Transformations of Sine and Cosine Functions

To graph

$$y = A\sin(Bx - C) + D = A\sin\left[B\left(x - \frac{C}{B}\right)\right] + D$$

and

$$y = A\cos(Bx - C) + D = A\cos\left[B\left(x - \frac{C}{B}\right)\right] + D,$$

follow the steps listed below in the order in which they are listed.

1. Stretch or shrink the graph horizontally according to B.

   $|B| < 1$ Stretch horizontally
   $|B| > 1$ Shrink horizontally
   $B < 0$ Reflect across the y-axis

   The *period* is $\left|\frac{2\pi}{B}\right|$.*

2. Stretch or shrink the graph vertically according to A.

   $|A| < 1$ Shrink vertically
   $|A| > 1$ Stretch vertically
   $A < 0$ Reflect across the x-axis

   The *amplitude* is $|A|$.

3. Translate the graph horizontally according to C/B.

   $\frac{C}{B} < 0$ $\left|\frac{C}{B}\right|$ units to the left
   $\frac{C}{B} > 0$ $\frac{C}{B}$ units to the right

   The *phase shift* is $\frac{C}{B}$.

4. Translate the graph vertically according to D.

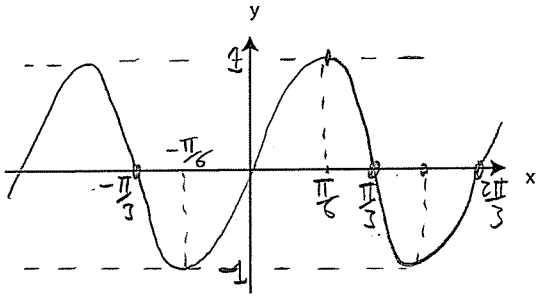
   $D < 0$ $|D|$ units down
   $D > 0$ D units up

In all of your plots of the graphs you need to mark where:

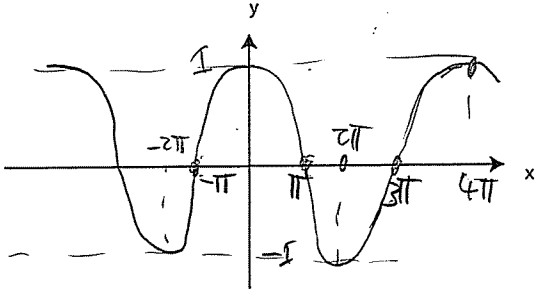
- the graph crosses the x-axis;
- where the maxima and minima are;
- the maximum and minimum y-values.

Examples

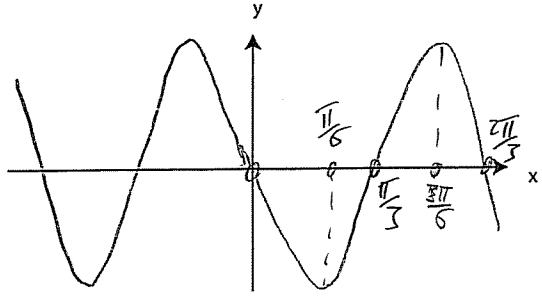
$\sin(3x)$:



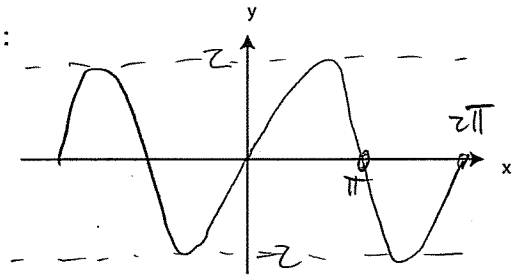
$\cos(\frac{1}{2}x)$:



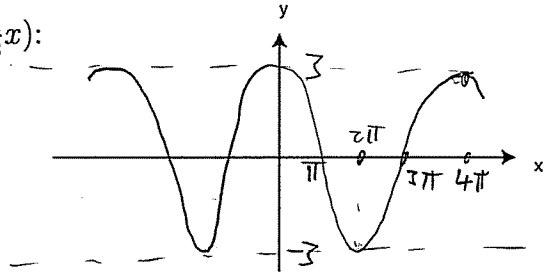
$\sin(-3x)$:



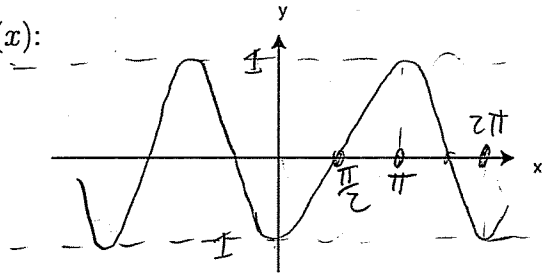
$2\sin(x)$:



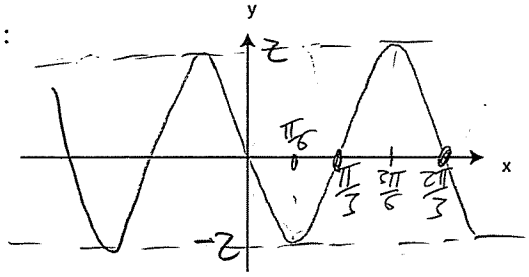
$3\cos(\frac{1}{2}x)$:



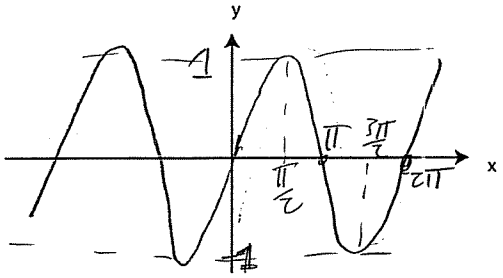
$-\cos(x)$:



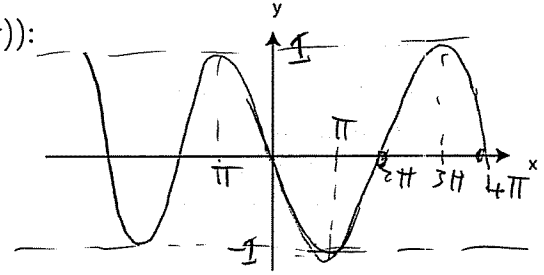
$-2\sin(3x)$:



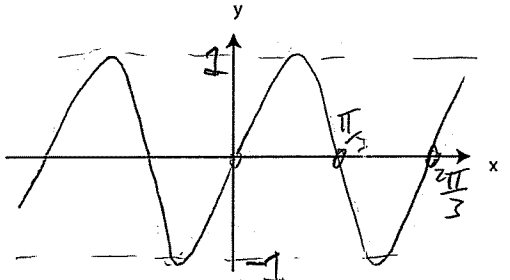
$\cos(x-\frac{\pi}{2})$:



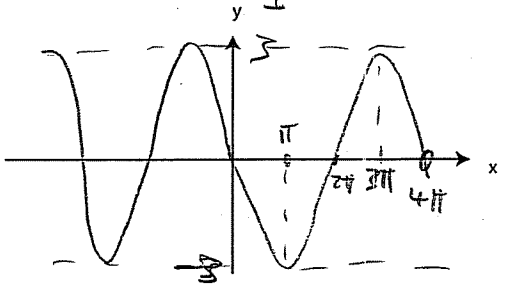
$\cos(\frac{1}{2}(x+\pi))$:



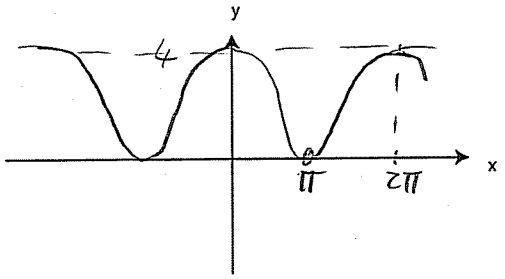
$\sin(-3(x-\frac{\pi}{3}))$:



