1 Introduction

One justification for government intervention is market failures. With market failures the first theorem of welfare economics breaks down and the decentralized market equilibrium will fail to be Pareto optimal. There may then be a government intervention to improve efficiency.

In this lecture we will consider two particular types of market failures: public goods and externalities. No doubt you are all aware of what we mean by public goods and externalities, so I assume that the topics need very little introduction. We will start by looking a public goods. So what will we be saying about public goods?

2 Public Goods: An Overview

The fundamental problem with public goods is how to design institutions such that maximum efficiency obtains. We might want to consider question such as: How bad is the market mechanism? Can other institutions be designed that generate better allocations? Our first tasks are thus as follows:

- Establish benchmark case: Characterize Pareto optimal allocations
- Consider institutions that determine public good provision:
  - Voluntary provision
  - Collective action, voting.
We will consider what happens when the consumers make voluntary contributions to a public good; this is effectively the competitive market equilibrium. As we will see there will be a substantial free rider problem. It is frequently argued that public goods ought to be publicly provided. If so, one can imagine either that the provision problem is solved by a benevolent policy maker (in which case we can expect the policy maker to select a Pareto optimal allocation); alternatively, one can imagine that the provision problem is determined through collective action e.g. through a democratic process such as majority voting. Hence we will consider the outcome of majority voting.

We will also consider extensions of the pure public goods model that are of great practical importance: These include congestion, “club goods”, “local public goods”. The theory of local public goods has recently been on the research agenda, because it can be used to study a range of interesting phenomena. In the US debate, a big debate is e.g. schooling and segregation; in Europe the theory of local public goods has become important for the study of European integration.

A fundamental problem associate with public goods is that the consumers do not have the incentives to reveal their preferences; this is what causes free-riding in the market equilibrium. It is also what causes inefficiencies associated with voting. This raises the question if there is any way to design mechanisms for determining public good supply which provide the consumers the incentives to truthfully reveal their preferences. Hence we will briefly consider the whether and how one can design mechanism to reveal the individuals’ preferences.

The preference-revelation problem was very much on the research agenda a decade or two ago. What emerged from that research was that it is possible to come up with well designed mechanisms whereby each consumer would, in fact, have an incentive to truthfully reveal their incentives. However, the mechanisms also have limitations.
3 Public Goods: Pareto Efficiency

3.1 Characterizing Features

So far we have been considering private goods; private goods have two characterizing features. The first is that there is rivalry among consumers in the sense that consumption by one consumer reduces the amount available to others. The second characterizing feature is that private goods are subject to exclusion – one has to own a good in order to consume it: if it is not yours you are effectively prevented from enjoying it. The characterizing feature of a pure public good is, on the other hand, the exact opposite.

**Definition 1** Non-rivalry. Consumption of a good by one consumer does not reduce the amount available to other consumers.

**Definition 2** Non-excludability. If a good is supplied, then no consumer can be excluded from consuming it.

**Definition 3** A pure public good has both the non-rivalry property and the non-excludability property.

However, some goods are non-rivalrous but still excludable. Consider e.g. a bridge: assuming that there is no congestion it is non-rivalrous; it is nevertheless easy to exclude consumers from using it. Some goods, may partially fail the non-rivalry property; in that case we say that there is congestion. Congestion will be important when we consider club goods.

3.2 The Samuelson Rule

The first logical step is to characterize the efficient allocation. Thus consider the following economy. There is a set of consumers \( i \in I = \{1, 2, \ldots, n\} \). There is one private good \( x \) and one public good \( z \). The assumption that there is only one public good and one private good is made for simplicity – the extension to several goods of each type is straightforward. Consumer \( i \)'s preferences are represented by a utility function \( u_i (x_i, z_i) \). Each consumer is assumed to have strictly convex and strictly monotonic preferences.
We can use a simple production function formulation to summarize the economy’s technology. Hence suppose that the public good $z$ is produced using the private good as input. Let the technology be summarized by $z = f(x)$ where $f$ is strictly increasing, continuous, and (weakly) concave and where $x$ is the amount of the private goods used as input in the production the public good.

Let there be an initial aggregate endowment $\omega_x$ units of $x$. Also let we use the vector notation $\mathbf{x} = (x_i)_{i \in I}$ to denote the level of consumption of the private good by each consumer; thus $\mathbf{x}$ a vector of length $n$. Similarly let $\mathbf{z} = (z_i)_{i \in I}$ describe the level of consumption of the private good by each consumer. Note that we are not assuming that each consumer will automatically consume the same amount; rather we will impose as feasibility constraint the each consumer consumes at most $f(x)$ of the public good, i.e. the amount produced.

