note $\sqrt{x} = x^{\frac{1}{2}}$

Example 3: Calculate the derivative of $f(x) = 2e^x(x^3 - \sqrt{x})$.

Solution: $f(x) = 2e^x(x^3 - \sqrt{x})$ is a product of $g(x) = 2e^x$ and $h(x) = x^3 - \sqrt{x}$. The product law is

$$(g \cdot h)' = g' \cdot h + g \cdot h'.$$

$g'(x) = 2e^x$

$h'(x) = 3x^2 - \frac{1}{2}x^{-\frac{1}{2}}$

so

$f'(x) = (g \cdot h)'(x) = g'(x) \cdot h(x) + g(x) \cdot h'(x) = 2e^x(x^3 - \sqrt{x}) + 2e^x\left(3x^2 - \frac{1}{2}x^{-\frac{1}{2}}\right)$

$= 2e^x\left(x^3 + 3x^2 - \sqrt{x} - \frac{1}{2\sqrt{x}}\right)$

Example 4: Calculate the derivative of $f(x) = \frac{3x^2-1}{x^3+2x}$.

Solution: $f(x) = \frac{g(x)}{h(x)}$ where $g(x) = 3x^2 - 1$ and $h(x) = x^3 + 2x$. The quotient law is

$$\left(\frac{g}{h}\right)' = \frac{g' \cdot h - g \cdot h'}{h^2}.$$

$g'(x) = 6x$ and $h'(x) = 3x^2 + 2$

so

$f'(x) = \left(\frac{g}{h}\right)'(x) = \frac{g'(x) \cdot h(x) - g(x) \cdot h'(x)}{h(x)^2} = \frac{6x(x^3+2x) - (3x^2-1)(3x^2+2)}{(x^3+2x)^2}$

$= \frac{-3x^4 + 9x^2 + 2}{(x^3+2x)^2}$

Example 5: Calculate the derivative of $\sin(\theta)\cos(\theta)$ with respect to θ.

Solution:

$\frac{d}{d\theta}\sin(\theta)\cos(\theta) = \left(\frac{d}{d\theta}\sin(\theta)\right) \cdot \cos(\theta) + \sin(\theta) \cdot \left(\frac{d}{d\theta}\cos(\theta)\right) = \left(\cos(\theta)\right)\cos\theta + \sin\theta(-\sin\theta))$

$= \cos^2\theta - \sin^2\theta$

$= 2\cos^2\theta - 1$

Example 5: Calculate the derivative of $2^x \cdot x^2 \cdot \cos(x)$ with respect to x.

Solution: $2^x \cdot x^2 \cdot \cos(x) = (2^x) \cdot (x^2 \cdot \cos(x))$, so by the product rule

$$\frac{d}{dx}\left(2^x \cdot x^2 \cdot \cos(x)\right) = \left(\frac{d}{dx}2^x\right) \cdot \left(x^2 \cdot \cos(x)\right) + (2^x) \cdot \left(\frac{d}{dx}x^2 \cdot \cos(x)\right)$$

We need to calculate $\frac{d}{dx}2^x$ and $\frac{d}{dx}x^2 \cdot \cos(x)$.

$\frac{d}{dx}2^x = \ln(2)2^x$

To calculate $\frac{d}{dx}x^2 \cdot \cos(x)$ we need to use the product rule again:

$\frac{d}{dx}x^2 \cdot \cos(x) = \left(\frac{d}{dx}x^2\right)\cos(x) + x^2\left(\frac{d}{dx}\cos x\right) = 2x\cos(x) - x^2\sin x$

Finally, we can calculate $\frac{d}{dx}\left(2^x \cdot x^2 \cdot \cos(x)\right)$:

$\frac{d}{dx}\left(2^x \cdot x^2 \cdot \cos(x)\right) = \left(\frac{d}{dx}2^x\right) \cdot \left(x^2 \cdot \cos(x)\right) + (2^x) \cdot \left(\frac{d}{dx}x^2 \cdot \cos(x)\right) =$

$= 2^x\left[x^2\cos(x) + 2x\cos(x) - x^2\sin(x)\right]$

$= x2^x\left[(x+2)\cos(x) - x\sin(x)\right]$

Example 6: Calculate the derivative of $\frac{e^x \sin(x)}{x^2+7}$ with respect to x.

Solution: By the quotient rule we have

$$\frac{d}{dx}\frac{e^x \sin(x)}{x^2+7} = \frac{\left(\frac{d}{dx}e^x \sin(x)\right) \cdot (x^2+7) - (e^x \sin(x)) \cdot \left(\frac{d}{dx}(x^2+7)\right)}{(x^2+7)^2}$$

We need to calculate $\frac{d}{dx}e^x \sin(x)$ and $\frac{d}{dx}(x^2+7)$.

$\frac{d}{dx}(x^2+7) = 2x$

Using the product rule we see that

$\frac{d}{dx}e^x \sin(x) = \left(\frac{d}{dx}e^x\right)\sin(x) + e^x\left(\frac{d}{dx}\sin(x)\right) = e^x\sin x + e^x\cos(x) = e^x(\sin(x) + \cos(x))$

Finally, we have

$\frac{d}{dx}\frac{e^x \sin(x)}{2x^3-4x^2+7} = \frac{\left(\frac{d}{dx}e^x \sin(x)\right)\cdot(x^2+7) - (e^x \sin(x))\cdot\left(\frac{d}{dx}(x^2+7)\right)}{(x^2+7)^2} = \frac{e^x(\sin x + \cos(x))(x^2+7) - 2xe^x\sin x}{(x^2+7)^2}$

$= \frac{e^x\left[\sin(x)(x^2-2x+7) + \cos(x)(x^2+7)\right]}{(x^2+7)^2}$