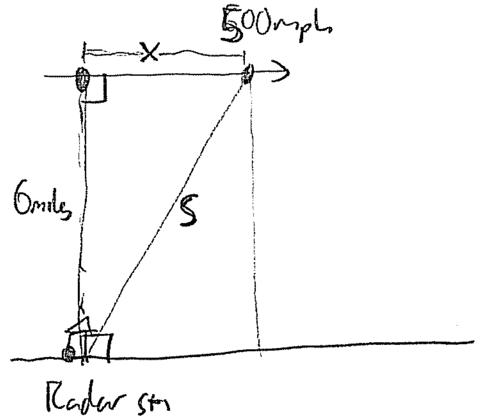
§3.4

15



Let $x(t)$ horizontal distance from radar station at time t
$s(t)$ distance from radar station at time t

$$s(t)^2 = x(t)^2 + 36$$

$$\frac{dx}{dt} = 500$$

we want to find $\left.\frac{ds}{dt}\right|_{t=1/2}$ (and $\left.\frac{ds}{dt}\right|_{t=0}$ for part (b))

$$2\, s(t) \frac{ds}{dt} = 2\, x(t) \frac{dx}{dt}$$

$$\Rightarrow \frac{ds}{dt} = \frac{x(t)}{s(t)} \frac{dx}{dt}$$

(a) After 30 mins i, $x(0.5) = 250$ miles, $s(0.5) = \sqrt{6^2 + (250)^2}$

So $\frac{ds}{dt} = \frac{250}{\sqrt{6^2+(250)^2}} \times 500 = 499.86$ mph

(b) when $t=0$: $x(0) = 0$, $s(0) \neq 0$

so $\left.\frac{ds}{dt}\right|_{t=0} = 0$ mph

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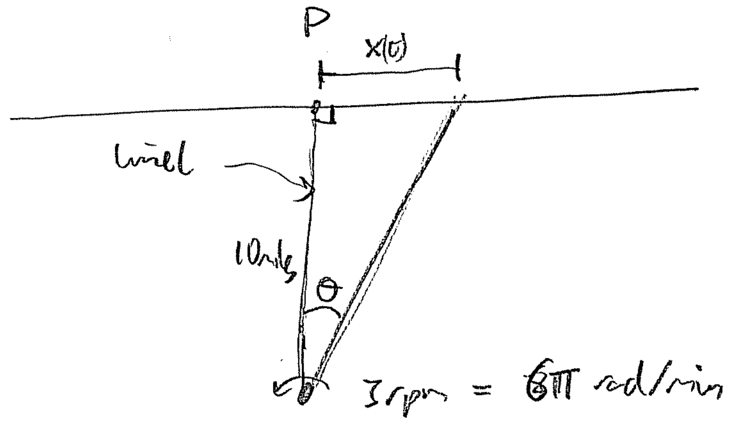

Let l be the line between spotlight and wall that is perpendicular to the wall.
Let P be the point where l meets the wall
Then our variables are
$x(t)$ = distance of dot from P at time t
$\theta(t)$ = angle between l and the light beam.

$\frac{d\theta}{dt} = 6\pi$ rad/min $(= 3\text{rpm})$

we want to find $\left.\frac{dx}{dt}\right|_{\theta=\pi/6}$

$10\tan(\theta(t)) = x(t)$

so

$10\sec^2(\theta(t))\frac{d\theta}{dt} = \frac{dx}{dt}$

so if $\theta(t) = \frac{\pi}{6}$

$\frac{dx}{dt} = 60\pi\sec^2\left(\frac{\pi}{6}\right) = 80\pi$

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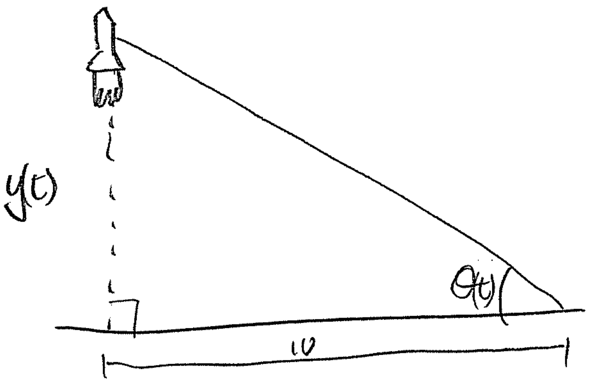


given $\frac{dy}{dt} = 800 \text{ mph}$

want to find $\left.\frac{d\theta}{dt}\right|_{t=3\text{ mins} = \frac{1}{20}h}$

let $y(t)$ = height at time t

$\theta(t)$ = angle at time t

we see

$$\tan\theta(t) = \frac{y(t)}{10}$$

so

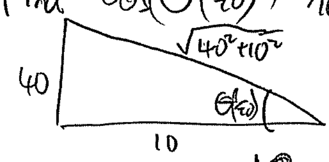
$$\sec^2(\theta(t))\frac{d\theta}{dt} = \frac{1}{10}\frac{dy}{dt}$$

so $$\frac{d\theta}{dt} = \frac{\cos^2(\theta(t))}{10}\frac{dy}{dt}$$

so $$\left.\frac{d\theta}{dt}\right|_{t=\frac{1}{20}} = \frac{\cos^2(\theta(\frac{1}{20}))}{10}\cdot 800$$