**Definition 4** An allocation is a pair $(\mathbf{x}, \mathbf{z})$.

Consider the feasibility constraints for this economy.

**Definition 5** An allocation $(\mathbf{x}, \mathbf{z})$ is feasible if

$$\sum_{i \in I} x_i \leq \omega_x - x, \text{ and }$$

$$z_i \leq f(x) \text{ for all } i \in I.$$  

The second constraint captures the non-rivalry of $z$. In principle, a consumer can consume less than the available amount of $z$, $z_i \leq f(x)$. However, since preferences are strongly monotone, this will never be efficient. Hence, given that we are seeking to characterize efficient allocation, we can focus on $z_i = z = f(x)$ for all $i \in I$.

Pareto optimal allocations can be characterized as the solution to an optimization problem. In particular, consider the problem of maximizing the utility of individual $i$ given a set of required utilities for the other individuals and given the aggregate resource constraint.

Figure 3.1 illustrate the case where there are two consumers; for a given level of utility to individual 1, denoted $u_1$, we maximize the utility of individual 2 given the feasibility
constraint. The value of that problem is the value of the utility possibility frontier at that specific value of $u_1$. Then as we vary the required utility for individual 1 we trace out the utility possibility frontier (UPF).

**FIG 3.1**

When we have $n$ individuals we fix the utility for all individuals except one, individual $i$, and maximize the utility of this last individual subject to the fixed utility for everyone else and subject to the feasibility constraint. Letting $g(\cdot)$ be the inverse of $f(\cdot)$, $g(z)$ measures the amount of the private goods required as input in order to produce $z$ units of the public good; using this we can simplify the feasibility constraint to the inequality $\sum_{i \in I} x_i \leq \overline{x} - g(z)$ (which allows us to eliminate the input level $x$ from the feasibility constraint).

Pareto optimality can then be characterize as follows:

**LEMMA 1** A feasible allocation $(x^*, z^*)$ is Pareto optimal if and only if it solves the following problem for every $i \in I$

$$\begin{align*}
\max_{x,z} & \quad u_i(x_i, z) \\
\text{s.t.} & \quad u_j(x_j^*, z^*) \leq u_j(x_j, z), \quad j \neq i \quad (\gamma_i) \\
\text{and} & \quad \sum_{i \in I} x_i \leq \overline{x} - g(z) \quad (\lambda)
\end{align*}$$

(3)

Note that in this problem we have as utility requirements for the “other individuals” the utilities that they enjoy at the Pareto optimal allocation $(x^*, z^*)$. The Lagrangean for this problem, for $i = 1$ (say)

$$\mathcal{L} = u_1(x_1, z) - \gamma_j \sum_{i=2}^{n} [u_j(x_j^*, z^*) - u_j(x_j, z)] - \lambda \left[ \sum_{i \in I} x_i - \overline{x} + g(z) \right].$$

Since $(x^*, z^*)$ is a solution to (3), by the Kuhn-Tucker theorem, there exists $(\gamma_j^*)_{j=2}^{n}$ and $\lambda^*$ such that all derivatives of $\mathcal{L}$ are zero at $(x^*, z^*, \gamma^*, \lambda^*)$. If we also define $\gamma_1^* \equiv 1$ we can write the first-order conditions

$$\frac{\partial \mathcal{L}}{\partial x_i} = \gamma_i \frac{\partial u_i}{\partial x_i} - \lambda^* = 0, \quad i \in I$$

(4)

$$\frac{\partial \mathcal{L}}{\partial z} = \sum_{i \in I} \gamma_i \frac{\partial u_i}{\partial z} - \lambda^* g'(z^*) = 0.$$
Solving (4) for $\gamma^*_i$ and then substituting in (5) we can eliminate the multiplier: this yields

$$\sum_{i \in I} \frac{\partial u_i}{\partial z} \frac{\partial u_i}{\partial x_i} = g'(z^*) \quad \text{(Samuelson Condition).}$$

The interpretation of the Samuelson (1954, 1955) condition is straightforward. Note that

$$\frac{\partial u_i}{\partial z} \frac{\partial u_i}{\partial x_i}$$

can be interpreted as consumer $i$’s *marginal willingness to pay* for $z$ (in terms of the private good), i.e. the individual’s marginal rate of substitution. The right hand side of (6) measures the marginal cost of $z$ (again in terms of the private good) – the marginal rate of transformation. Hence the condition states that the sum of the consumers’ marginal willingness to pay for $z$ should equal the marginal cost.

