SOLUTIONS

Name:_____

Signature:_____
J. Number:_____

MA 113- 101 Precalculus Trigonometry
Test 1
13:25-14:15 Wednesday 24th September 2008

1. **Do any eight of the nine questions.**
2. Write your name and J. number at the top of this page.
3. Answer the questions in the spaces provided.
4. Show all your work required to obtain your answers.
5. **No calculators are allowed.**
6. This is a closed book test.

Question	Mark
1	/5
2	/5
3	/5
4	/5
5	/5
6	/5
7	/5
8	/5
9	/5
total	/40

1) a) Fill in the missing entries in the following table so that the two entries in each row describe the same angle.

Degrees	Radians
90°	$\pi/2$
60°	$\frac{\pi}{3}$
$-150°$	$-\frac{5\pi}{6}$
270°	$\frac{3\pi}{2}$
45°	$\frac{\pi}{4}$
405°	$\frac{9\pi}{4}$

½ ½ ½ ½ ½ ½

$\frac{\pi}{2} + \frac{\pi}{3} = \frac{3\pi}{6} + \frac{2\pi}{6} = \frac{5\pi}{6}$

$405^\circ = 360^\circ + 45^\circ$

$= 2\pi + \frac{\pi}{4} = \frac{8\pi}{4} + \frac{\pi}{4} = \frac{9\pi}{4}$

b) Write the angle 12°20′ in decimal form. (Work to two decimal places.)

$12^\circ \longrightarrow 12^\circ$

$20' \longrightarrow \left(\frac{20}{60}\right)^\circ = \left(\frac{2}{6}\right)^\circ = \left(\frac{1}{3}\right)^\circ = 0.33^\circ$

(1)

so $12^\circ 20' = 12.33^\circ$

c) Write the angle 67.25° in D°M′S″ form.

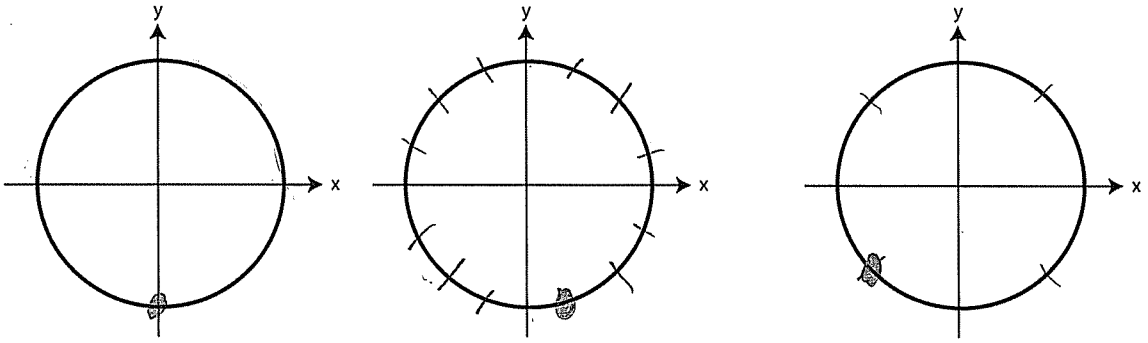
$67^\circ \longrightarrow 67^\circ$

$0.25^\circ \longrightarrow (0.25 \times 60)' = \left(\frac{60}{4}\right)' = 15'$

$67.25^\circ = 67^\circ 15'$

(1)

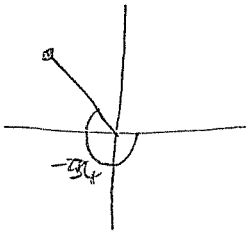
2) Mark the point defined by the angle **(a)** $\frac{7\pi}{2}$, **(b)** $\frac{-3\pi}{8}$ and **(c)** $\frac{13\pi}{4}$ on the unit circles below.



(a) (b) (c)

d) Complete the table below by indicating if the given function is positive or negative at the specified angle.