To find $\cos(\theta(\frac{1}{20}))$ note $y(\frac{1}{20}) = \frac{800}{20} = 40$

so



so $$\cos(\theta(\tfrac{1}{20})) = \frac{10}{\sqrt{40^2+10^2}}$$

Then

$$\left.\frac{d\theta}{dt}\right|_{t=\frac{1}{20}} = \frac{800}{(\sqrt{40^2+10^2})^2} = \frac{800}{1700} = \frac{80}{17} \text{ rad/hr.}$$

A3

Q30 $S(t) = 0 + 200t - \frac{1}{2}gt^2$

$V(t) = 200 - gt$

pebble reaches max height when $V(t) = 0$ so when $t = \frac{200}{g}$

So max height is $S\left(\frac{200}{g}\right) = \frac{(200)^2}{g} - \frac{1}{2}g\frac{200^2}{g^2} = \frac{(200)^2}{2g} = \frac{20000}{g}$ m

To find max velocity we need to find max value of $v(t) = 200 - gt$.

Note $0 \le t \le \frac{400}{g}$. $V(t)$ has no critical points $V(0) = 200$, $V\left(\frac{400}{g}\right) = -200$

so max speed is ± 200 m/s

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32 $S(t) = S_0 + 0t - \frac{1}{2}gt^2 = S_0 - \frac{1}{2}gt^2$

$S(3) = 0 \Rightarrow 0 = S_0 - \frac{9}{2}g \Rightarrow S_0 = \frac{9}{2}g$ m

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34 $S(t) = V_0 t - \frac{1}{2}gt^2$

$S(T) = 0 \Rightarrow V_0 T - \frac{1}{2}gT^2 = 0 \Rightarrow V_0 = \frac{1}{2}gT$

max height is attained when $V(t) = 0$

$V(t) = V_0 - gt = \frac{1}{2}gT - gt$.

$V(t) = 0 \Leftrightarrow \frac{1}{2}gT = gt \Leftrightarrow t = \frac{1}{2}T$

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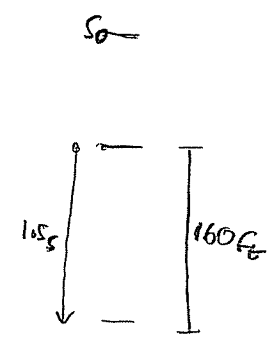
35 OK, so there really isn't enough information given in this question. In particular, although we know the observer was on the 10th floor, we don't know the height of the window or his eye level.

Let's assume it is 160ft so that the bucket takes 1.5s to fall 16ft.



First find the speed of the bucket when it passes the observer:

V = speed of bucket when observer sees it

so $S(t) = 160 + Vt - \frac{1}{2}gt^2$

$S(1.5) = 0 \Rightarrow 160 + \frac{3}{2}V - \frac{1}{2}g\left(\frac{3}{2}\right)^2 = 0 \Rightarrow 160 + \frac{3}{2}V - \frac{99}{4} = 0 \Rightarrow V = \frac{2}{3}\left(\frac{99}{4} - 160\right)$

$= \frac{33}{2} - \frac{80}{3}$ ft/s

$= \frac{99 - 160}{6}$ ft/s

Now if H is the height the bucket fell from

$S(t) = H + 0t - \frac{1}{2}gt^2$ and $V(t) = -gt$

35 cont

$$V(t) = V = \frac{99-160}{6} \Rightarrow \frac{99-160}{6} = -gt \Rightarrow t = \frac{(160-99)}{6g} \text{ seconds}$$

So after $\frac{160-99}{6g}$ s the ladder was 160ft up

so $$160 = H - \frac{1}{2} g \left(\frac{160-99}{6g} \right)^2$$

solve for H and translate into floors (remember g is in ft/s^2)

37.

Average of instantaneous:

$$= \frac{V(t_1) + V(t_2)}{2} = \frac{(V_0 - gt_1) + (V_0 - gt_2)}{2} = \frac{2V_0 - g(t_2+t_1)}{2} = V_0 - \frac{g}{2}(t_2+t_1)$$

Av. velocity over $[t_1, t_2]$

$$= \frac{S(t_2) - S(t_1)}{t_2 - t_1} = \frac{(S_0 + V_0 t_2 - \frac{1}{2}gt_2^2) - (S_0 + V_0 t_1 - \frac{1}{2}gt_1^2)}{t_2 - t_1}$$

$$= \frac{V_0(t_2 - t_1) - \frac{1}{2}g(t_2^2 - t_1^2)}{t_2 - t_1} = V_0 - \frac{1}{2}g\frac{(t_2-t_1)(t_2+t_1)}{t_2-t_1} = V_0 - \frac{g}{2}(t_2+t_1)$$