Note that the Samuelson rule does not rely on non-excludability, only non-rivalry. Simply – since the good is non-rivalrous, and since the consumers have monotone preferences optimality requires that no one is excluded, so whether exclusion is possible is irrelevant – it would never be optimal. However, excludability may become important when we consider different institutions for determining public good supply.

## 4 Equilibria with Private Provision of Public Goods

If there was a benevolent government with unrestricted policy instruments and perfect information about preferences, then we would expect the Samuelson rule to be implemented. However, suppose instead that there is no government at all, but only a private market. On that private market each consumer can buy units of the public good; since the public good is non-excludable if a consumer does decide to buys some amount of the private good, that amount will automatically be available to all other consumers as well. This suggests that there may be free riding: every consumer would happy to enjoy the contributions to the public good done by other and would be less inclined to make own contributions. A natural conjecture would thus be that there will be too little voluntary public good provision. Moreover, we would expect that problem to be worse in
large economies than in small economies. Thus let’s consider private provision equilibria formally.

In order to do so, let’s modify the model a little bit. We will do this in order to bring in “individual incomes” into the model. One factor of production (labour) is supplied inelastically by each consumer; this gives an income $R_i$ to consumer $i \in I$. The income of each consumer represents the amount of “effective” labour supplied. The production technology of the economy is assumed to be the simplest possible: the private good and the public good are both produced using effective labour as input; the technology for each good is linear where one unit of effective labour produces one unit of output of either good. Given this assumption on technology we can set the prices of each good equal to unity:

$$p_x = p_z = 1. \quad (7)$$

Thus we have a general equilibrium model working in the background; however, by assuming constant returns to scale we are largely short-circuiting the model, effectively throwing out effects feeding through prices. Note also that since labour is supplied inelastically we do not need to include it in the utility function – it simply does not vary.

Now let $z_i$ denote consumer $i$’s purchases of (or “contribution” to) the public good and let

$$z = \sum_{i \in I} z_i \quad (8)$$

denote the aggregate contribution. We will also use the notation

$$z_{-i} = \sum_{j \neq i} z_j \quad (9)$$

denote the contributions of all consumers except $i$.

We will now consider a simple Nash equilibrium: Hence we make the Nash assumption that each consumer takes $z_{-i}$ as given and maximize her own utility. Consumer $i$’s budget is

$$x_i + z_i = R_i. \quad (10)$$

She maximizes her utility $u_i(x_i, z)$; using (10) and that $z = z_i + z_{-i}$ and that $x_i = R_i - z_i$ we can write the utility maximization problem (UMP) as an unconstrained problem:

$$\max_{0 \leq z_i \leq R_i} u_i(R_i - z_i, z_i + z_{-i}). \quad (11)$$
Indifference curves can be drawn in the \((z_i, z_{-i})\)-space. By convexity of the consumer’s preferences the preferred set is convex.

**FIG 3.2**

Consumer \(i\)’s optimal choice given \(z_{-i}\) is then given by the point of tangency. The “Nash reaction function” traces out consumer \(i\)’s optimal contributions as \(z_{-i}\) is varied.

The first order condition for problem (11) (ignoring the possibility that the consumer will spend her entire income on the public good) is

\[
\frac{\partial u_i}{\partial z} - \frac{\partial u_i}{\partial x_i} \leq 0, \text{ with } z_i \left( \frac{\partial u_i}{\partial z} - \frac{\partial u_i}{\partial x_i} \right) = 0. \tag{12}
\]

Thus, if the individual makes a strictly positive contribution, her marginal utilities of the two goods will be equal (since the price of the two goods are equal); in contrast, if she choose not to contribute to the public good, \(z_i = 0\), then she has a marginal utility of the private good that is at least as large as the marginal utility of the public good.

“Solving” (12) yields individual \(i\)’s contribution as a function of the total contribution by everyone else,

\[z_i = \zeta_i (z_{-i}),\]

which is generally (weakly) decreasing.

We can now define what we mean by a (Nash) private provision equilibrium:

**Definition 6** A private provision equilibrium (PPE) is a set of contributions \(z = (z_i)_{i \in I}\) such that \(z_i = \zeta_i (z_{-i})\) for all \(i \in I\).