Angle θ	sign of $\sin(\theta)$	sign of $\cos(\theta)$	sign of $\tan(\theta)$
$\theta = -234°$	+	−	−
$\theta = \frac{13\pi}{4}$	−	−	+



3) a) Find a positive and a negative angle which is coterminal to $\frac{4\pi}{7}$.

$$\frac{4\pi}{7} + 2\pi = \frac{4\pi}{7} + \frac{14\pi}{7} = \frac{18\pi}{7} \quad (1)$$

$$\frac{4\pi}{7} - 2\pi = \frac{4\pi}{7} - \frac{14\pi}{7} = \frac{-10\pi}{7} \quad (1)$$

b) Find a positive coterminal angle to $-800°$.

$$-800° + (3 \times 360)° = -800 + 1080° = 280° \quad (1)$$

c) Find the angle which is supplementary to $10°20'30''$.

$$180° - 10°\,20'\,30'' = 169°\,39'\,30'' \quad (1)$$

d) Find the angle which is complementary to $\pi/5$

$$\frac{\pi}{2} - \frac{\pi}{5} = \frac{5\pi}{10} - \frac{2\pi}{10} = \frac{3\pi}{10} \quad (1)$$

4) If $\cos(\theta) = \frac{3}{5}$, find the other five trigonometric functions:

$\sin(\theta) = \frac{4}{5}$ (1) $\quad \tan(\theta) = \frac{4}{3}$ (1) $\quad \csc(\theta) = \frac{5}{4}$ (1)

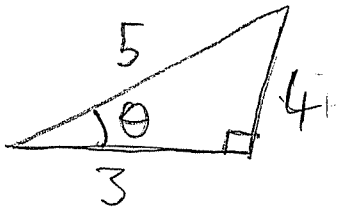
$\sec(\theta) = \frac{5}{3}$ (1/2) $\quad \cot(\theta) = \frac{3}{4}$ (1/2)

$\cos(\theta) = \frac{adj}{hyp} = \frac{3}{5}$

$5^2 = 3^2 + a^2$

$a = \sqrt{5^2 - 3^2} = \sqrt{25-9} = \sqrt{16}$

$= 4$

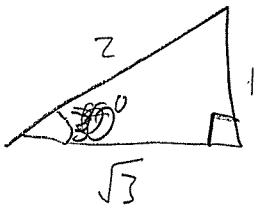


(1)

5) a) If $\tan(\theta) = \frac{\sqrt{12}}{6}$, and θ is an acute angle, what is θ?

$$\frac{\sqrt{12}}{6} = \frac{\sqrt{4\times3}}{6} = \frac{\sqrt{4}\sqrt{3}}{6} = \frac{2\sqrt{3}}{6} = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}} = \frac{opp}{adj}$$

So



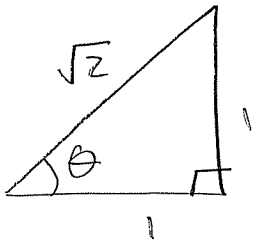
$$\theta = 30^\circ = \frac{\pi}{6}$$

2

b) If $\csc(\theta) = \sqrt{\frac{8}{4}}$, and θ is an acute angle, what is θ?

$$\sqrt{\frac{8}{4}} = \frac{\sqrt{8}}{\sqrt{4}} = \frac{2\sqrt{2}}{2} = \frac{\sqrt{2}}{1} = \frac{hyp}{opp}$$

so

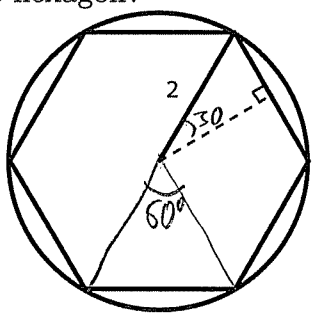


ad

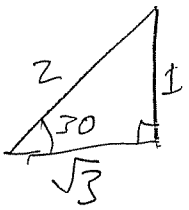
$$\theta = 45^\circ = \frac{\pi}{4}$$

3

6) If a regular hexagon is circumscribed by a circle of radius 2 in., what is the perimeter of the hexagon?



either

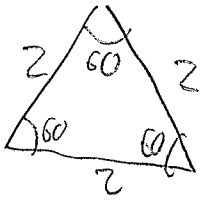


So each side of the hexagon is of length 2

So perimeter is

$6 \times 2 = 12$

or



So each side is of length 2 and the perimeter is

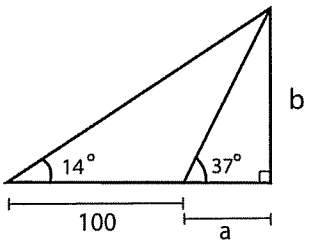
$6 \times 2 = 12$

7) The top of a building is at elevation 14°. When I walk 100 ft closer to the building the elevation is 37°. What is the height of the building?

