HW3 PARTIAL SOLUTIONS

A1

1) $f(x) = x^3 + x$

First show $f(x)$ is cts:
x is cts so x^3 is cts by product law,
Therefore $x^3 + x$ is cts by sum law.

Now $f(1) = 0$
$f(2) = 10$

So by IVT, $f(c) = 9$ for some c in $[1,2]$

7) Let $f(x) = \sqrt{x} + \sqrt{x+1}$
we need to show that $f(c) = 2$ for some c.
Since $x, x+1, \sqrt{x}$ are cts, $\sqrt{x}$ is cts for $x \geq 0$, $\sqrt{x+1}$ is cts for $x \geq -1$ by composition law.
So $\sqrt{x} + \sqrt{x+1}$ is cts by sum law.

$f(0) = 1$
$f(4) = \sqrt{4} + \sqrt{5} = 2 + \sqrt{5} > 2$
so $f(c) = 2$, for some c in $[0,4]$

8) let $f(x) = \sin(nx) - \cos(x)$
we need to show $f(c) = 0$ for some c in $[0,\pi]$
First show $f(x)$ is cts: $\sin(x)$ and $\cos(x)$ are cts, and nx is cts as it is a polynomial,
so $\sin(nx)$ is cts by composition law, $-\cos(x)$ is cts by const. mult law.
so $\sin(nx) - \cos(x)$ is cts by sum law.

$f(0) = -1$
$f(\pi) = \sin(n\pi) - \cos(\pi) = 0 - (-1) = 0$
so by IVT $f(c) = 0$ for some c in $[0,\pi]$

15) e^x is cts, so $\ln(x)$ is cts as it is the inverse of e^x
Then $e^x + \ln(x)$ is cts by sum law.

let $f(x) = e^x + \ln(x)$

$f(1) = e^1 + \ln(1) = 1 + 0 = 1$

Observe that $f(0)$ does not exist as $\ln(0)$ is not defined.

So choose a number close to zero

$f(0.0001) = e^{0.0001} + \ln(0.0001) \approx 1.0001 - 9.2103 < 0$

So by IVT $f(c) = 0$ for some c in $(0,1)$

A2)

5) $-1 \leq \cos(x) \leq 1$ so $-1 \leq \cos(\frac{1}{x}) \leq 1$ so $-|x| \leq x\cos(\frac{1}{x}) \leq |x|$

$\lim_{x \to 0} |x| = 0 = \lim_{x \to 0} -|x|$ so by squeeze thm $\lim_{x \to 0} x\cos(\frac{1}{x}) = 0$

6) $-1 \leq \sin(x) \leq 1$ so $-1 \leq \sin(\frac{1}{x}) \leq 1$ so $-x^2 \leq x^2\sin(\frac{1}{x}) \leq x^2$

(note $x^2 \geq 0$). Then $\lim_{x \to 0} x^2 = 0 = \lim_{x \to 0} -x^2$ so by squeeze Thm

$\lim_{x \to 0} x^2 \sin(\frac{1}{x}) = 0$