A1

30

$\Delta R \approx R'(20)\,\Delta T = 0.06\times 2 = 0.12\ \Omega$

so The resistance at $T=22\,^\circ C$ is approx $15.12\ \Omega$

35

$\Delta F \approx F'(35)\,\Delta S = F'(35)\times 1 = 4.88$ ft.

$\Delta F \approx F'(55)\,\Delta S = F'(55)\times 1 = 7.04$ ft

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64

$S = \frac{1}{32} V^2 \sin(2\theta)$

$V = 25$ ft/s , $34^\circ = \frac{34\pi}{180} = \frac{17\pi}{90}$ radians

so

$S(\theta) = \frac{625}{32}\sin(2\theta)$

(a)

$\Delta S \approx S'\left(\frac{17\pi}{90}\right)\Delta\theta = \frac{1250}{32}\cos\left(2\times\frac{17\pi}{90}\right)\Delta\theta \approx 14.63\,\Delta\theta$ where θ is in radians

In terms of degrees $\Delta S = 14.63\,\Delta\theta \times \frac{\pi}{180} = 0.2550\,\theta$, θ in degrees

(b)

If $\Delta\theta = 2^\circ$

$\Delta S \approx 0.51$ ft

B1 (note that in Q15 we need $|a|$ since a might be negative)

15 Let $\varepsilon > 0$ be given,

and let $0 < |x-c| < \frac{\varepsilon}{|a|}$

Then

$$|f(x) - L| = |(ax+b) - (ac+b)| = |ax - ac| = |a||x-c| < |a|\frac{\varepsilon}{|a|} = \varepsilon$$

so $\lim_{x \to c} (ax+b) = ac+b$

16

First observe that

$$|f(x) - L| = |x^2 - c^2| = |x+c||x-c|$$

and if $|x-c| < |c|$ (note that we can use 1 here, but $|c|$ turns out to be easier)

then $-|c| < x - c < |c|$

$-|c| + 2c < x+c < |c| + 2c$

so

$-|c| - |2c| < x+c < |c| + |2c|$

so $-3|c| < x+c < 3|c|$

so

$|x+c| < 3|c|$

(note you need to take care here incase c is negative)

so

Let $\varepsilon > 0$ be given

and let $|x-c| < \frac{\varepsilon}{3|c|} < |c|$

then

$$|f(x) - L| = |x^2 - c^2| = |x+c||x-c| < 3|c||x-c| < 3|c|\frac{\varepsilon}{3|c|} = \varepsilon$$

so $\lim_{x \to c} x^2 = c^2$

81

17

$|f(x)-L| = \left|\frac{1}{x}-\frac{1}{c}\right| = \left|\frac{x-c}{xc}\right| = \left|\frac{1}{xc}\right||x-c|$

let $|x-c|<|c|$

then $-|c| < x-c < |c|$

so $-|c|+c < x < |c|+c$

then $-2|c| < x < 2|c|$

so $|x| < 2|c|$

so $\frac{1}{|x|} < \frac{1}{2|c|}$

so $\frac{1}{|xc|} = \frac{1}{|x|}\frac{1}{|c|} < \frac{1}{2|c|}\cdot\frac{1}{|c|} = \frac{1}{2|c|^2}$

Then let $\varepsilon>0$ be given and let $|x-c| < 2|c|^2\varepsilon$

$|f(x)-L| = \left|\frac{1}{x}-\frac{1}{c}\right| = \left|\frac{1}{xc}\right||x-c| < \frac{1}{2|c|^2}|x-c| < \frac{2|c|^2\varepsilon}{2|c|^2} = \varepsilon$

so $\lim_{x\to c}\frac{1}{x} = \frac{1}{c}$.