You may find some of the following values useful:

$$\sin(14°) \approx \frac{\sqrt{17}}{17} \quad \cos(14°) \approx \frac{4\sqrt{17}}{17} \quad \tan(14°) \approx \frac{1}{4}$$
$$\sin(37°) \approx \frac{3}{5} \quad \cos(37°) \approx \frac{4}{5} \quad \tan(37°) \approx \frac{3}{4}$$

(Warrning: remember that $\frac{a}{b} = \frac{3}{4}$, does not imply $a = 3$ and $b = 4$.)



$\tan(37^\circ) = \frac{b}{a} \Rightarrow \frac{3}{4} = \frac{b}{a} \Rightarrow a = \frac{4b}{3}$ (1)

$\tan(14^\circ) = \frac{b}{100+a} \Rightarrow \frac{1}{4} = \frac{b}{100+a}$ (1)

$\Rightarrow 4b = 100 + a$

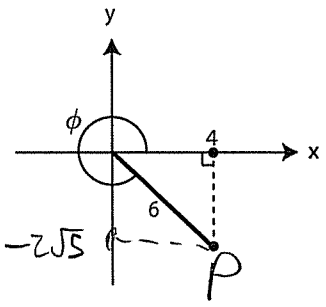
$\Rightarrow 4b = 100 + \frac{4b}{3}$

$\Rightarrow 4b - \frac{4b}{3} = 100$ (3)

$\Rightarrow (12-4)b = 300$

$\Rightarrow b = \frac{300}{8} = \frac{75}{2}$

8) Find the six trigonometric functions for the angle ϕ given in the figure:

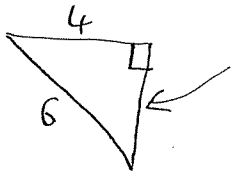


$\sin(\phi) = \frac{-2\sqrt{5}}{6} = \frac{-\sqrt{5}}{3}$ $\tan(\phi) = \frac{-2\sqrt{5}}{4} = \frac{-\sqrt{5}}{2}$ $\cos(\phi) = \frac{4}{6} = \frac{2}{3}$

$\sec(\phi) = \frac{3}{2}$ $\cot(\phi) = \frac{-2}{\sqrt{5}} = \frac{-2\sqrt{5}}{5}$ $\csc(\phi) = \frac{-3}{\sqrt{5}} = \frac{-3\sqrt{5}}{5}$

(3)

Find the coordinates of P:



$a = \sqrt{6^2 - 4^2}$
$= \sqrt{36 - 16}$
$= \sqrt{20}$
$= 2\sqrt{5}$

so $P = (4, -2\sqrt{5})$ (2)

-2 for sign errors
-1 for wrong point if small error
-1 for wrong trig functions si cos ta
-1 for wrong sec cot csc

9) Let $\theta = -225°$.

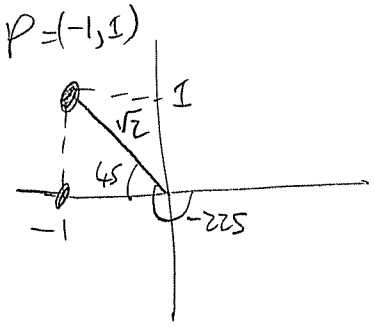
(a) Find the reference angle of θ.

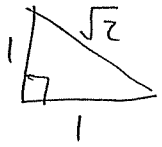
$-225° = -(180° + 45°)$

$45°$

(2)

(b) Find $\csc(-225°)$.





(3)

So $\csc(-225°) = \frac{\sqrt{2}}{1} = \sqrt{2}$