A1

a) $$f'(x) = \frac{x^4 - x^2 + 2}{(x^2-1)^2}$$

$f'(x) = 0 \iff x^4 - x^2 + 2 = 0$ This is a quadratic in x^2 and $(-1)^2 - 4\times1\times2 < 0$ so there are no real solutions

(i) $f(2) = \frac{4}{3}$ $f(3) = \frac{21}{8}$ max $f(3) = \frac{21}{8}$, min $f(2) = \frac{4}{3}$

(ii) $\lim_{x\to-\infty} f(x) = -\infty$, $f(0) = 0$ so no min max is $f(0) = 0$

(iii) $\lim_{x\to1_+} f(x) = -\infty$ $f(4) = \frac{56}{15}$ no min max is $f(4) = \frac{56}{15}$

(iv) $f(x)$ is not defined on $(-\infty, 1)$ but is defined on $(-\infty,-1)\cup(-1,1)$ on this domain $\lim_{x\to-\infty} = -\infty$, $\lim_{x\to1_-} = \infty$ so no max no min

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(b) $$f'(x) = \frac{-4x}{(x^2-1)^2}$$

$f'(x) = 0 \iff x = 0$

(i) max $f(2)$, min $f(3)$

(ii) $\lim_{x\to\infty} f(x) = 2$, max is $f(2)$, no min

(iii) $\lim_{x\to1_+} f(x) = -\infty$, $\lim_{x\to1_-} = -\infty$ no min max is $f(0) = 0$

(iv) $\lim_{x\to0+} f(x) = 0$, $\lim_{x\to\frac{1}{2}-} = f(\frac{1}{2})$, $\lim_{x\to1_+} f(x) = \infty$, $\lim_{x\to\infty} f(x) = 2$ no max no min

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(c) $$f'(x) = \frac{x(2-3x)}{2\sqrt{x^2-x^3}}$$ $f'(x) = 0 \iff x = 0$ or $x = \frac{2}{3}$

(i) max: $f(-1)$, min: $f(0)$

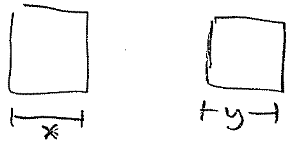
(ii) $\lim_{x\to-\infty} f(x) = \infty$, no max, min $= f(0)$

(iii) $\lim_{x\to1_-} f(x) = 0$, max is $f(-1) = \sqrt{2}$ min is $f(0) = 0$

(iv) $\lim_{x\to-\infty} f(x) = \infty$, $\lim_{x\to1_-} f(x) = 0$ no max, min is $f(0) = 0$

B1

2.



A = area
x = width of square
y = width of other square
$A(x,y) = x^2+y^2$

$4x+4y=100$

$0 \leq x \leq \frac{100}{4} = 25$

$y = 25 - x$

so $A(x) = x^2 + (25-x)^2 = x^2 + 625 - 50x + x^2 = 2x^2 - 50x + 625$

maximize $A(x)$ over $[0, 25]$

$A'(x) = 4x - 50$

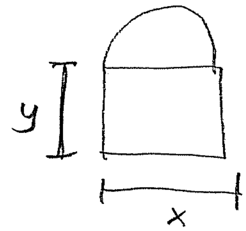
$A'(x) = 0 \iff x = \frac{50}{4} = \frac{25}{2}$

$A(0) = 625 \text{ in}^2$
$A(\frac{25}{2}) = 312.5 \text{ in}^2$
$A(25) = 625 \text{ in}^2$

so area is minimized when $x = y = \frac{25}{2}$
so wire is cut in half.

a) Let x = width , y = height

Then length of semicircle is $\frac{\pi x}{2}$



$$A = xy + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2 = xy + \frac{\pi}{8}x^2$$

$$\text{Perim.} = x + 2y + \frac{\pi x}{2} = 600$$

$$\Rightarrow \quad y = 300 - \frac{x}{2} - \frac{\pi x}{4}$$

So

$$A = 300x - \left(\frac{1}{2} + \frac{\pi}{4}\right)x^2 + \frac{\pi}{8}x^2$$

and $0 \leq x \leq \frac{600}{1 - \frac{\pi}{2}}$ (= perim. when $y=0$)

$$A'(x) = 300 - \left(1 + \frac{\pi}{2}\right)x + \frac{\pi}{4}x$$

$$A'(x) = 0 \Leftrightarrow x = \frac{300}{1 + \frac{\pi}{4}}$$

$A(0) = 0 \text{ ft}^2$

$A\left(\frac{300}{1+\frac{\pi}{4}}\right) \approx 25204.5 \text{ ft}^2$

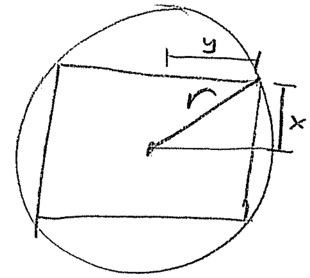
$A\left(\frac{600}{1+\frac{\pi}{2}}\right) \approx 21390.8 \text{ fts}$