Note that we are now using \(z = (z_i)_{i \in I}\) to denote the contributions by the individuals in the economy rather than their consumption. We will say that consumer \(i\) is a contributor if (at the equilibrium) \(z_i > 0\) and a non-contributor is \(z_i = 0\) and we will use \(C \subset I\) to denote the set of contributors.

\(^1\)A sufficient condition for \(\zeta’(z_i) < 0\) at interior \(z_i\) is that \(\frac{\partial^2 u_i}{\partial x_i \partial z} > 0\).
4.1 Existence*

The first question is if a private provision equilibrium generally exists. The answer is yes.

**Proposition 1** A private provision equilibrium exists.

**Proof.** Let \( n = 2 \) (the generalization to \( n \geq 2 \) is obvious) and let \( z = (z_1, z_2) \) and let \( H_i = [0, R_i] \) (the set of possible contributions for \( i \), \( i = 1, 2 \). Note that \( \zeta_i : H_j \to H_i, j \neq i \).

\( \zeta_i (\cdot) \) here is the Nash “reaction function”, which tells us what is \( i \)'s optimal choice for every possible choice for \( j \). \( \zeta_i (\cdot) \) is a continuous function by the fact that it is derived from a well-behaved optimization problem. In that problem, convexity of consumer \( i \)'s preferences implies that \( i \)'s utility maximisation problem has a unique solution – hence \( \zeta_i (\cdot) \) is single-valued; moreover, continuity of the utility function \( u_i (\cdot) \) (in \( x_i, z_i \) and \( z_j \)) implies that \( \zeta_i (\cdot) \) is a continuous function.

The vector \( z^* \in H_1 \times H_2 \) is a PPE if

\[ z_1^* = \zeta_1 (z_2^*), \text{ and } z_2^* = \zeta_2 (z_1^*) . \]

Define

\[ \hat{\zeta}_1 (z) = \zeta_1 (z_2) \text{ and } \hat{\zeta}_2 (z) = \zeta_2 (z_1) . \]

(This simply extends the definition of \( \zeta_1 (\cdot) \) to have the own contribution as an “irrelevant” argument; this is done so as to ensure that each function becomes a function of the entire vector of contributions.) Note that \( \hat{\zeta}_i : H_1 \times H_2 \to H_i, i = 1, 2 \). Then define the vector valued function

\[ \hat{\zeta} (z) = \left( \hat{\zeta}_1 (z), \hat{\zeta}_2 (z) \right) . \]

The vector valued function \( \hat{\zeta} (\cdot) \) maps the set \( H_1 \times H_2 \) into itself and is continuous. Since \( H_1 \times H_2 \) is a nonempty, compact, convex subset of \( R^2 \), by Brouwer’s theorem \( \hat{\zeta} \) has a fixed-point \( z^* \), which, by construction constitutes a PPE. ■

In the case of two consumers there would be a faster route: simply construct the composite function \( \zeta_1 \circ \zeta_2 : H_1 \to H_1 \); this function is continuous since \( \zeta_1 \) and \( \zeta_2 \) are both continuous. Thus it has a fixed point which identifies a PPE. However, the above version makes it easier to see how an extension to \( n \geq 2 \) would work. One can also show that normality of both the private and the public good implies that the PPE is unique.
4.2 Inefficiency

To gain more insight we can consider the case of two consumers, $I = \{1, 2\}$. Hence consider the reaction function $\zeta_1(z_2)$ and $\zeta_2(z_1)$.

![FIG 3.3](image)

The PPE, $z^*$, occurs at the intersection of the two reaction functions. FIG 3.4 shows that the PPE is not Pareto efficient:

![FIG 3.4](image)

To see this, note that consumer 1’s indifference curve is horizontal at $z^*$ while that of consumer 2 is vertical. Hence there will be allocations to the north-east that would entail Pareto improvements. In contrast, the Pareto efficient allocations are identified through the points of tangency of the consumers’ indifference curves. More formally, at the PPE,

$$\frac{\partial u_i/\partial z}{\partial u_i/\partial x_i} = 1 \text{ for } i = 1, 2$$

while the Samuelson condition would require

$$\sum_{i=1,2} \frac{\partial u_i/\partial z}{\partial u_i/\partial x_i} = 1.$$ 

where we used that the marginal rate of transformation between $x$ and $z$ is unity (both have constant returns to scale technologies with the same output rate, reflected in the prices $p_x = p_z = 1$).

