A3

Q29

let $G(x) = \int_0^{x^2} \sin^2(t)\,dt$

and $A(x) = \int_0^{x} \sin^2(t)\,dt$

then $A'(x) = \sin^2(x)$

$G(x) = A(x^2)$

so $G'(x) = A'(x^2) \cdot 2x = 2x \sin^2(x^2)$

Q33

$\frac{d}{dx} \int_{x^3}^{0} \sin^2(t)\,dt = -\frac{d}{dt} \int_0^{x^3} \sin^2 t\,dt$

$G(x) = \int_0^{x^3} \sin^2(t)\,dt$

$A(x) = \int_0^{x} \sin^2(t)\,dt$

$G(x) = A(x^3)$

so

$G'(x) = A'(x^3)\,3x^2 = 3x^2 \cdot \sin^2(x^3)$

and

$\frac{d}{dx} \int_{x^3}^{0} \sin^2(t)\,dt = -3x^2 \sin^2(x^3)$

B1
Q40

$\lim_{x\to 0} a^x - 1 = 1 - 1 = 0$ and $\lim_{x\to 0} x = 0$ so l'Hopital's applies

$$\lim_{x\to 0} \frac{a^x - 1}{x} = \lim_{x\to 0} \frac{\ln(a)a^x}{1} = \ln(a)$$

Q46

$$\lim_{x\to 0_+} x^{\sin(x)} = \lim_{x\to 0_+} e^{\ln(x^{\sin x})} = \lim_{x\to 0_+} e^{(\sin x \ln(x))}$$

$$= e^{\lim_{x\to 0_+}(\sin(x)\ln(x))}$$

$$\lim_{x\to 0_+} \sin(x)\ln(x) = \lim_{x\to 0_+} \frac{\ln(x)}{\frac{1}{\sin(x)}}$$

$\lim_{x\to 0_+} \ln(x) = -\infty$, $\lim_{x\to 0_+} \frac{1}{\sin(x)} = +\infty$ so l'Hopital's applies

Then

$$\lim_{x\to 0_+} \frac{\ln(x)}{1/\sin(x)} = \lim_{x\to 0_+} \frac{\left(\frac{1}{x}\right)}{\frac{-\cos(x)}{\sin^2(x)}} = \lim_{x\to 0_+} \frac{-\sin^2(x)}{x\cos(x)}$$

$\lim_{x\to 0_+} -\sin^2(x) = 0$ and $\lim_{x\to 0_+} x\cos(x) = 0$ so l'Hopital's applies again

so

$$\lim_{x\to 0_+} \frac{-\sin^2 x}{x\cos(x)} = \lim_{x\to 0} \frac{+2\cos(x)\sin(x)}{\cos x - x\sin(x)} = 0$$

So

$$\lim_{x\to 0_+} x^{\sin(x)} = e^{\lim_{x\to 0_+}(\sin(x)\ln(x))} = e^0 = 1$$

B1

Q9 The two functions are ~~diff~~ diff and $\lim_{x\to 0}(\cos x - \cos^2(x)) = 0$

and $\lim_{x\to 0}\sin(x) = 0$ so L'Hopital's applies.

Then

$$\lim_{x\to 0}\left(\frac{\cos x - \cos^2 x}{\sin(x)}\right) = \lim_{x\to 0}\frac{-\sin(x) + 2\cos x\sin(x)}{\cos(x)} = \frac{0}{1} = 0$$

Q15 $\lim_{x\to\infty}(x) = \infty$ and $\lim_{x\to\infty} e^x = \infty$. L'Hopital's applies

so

$$\lim_{x\to\infty}\frac{x^2}{e^x} = \lim_{x\to\infty}\frac{2x}{e^x}.$$

since $\lim_{x\to\infty} 2x = \infty$, L'Hopital's applies again

so

$$\lim_{x\to\infty}\frac{2x}{e^x} = \lim_{x\to\infty}\frac{2}{e^x} = 0$$

Q22

$$\lim_{x\to 2}(e^{x^2} - e^4) = e^4 - e^4 = 0$$

$$\lim_{x\to 2}(x-2) = 0$$

Functions are both diff, so L'Hopital's applies

$$\lim_{x\to 2}\frac{e^{x^2} - e^4}{x-2} = \lim_{x\to 2}\frac{2xe^{x^2}}{1} = 4e^4$$

BI

38

$$\lim_{x\to 0} \sin(x) - x\cos(x) = 0$$

$$\lim_{x\to 0} x - \sin(x) = 0$$

so L'Hopital's applies

$$\lim_{x\to 0} \frac{\sin(x) - x\cos(x)}{x - \sin(x)} = \lim_{x\to 0} \frac{x\sin(x)}{1-\cos(x)}$$

$\lim_{x\to 0} x\sin(x) = 0$ $\lim_{x\to 0} (1-\cos(x)) = 0$ so L'Hopital's applies again

$$\lim_{x\to 0} \frac{x\sin(x)}{1-\cos(x)} = \lim_{x\to 0} \frac{\sin(x) + x\cos(x)}{\sin(x)}$$

$\lim_{x\to 0} (\sin x + x\cos x) = 0$ and $\lim_{x\to 0} \sin(x) = 0$ so L'Hopital's applies again

$$\lim_{x\to 0} \frac{\sin(x) + x\cos x}{\sin(x)} = \lim_{x\to 0} \frac{\cos x + \cos x - x\sin x}{\cos x} = 2$$