(a) x is cts, $\sin x$ is cts

so $3x$ and $4\sin(x)$ are both cts by scalar mult law

so $3x + 4\sin(x)$ is cts by sum law

(11) x is cts so x^2 is cts by product law

1 is cts so

x^2+1 is cts by sum law

since 1 and x^2+1 are cts and $x^2+1 \neq 0$ for all x (why?)

by quotient law $\frac{1}{x^2+1}$ is cts.

(13) 3^x, 4^x and 1 are all cts

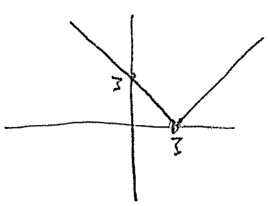
so $1+4^x$ is cts by sum law

$1+4^x \neq 0$ for all x (why?)

so $\frac{3^x}{1+4^x}$ is cts for all x, by quotient law

B2

21)



$\lim_{x\to 3_-} f(x) = \lim_{x\to 3_-} |x-3| = \lim_{x\to 3_-} -(x-3)$ since $x-3<0$ when $x<3$

$= \left(\lim_{x\to 3_-} -x\right) + \lim_{x\to 3_-}(3) = -3+3 = 0 = f(3)$

$\lim_{x\to 3_+} f(x) = \lim_{x\to 3_+} x-3 = 0 = f(3)$

so $f(x)$ is cts at $x=3$

$f(x)$ is cts at $x \neq 3$ since f is given by a polynomial on these values.

B2

83) $f(x)$ is cts for all value of c when $x \neq 5$

$f(x)$ is cts at $x=5$ if

$$\lim_{x\to 5_-} f(x) = \lim_{x\to 5_+} f(x) = f(5)$$

So we need to find the value of c for which this is true.

$$\lim_{x\to 5_-} f(x) = \lim_{x\to 5_-} x^2 - c = 25 - c$$

$$\lim_{x\to 5_+} f(x) = \lim_{x\to 5_+} 4x + c = 20 + 2c = f(5)$$

Now

$$f(5) = \lim_{x\to 5_-} f(x) = \lim_{x\to 5_+} f(x) \iff 25 - c = 20 + 2c$$

$$\iff 5 - c = 2c$$

$$\iff 5 = 3c$$

$$\iff c = 5/3$$

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85) By laws of continuity $ax + \cos(x)$ and $bx+2$ are cts for all a, b and x (why?)

so $f(x)$ is cts when $x \neq \frac{\pi}{4}$

$$\lim_{x\to \frac{\pi}{4}-} f(x) = \lim_{x\to \frac{\pi}{4}-} ax + \cos x = \frac{a\pi}{4} + \cos\left(\frac{\pi}{4}\right) = \frac{a\pi}{4} + \frac{1}{\sqrt{2}} = f\left(\frac{\pi}{4}\right)$$

$$\lim_{x\to \frac{\pi}{4}+} f(x) = \lim_{x\to \frac{\pi}{4}+} (bx+2) = \frac{b\pi}{4} + 2$$

So f is cts if and only if

$$f\left(\frac{\pi}{4}\right) = \lim_{x\to \frac{\pi}{4}-} f(x) = \lim_{x\to \frac{\pi}{4}+} f(x) \iff \frac{a\pi}{4} + \frac{1}{\sqrt{2}} = \frac{b\pi}{4} + 2 \iff (a-b)\frac{\pi}{4} = 2 - \frac{1}{\sqrt{2}}$$

$$\iff a - b = \frac{4(2\sqrt{2}-1)}{\sqrt{2}\pi}$$

D1)

(i) put $\theta = ax$

$$\lim_{x\to 0} \frac{\sin(ax)}{bx} = \lim_{\theta\to 0} \frac{\sin\theta}{\frac{b}{a}\theta} = \frac{a}{b} \lim_{\theta\to 0} \frac{\sin\theta}{\theta} = \frac{a}{b}$$

(ii) $$\lim_{t\to 0} \frac{\sin(at)}{\sin(bt)} = \lim_{t\to 0}\left(\frac{\sin(at)}{t}\cdot\frac{t}{\sin(bt)}\right) = \left(\lim_{t\to 0} \frac{\sin(at)}{t}\right)\left(\lim_{t\to 0}\left(\frac{t}{\sin(bt)}\right)\right)$$ by product law.

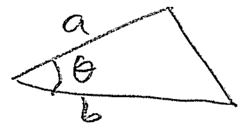
$$= \left(\lim_{t\to 0} \frac{\sin(at)}{t}\right)\left(\lim_{t\to 0} \frac{1}{\left(\frac{\sin(bt)}{t}\right)}\right) = \left(\lim_{t\to 0} \frac{\sin at}{t}\right)\frac{1}{\left(\lim_{t\to 0} \frac{\sin(bt)}{t}\right)}$$ by quotient law

$$= a \cdot \frac{1}{b}$$ by part 1

(iii) put $\theta = x+2$ then as $x \to -2$, $\theta \to 0$

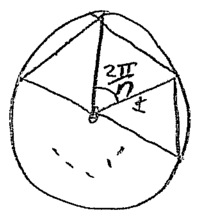
so $$\lim_{x\to -2} \frac{\sin(x+2)}{x+2} = \lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$$

D2

Recall the area of a triangle 

is $\frac{1}{2}ab\sin(\theta)$

(i) To find $A(n)$ split the n-gon into n triangles by joining the center point to the vertices:



The area of each triangle is $\frac{1}{2}\cdot 1\cdot 1\sin\left(\frac{2\pi}{n}\right) = \frac{1}{2}\sin\left(\frac{2\pi}{n}\right)$

The n-gon is made of n triangles so

$$A(n) = \frac{n}{2}\sin\left(\frac{2\pi}{n}\right)$$

(ii)

put $h = \frac{2\pi}{n}$ so as $n\to\infty$, $h\to 0$

Then

$$\lim_{n\to\infty} A(n) = \lim_{n\to\infty}\frac{n}{2}\sin\left(\frac{2\pi}{n}\right) = \lim_{h\to 0}\frac{2\pi\sin(h)}{2h} = \pi\lim_{h\to 0}\frac{\sin h}{h} = \pi$$