So max area has dimension $x = \frac{300}{1 + \frac{\pi}{4}}$

(16)

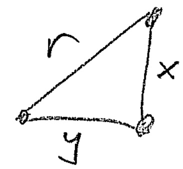
let $x = \frac{1}{2}$(height of rectangle)

$y = \frac{1}{2}$(width of rectangle)



$$r^2 = x^2+y^2 \Rightarrow y = \sqrt{r^2-x^2}$$

$$A = 4xy$$



$$0 \le x \le r$$

$$A = 4xy = 4x\sqrt{r^2-x^2}$$

$$A'(x) = \frac{4(r^2-x^2)-4x^2}{\sqrt{r^2-x^2}}$$

$A'(x) = 0 \Leftrightarrow r^2 = 2x^2 \Rightarrow x = \frac{r}{\sqrt{2}}$ (since $x>0$)

$A(0) = 0$

$A(r) = 0$

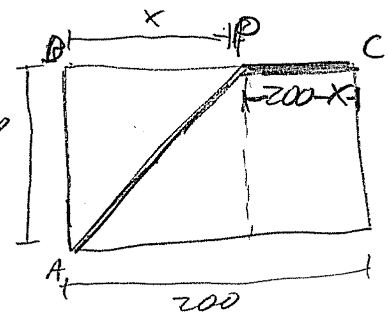
So max is $A\left(\frac{r}{\sqrt{2}}\right) = 2r^2$

36)

(a)

C = cost of pipe

x = length of PB



length of the diagonal is

$\sqrt{100^2+x^2}$

so

$$\text{cost} = f(x) = \begin{cases} 30\sqrt{100^2+x^2} + 15(200-x) & \text{if } 0<x<200 \\ 15(200+100) = 4500 & \text{if } x=0 \end{cases}$$

(b)

$$f'(x) = \frac{30x}{\sqrt{100^2+x^2}} - 15$$

$f'(x) = 0 \Leftrightarrow x = \pm\frac{100}{\sqrt{3}}$ but $x>0$ so $x = \frac{100}{\sqrt{3}}$

$f(200) = \$6708$

$f(0) = \$4500$

$f\left(\frac{100}{\sqrt{3}}\right) \approx \$5598$

so most economical is when $x=0$

<u>if</u> cost along sides is \$24 ft

$$f(x) = \begin{cases} 30\sqrt{100^2+x^2} + 24(200-x) & 0<x<200 \\ 24(300) = 72000 \end{cases}$$

$$f'(x) = \frac{30x}{\sqrt{100^2+x^2}} - 24$$

$f'(x) = 0 \Leftrightarrow x = \frac{400}{3}$

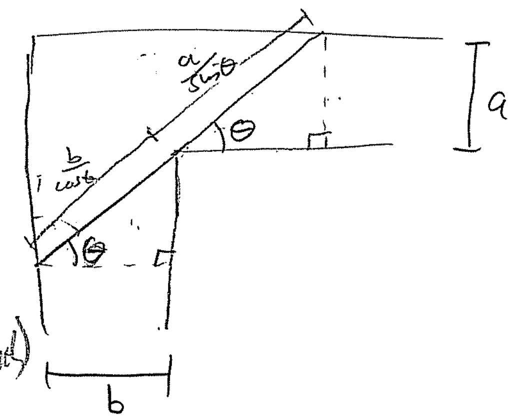
$f(0) = \$72000$

$f\left(\frac{400}{3}\right) = \$6600$

$f(200) = \$6708$ So least cost is when $x = \frac{400}{3}$

60)

L = length of pole
θ = angle shown in figure



$$L(\theta) = \frac{a}{\sin(\theta)} + \frac{b}{\cos\theta}$$

Need to minimize $L(\theta)$ (since longest pole fits in shortest length)

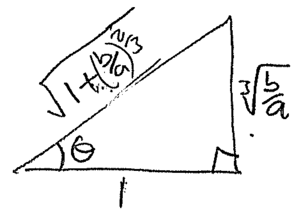
assume
$$0 < \theta < \frac{\pi}{2}$$

$$L'(\theta) = \frac{b\sin(\theta)}{\cos^2(\theta)} - \frac{a\cos(\theta)}{\sin^2\theta}$$

$$L'(\theta) = 0 \iff b\sin^3(\theta) = a\cos^3(\theta) \iff \sqrt[3]{b}\,\sin\theta = \sqrt[3]{a}\,\cos\theta$$

$$\iff \tan\theta = \sqrt[3]{\frac{b}{a}}$$

so θ is the angle given in the triangle



$$\lim_{\theta\to 0^+} L(\theta) = \infty$$

$$\lim_{\theta\to \frac{\pi}{2}} = \infty$$

So min is at the critical point

using the triangle

$$L(\theta) = \frac{a\sqrt{1+\left(\frac{b}{a}\right)^{2/3}}}{\sqrt[3]{\frac{b}{a}}} + b\sqrt{1+\left(\frac{b}{a}\right)^{2/3}} = \left(a^{\frac{2}{3}}+b^{\frac{2}{3}}\right)^{\frac{3}{2}}$$