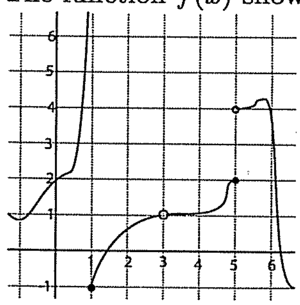
1) The function $f(x)$ shown in the figure below has three points of discontinuity.



(a) write down that values of the following limits or state that they do not exist. [3]

$$\lim_{x\to 3} f(x) = 1 \quad , \lim_{x\to 1_+} f(x) = -1 \quad , \lim_{x\to 5} f(x) = \text{DNE} ,$$

1 1 1

(b) For each of the three discontinuities, state whether the discontinuity is a jump, removable, infinite or none of these. [3]

$x=1$ INFINITE 1

$x=3$ NONE 1

$x=5$ JUMP 1

2) (a) Using the facts that $\cos(x)$, e^x and x are continuous functions and the laws of continuity, show that $x\cos(\pi e^x)$ is a continuous function. **State clearly which laws of continuity you are using in your solution.** [4]

x, $\cos(x)$, e^x are cts 1

πe^x cts by ~~const~~ const mult rule 1

$\cos(\pi e^x)$ cts by composition 1

$x\cos(\pi e^x)$ cts by prod rule 1

(b) Using part (a) or otherwise, find $\lim_{x\to 0} x\cos(\pi e^x)$. **Justify your answer.** [2]

0 1

Since $x\cos(\pi e^x)$ is cts 1

3) Using any of the results and methods (except for numerical estimates) you have seen in the course, determine the following limits. **Justify your answer.**

(a) $\lim_{x\to 0} \frac{1+x}{1-x^2}$. [2]

$\frac{1+x}{1-x^2}$ is cts (1)

so

$\lim_{x\to 0} \frac{1+x}{1-x^2} = 1$ (1) by evaluation

(b) $\lim_{x\to 2} \frac{x^2-3x+2}{x-2}$. $= \lim_{x\to 2} \frac{(x-2)(x-1)}{x-2}$ (1) [3]

$= \lim_{x\to 2} (x-1)$ (1)

$= 1$ (1)

since $x-1$ is cts. (1)

(c) $\lim_{x\to 4} \left(\frac{1}{\sqrt{x}-2} - \frac{4}{x-4}\right)$. $= \lim_{x\to 4} \left(\frac{\sqrt{x}+2}{x-4} - \frac{4}{x-4}\right)$ (1) [4]

$= \lim_{x\to 4} \left(\frac{\sqrt{x}-2}{(\sqrt{x}+2)(\sqrt{x}-2)}\right)$ (1)

$= \lim_{x\to 4} \left(\frac{1}{\sqrt{x}+2}\right) = \frac{1}{4}$ (1)

since $\frac{1}{\sqrt{x}+2}$ is cts. (1)

(d) $\lim\limits_{x\to -1} \frac{\sin(2x+2)}{2(x+1)}$. [3]

put $\theta = 2x+2$ (1)

$$\text{limit} = \lim_{\theta\to 0} \frac{\sin\theta}{\theta} \quad (1)$$

$$= 1 \quad (1)$$

(e) $\lim\limits_{x\to 0} \frac{e^x - e^{2x}}{1-e^x}$. $= \lim\limits_{x\to 0} \frac{e^x(1-e^x)}{1-e^x}$ (1) [3]

$$= \lim_{x\to 0} e^x = 1 \quad (1)$$

by evaluation since e^x is cts (1)

(f) $\lim\limits_{h\to 0} \frac{h^2}{\sin^2(h)}$. [4]

$$= \left(\lim_{h\to 0} \frac{h}{\sin h}\right)^2$$ product law (1)

$$= \left(\lim_{h\to 0} \frac{1}{\left(\frac{\sin h}{h}\right)}\right)^2 \quad (1)$$

$$= \left(\frac{1}{\lim\limits_{h\to 0}\left(\frac{\sin h}{h}\right)}\right)$$ quotient law (1)

$$= 1 \quad (1)$$

4. **(a)** Use the limit laws and the facts that $\lim_{x\to\infty} k = k$ and $\lim_{x\to\infty} 1/x = 0$ to calculate

$$\lim_{x\to\infty} \frac{x-1}{x^2-3x+2}.$$

[3]