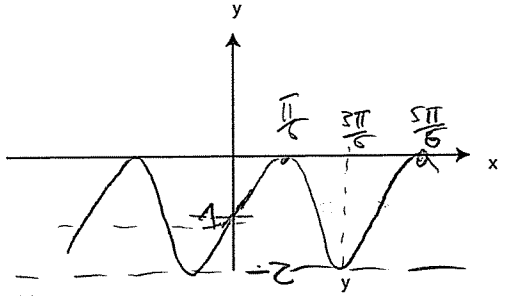
$3\cos(\frac{1}{2}(x+\pi))$:



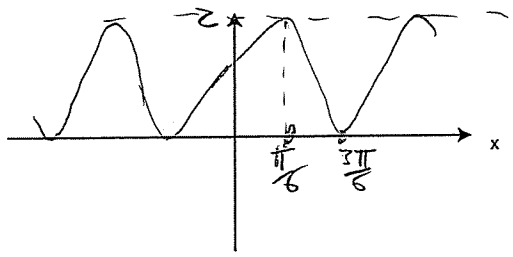
$\cos(x) + 2$:



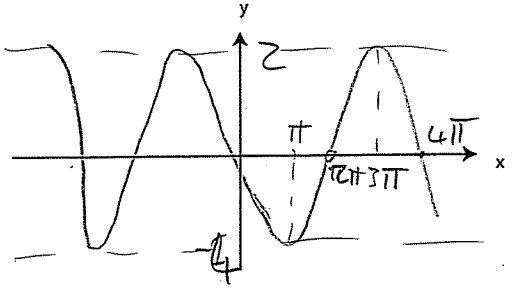
$\sin(3x) - 1$:



$\sin(-3(x - \frac{\pi}{3})) + 1$:

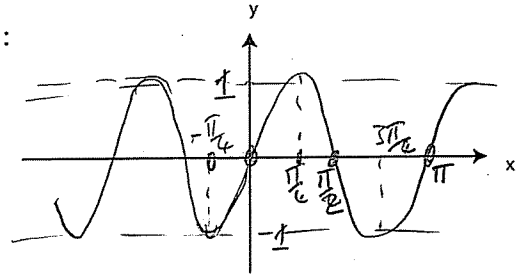


$3\cos(\frac{1}{2}(x + \pi)) - 1$:

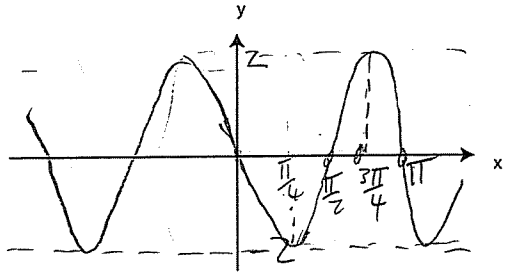


Plot $-2\sin(2x - \frac{\pi}{2}) + 1 = -2\sin(2[x - (\frac{\pi}{4})]) + 1$

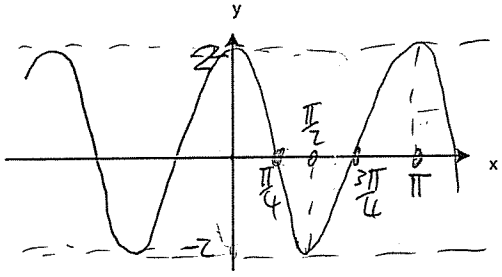
$\sin(2x)$:



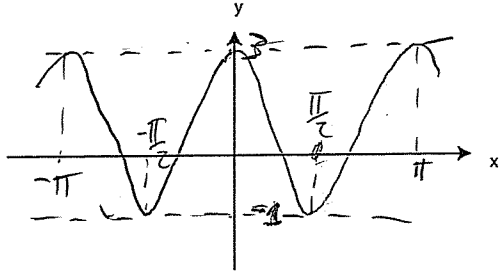
$-2\sin(2x)$:



$-2\sin(2(x - \frac{\pi}{4})$:

