

CZ QSS

Area of triangle:

Area of triangle = $\frac{1}{2}$ base \times height

$$\text{base} = b - a$$

$$\text{height} = y\left(\frac{a+b}{2}\right) = \left(\frac{a+b}{2} - a\right)\left(b - \frac{a+b}{2}\right) = \left(\frac{b-a}{2}\right)^2$$

$$\text{So area of triangle is } \frac{1}{8}(b-a)^3$$

$$\text{Area under arch} = \int_a^b (x-a)(b-x) dx = \int_a^b (bx - x^2 - ab + ax) dx$$

$$= \left(\frac{b}{2}x^2 - \frac{1}{3}x^3 - abx + \frac{a}{2}x^2 \right) \Big|_a^b$$

$$= \dots \text{ algebra } \dots = \frac{1}{6}(b-a)^3$$

and

$$\frac{1}{8}(b-a)^3 = \frac{4}{3} \cdot \frac{1}{8}(b-a)^3$$