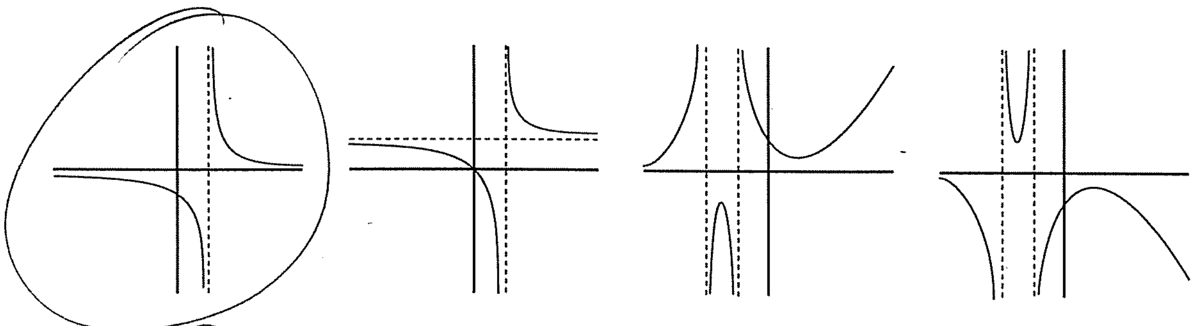
$$= \lim_{x\to\infty}\left(\frac{x-1}{x^2-3x+2}\,\frac{\frac{1}{x^2}}{\frac{1}{x^2}}\right) \quad (1)$$

$$= \lim_{x\to\infty}\left(\frac{\frac{1}{x}-\frac{1}{x^2}}{1-\frac{3}{x}+\frac{2}{x^2}}\right) \quad (1)$$

$= 0$ by limit laws (1)

(b) One of the graphs below is a plot of $y = \frac{x-1}{x^2-3x+2}$. Circle this graph.

[2]



(2)

5) (a) Find **all** of the solutions in $[0, 2\pi]$ to the equation

$$\cos(x)\sin(x) + \cos(x) = 0.$$

(Hint: start by factorizing.)

[4]

$\Leftrightarrow \cos(x)(\sin(x)+1) = 0$ 1

$\Leftrightarrow \cos(x) = 0$ or $\sin(x) = -1$ 1

$\Leftrightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$ or $x = \frac{3\pi}{2}$

1 1

(b) Let $f(x) = \tan(e^{5-6x})$. The function $f(x)$ can be expressed as a composite of three functions $f(x) = (p \circ q \circ r)(x)$. Write down the functions p, q, r.

$$p(x) = \tan(x), \; q(x) = e^x, \; r(x) = 5 - 6x$$

1 1 1

[3]

6) (a) State what it means for a function $f(x)$ to be continuous at a point c. [3]

$f(x)$ is defined in an interval containing c

$\lim_{x \to c} f(x) = f(c)$

(3) if correct

(1) if partially correct

(b) For what value of c is the function $f(x) = \begin{cases} 2x+9 & \text{if } x \le 3 \\ -4x+c & \text{if } x > 3 \end{cases}$ continuous at x=3? Justify your answer. [4]

cts at 3 iff

$\lim_{x \to 3_-} f(x) = \lim_{x \to 3_+} f(x) = f(3) = 15$ (1)

$\lim_{x \to 3_-} f(x) = \lim_{x \to 3_-} 2x+9 = 15$ (1)

$\lim_{x \to 3_+} f(x) = \lim_{x \to 3_+} -4x+c = -12+c$ (1)

cts iff $15 = -12+c$

if $c = 27$ (1)