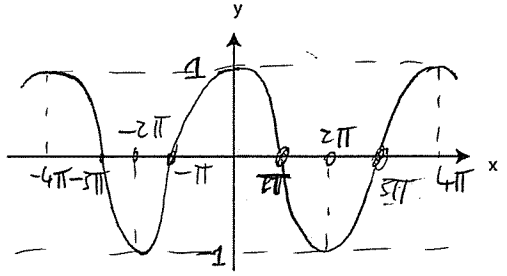


$-2\sin(2(x - \frac{\pi}{4}) + 1$:

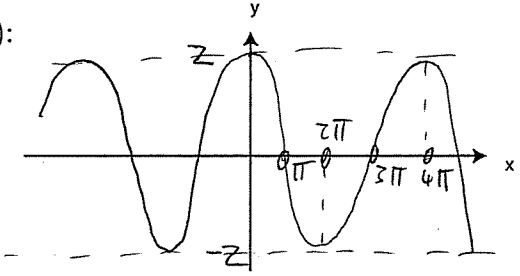


Plot $2\cos(\frac{1}{2}x+\pi)-2 = 2\cos(\frac{1}{2}[\ x-(\frac{-2\pi}{1})])-2$

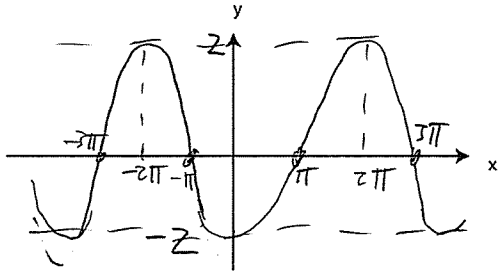
$\cos(\frac{1}{2}x)$:



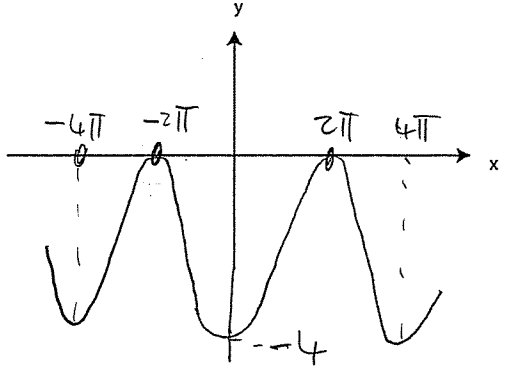
$2\cos(\frac{1}{2}x)$:



$2\cos(\frac{1}{2}[x+(2\pi)])$:

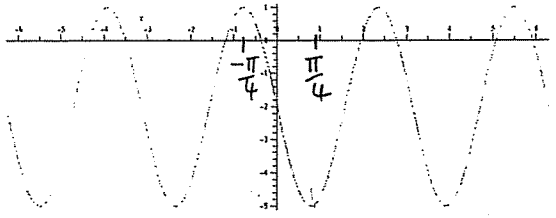


$2\cos(\frac{1}{2}[x+(2\pi)])-2$:

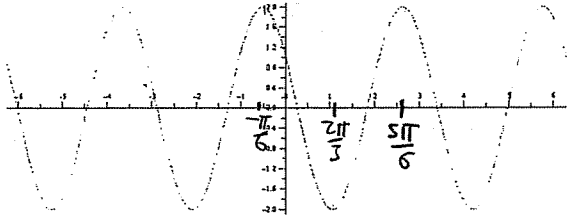


The definitions of the *period*, *amplitude* and the *phase shift* are given on the first page.

Intuitively, the period is the distance the graph takes to make a complete "cycle", the amplitude is the height between the midpoint and a maxima, and the phase shift is how for the graph is shifted to the right or left.



For the above graph, amplitude= 3 , period= π



For the above graph, amplitude= , period=

For $2\cos(\frac{1}{2}x+\pi)-2$.
amplitude= 2 , period= 4π , phase shift= -2π

For $1\sin(-2x-\frac{\pi}{2})-2$. = $\sin(-2(x-(\frac{-\pi}{4})))-2$
amplitude= 1 , period= π , phase shift= $\frac{-\pi}{4}$