MATH 125 - 104/109 HOMEWORK FOR WEEK 1
TOPICS COVERED: ESTIMATING LIMITS NUMERICALLY AND LIMIT LAWS.

1. Do Questions **1, 2, 7, 8, 21 and 31** of Rogawski Section 2.2 p76-77.

2. By using your calculator, estimate the following limits or state that the limit does not exist.

(a) $\lim\limits_{x \to 0_\pm} \frac{1}{x}$ and $\lim\limits_{x \to \pm\infty} \frac{1}{x}$. Sketch the graph of $f(x) = \frac{1}{x}$

(b) $\lim\limits_{x \to 1} \frac{\sin(x)}{x^3-1}$

(c) $\lim\limits_{x \to -\infty} \frac{x^2+1}{x^3}$

(e) $\lim\limits_{x \to 0_+} e^{\frac{1}{x}}$ and $\lim\limits_{x \to 0_-} e^{\frac{1}{x}}$

3. Do Questions **44, 45 and 46** of Rogawski Section 2.2 p77.

4. The *greatest integer function* (or the *floor function*) will round a number **down** to the nearest integer. It can be defined formally by $[x] = n$ where n is the unique integer (i.e. "whole number") such that $n \leq x < n + 1$. For example, $[2.5] = 2$, and $[-\frac{1}{2}] = -1$.

(a) Calculate $[1.45]$, $[0.2]$, $[\pi]$ and $[2]$.

(b) Plot the graph of $y = [x]$.

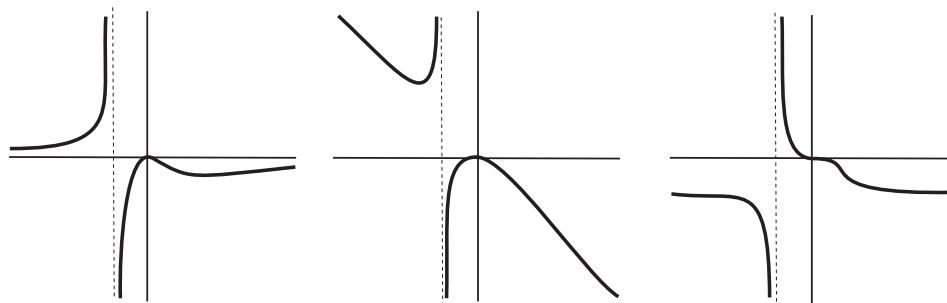**(c)** For which values of c does $\lim\limits_{x \to c_-} [x]$ exist?

(d) For which values of c does $\lim\limits_{x \to c_+} [x]$ exist?

(e) For which values of c does $\lim\limits_{x \to c}[x]$ exist?

5. The graphs below are the graphs of

$$y = \frac{-x^2}{x^3 + 1}, \quad y = \frac{-x^3}{x^3 + 1} \quad \text{and} \quad y = \frac{-x^4}{x^3 + 1}.$$

By considering the asymptotic behaviour of the functions, match each graph with its function.

6. Consider the function $Q(x) = \frac{x^p}{x^q}$, where p and q are positive integers. We know that either $p = q$, $p < q$ or $p > q$. In each of the three cases $p = q$, $p < q$ and $p > q$ find the limit $\lim_{x \to \infty} Q(x)$.

7. A function $f(x)$ is said to be *even* if $f(x) = f(-x)$. Suppose that $f(x)$ is an even function and that $\lim_{x \to 0_+} f(x)$ exists. Explain why it then follows that $\lim_{x \to 0} f(x)$ exists.

You should use the limit laws to answer the remaining questions.

8. Do Questions **9, 11, 13, 17, 25, 27 and 29** of Rogawski Section 2.3 p82.

9. Do Questions **31 and 32** of Rogawski Section 2.3 p82.