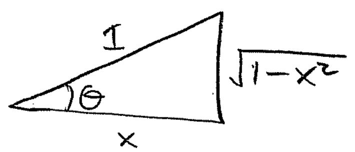
A2

38 Let $f(x) = \cos(x)$ so $f^{-1}(x) = \cos^{-1}(x)$

Then

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))} = \frac{-1}{\sin(\cos^{-1}(x))}$$

Let $\theta = \cos^{-1}(x)$ so $\cos(\theta) = x = \frac{x}{1}$ giving



then $\sin(\cos^{-1}(x)) = \sin(\theta) = \sqrt{1-x^2}$

so

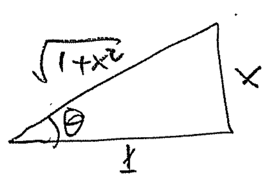
$$\frac{d}{dx}\cos^{-1}(x) = \frac{-1}{\sin(\cos^{-1}(x))} = \frac{-1}{\sqrt{1-x^2}}$$

39

Let $f(x) = \tan(x)$

Then $(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))} = \frac{1}{\sec^2(\tan^{-1}(x))} = \left(\frac{1}{\sec(\tan^{-1}(x))}\right)^2 = \cos^2(\tan^{-1}(x))$

Let $\theta = \tan^{-1}(x)$ so $\tan(\theta) = x$. This gives



so $\cos(\tan^{-1}(x)) = \cos(\theta) = \frac{1}{\sqrt{1+x^2}}$

So $\frac{d}{dx}\tan^{-1}(x) = \cos^2(\tan^{-1}(x)) = \frac{1}{1+x^2}$

B2

45 Let $f(x) = (2x+1)(4x^2)\sqrt{x-9}$

Then
$$\ln(f(x)) = \ln\left((2x+1)(4x^2)\sqrt{x-9}\right) = \ln(2x+1) + \ln(4x^2) + \frac{1}{2}\ln(x-9)$$

so $$\frac{d}{dx} f(x) = \frac{2}{2x+1} + \frac{8x}{4x^2} + \frac{1}{2(x-9)}$$

and
$$f'(x) = f(x)\frac{d}{dx}\ln(f(x)) = (2x+1)(4x^2)\sqrt{x-9}\left(\frac{2}{2x+1} + \frac{2}{x} + \frac{1}{2(x-9)}\right)$$

B3

36 $$\frac{d}{dx} x^{\cos(x)} = \frac{d}{dx} e^{\ln(x^{\cos(x)})} = \frac{d}{dx} e^{\cos(x)\ln(x)}$$

$$= \frac{d}{dx}(\cos(x)\ln(x))\, e^{\cos(x)\ln(x)} = \left(\frac{\cos(x)}{x} - \sin(x)\ln(x)\right) x^{\cos(x)}$$
