HW1 Partial Solutions

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2 (a) $\lim_{x\to 0_-} \frac{1}{x} = -\infty$, $\lim_{x\to 0_+} \frac{1}{x} = +\infty$, $\lim_{x\to +\infty} \frac{1}{x} = \lim_{x\to -\infty} \frac{1}{x} = 0$

(b) 0

(c) $\lim_{x\to 0_+} e^{\frac{1}{x}} = \infty$, $\lim_{x\to 0_-} e^{\frac{1}{x}} = 0$

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(4) in class

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(5) in class

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(6) if $p=q$ $\quad \lim_{x\to\infty} \frac{x^p}{x^q} = \lim_{x\to\infty} \frac{x^p}{x^p} = \lim_{x\to\infty} 1 = 1$

if $p<q$ $\quad \lim_{x\to\infty} \frac{x^p}{x^q} = \lim_{x\to\infty} \frac{1}{x^n}$, for some $n=q-p>0$

and $\lim_{x\to\infty} \frac{1}{x^n} = 0$

if $p>q$ $\quad \lim_{x\to\infty} \frac{x^p}{x^q} = \lim_{x\to\infty} x^m$, for $m=p-q>0$

and thus $\lim_{x\to\infty} x^m = \infty$

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7) $\lim_{x\to 0_+} f(x) = \lim_{x\to 0_-} f(-x) = \lim_{x\to 0_-} f(x)$, since $f(-x)=f(x)$

Then since $\lim_{x\to 0_+} f(x)$ exists, so does $\lim_{x\to 0_-} f(x)$

and they are equal, thus $\lim_{x\to 0} f(x)$ exists

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9) $\lim_{t\to 4}(3t-14) = 3\left(\lim_{t\to 4} t\right) - \lim_{t\to 4}(14) = 12-14=-2$

11) $\lim_{x\to \frac{1}{2}}(4x+1)(2x-1) = \left(\lim_{x\to\frac{1}{2}}(4x+1)\right)\left(\lim_{x\to\frac{1}{2}}(2x-1)\right) = \left[4\left(\lim_{x\to\frac{1}{2}}x\right)+\left(\lim_{x\to\frac{1}{2}}1\right)\right]\left[2\left(\lim_{x\to\frac{1}{2}}x\right)-\lim_{x\to\frac{1}{2}}1\right]=0$

17) $\lim_{x\to 3}\frac{1-x}{1+x} = \frac{\lim_{x\to 3}(1-x)}{\lim_{x\to 3}(1+x)}$ Since $\lim_{x\to 3}(1+x)\neq 0$

$= \frac{\left(\lim_{x\to 3}1\right)-\left(\lim_{x\to 3}x\right)}{\left(\lim_{x\to 3}1\right)+\left(\lim_{x\to 3}x\right)} = \frac{1-3}{1+3} = \frac{-2}{4} = -\frac{1}{2}$

27) $\lim_{x\to -4}\frac{g(x)}{x^2} = \frac{\lim_{x\to -4}g(x)}{\lim_{x\to -4}x^2} = \frac{1}{16}$

29) No. $\lim_{x\to 0}x=0$ so quotient law can not be applied.

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31) $f(x)=\frac{\sin^2 x}{x}$, $g(x) = -\frac{\sin^2(x)}{x}$ will do

32)

$L_{ab} = \lim_{x\to 0}\left(\frac{(ab)^x-1}{x}\right) = \lim_{x\to 0}\frac{a^x(b^x-1)+(a^x-1)}{x} = \left(\lim_{x\to 0}a^x\right)\left(\lim_{x\to 0}\frac{b^x-1}{x}\right)+\lim_{x\to 0}\left(\frac{a^x-1}{x}\right)$

$= 1\cdot L_b + L_a$