4.3 Invariance

One aspect of free-riding is that if one consumer contributes more, then other consumers contribute less. This was reflected in the Nash reaction functions shown in the previous figures. However, it turns out that the specific structure of the problem generates an even stronger and surprising result. It was demonstrated quite early on (Warr, 1983??, Bergstrom and Varian, 1985) that a redistribution of income will not affect the total level of public good provided as long as all consumers make positive contributions – in other
words – the overall equilibrium contribution would be invariant to income redistribution. However, the assumption that all individual are making positive contributions is rather restrictive – in many cases we would expect a large number of individuals to not contribution at all. The question is if then if the total level of contributions will still be invariant to income redistributions, or, if not, whether income redistribution will increase or decrease the total equilibrium level of the public good.

Hence the question we ask here is: How will a redistribution of income affect the total voluntary provision? To simplify the analysis we will make the assumption that all individual have identical preferences.

**Assumption 1** *All consumers have identical preferences: \( u_i = u \) for all \( i \in I \).*

Consider now consumer \( i \)’s problem; fully written out the problem is

\[
\begin{align*}
\max_{x_i, z_i} & \quad u(x_i, z_i + z_{-i}) \\
\text{s.t.} & \quad z_i + x_i = R_i \\
& \quad z_i \geq 0.
\end{align*}
\]

However, using that \( z = z_i + z_{-i} \) we can rewrite this problem as if \( i \) was choosing the final total contribution level \( z \): adding \( z_{-i} \) (which is taken as given the individual \( i \)) on both sides of both constraints we obtain the following version of the individual’s problem:

\[
\begin{align*}
\max_{x_i, z} & \quad u(x_i, z) \\
\text{s.t.} & \quad z + x_i = R_i + z_{-i} \quad (13) \\
& \quad z \geq z_{-i} \quad (\spadesuit)
\end{align*}
\]

Hence we have written consumer \( i \)’s problem as if he was choosing the total level \( z \), but we have included \( z_{-i} \) as part of his wealth.

**FIG 3.5**

Apart from the inequality constraint (\( \spadesuit \)) this is a standard consumption problem where the consumer’s full wealth is \( R_i + z_{-i} \). Hence we can view the solution to this problem as a standard Walrasian demand function. Specifically solving (13) ignoring (\( \spadesuit \)) gives \( z = \zeta(R_i + z_{-i}) \) where \( \zeta(\cdot) \) is the individual’s Walrasian demand as function of the full wealth (and where the prices have been suppressed since they are assumed to be fixed).
Since the individuals have identical utility functions the Walrasian demand function \( \zeta (\cdot) \) is not individual-specific. We will assume that both the public good \( z \) and the consumption good \( x \) are normal goods:

**Assumption 2** Normality of both goods: \( \zeta (\cdot) \) is strictly increasing with \( 0 < \zeta' < 1 \).

However, in order to obtain the solution to the individual’s true problem (13) we will need to take the constraint (\( \star \)) into account. Doing this yields that the individual’s optimal choice of \( z \) is the Walrasian demand \( \zeta (R_i + z_{-i}^*) \) whenever this exceeds the contribution already made by the other individuals and will otherwise just be the contributions already made the others; hence the optimal \( z \) from the point of view of individual \( i \), given \( z_{-i} \), can be written as \( z = \max \{ \zeta (R_i + z_{-i}), z_{-i} \} \). Moreover, if \( z^* \) is a PPE, this must be true for every individual: that is, \( z^* \) must be a solution to (13) for every individual. Hence we obtain the result that the vector of contributions \( z^* = (z^*_i)_{i \in I} \) is a PPE if and only if

\[
z^* = \max \{ \zeta (R_i + z_{-i}^*), z_{-i}^* \} \text{ for all } i \in I. \tag{14}
\]

Note that this is what the definition of the PPE reduces to: if (14) holds for all \( i \), then everyone is choosing a best rest response to the contribution of everyone else. It tells us that no individual will want to alter the final total contribution by altering her own contribution. Hence either an individual is contributing, in which case the individual is on her Walrasian demand (with full wealth \( R_i + z_{-i}^* \)) or she does not contribute, in which case \( z^* = z_{-i}^* \).

Suppose that individual \( i \) is a contributor; then, at the PPE, the individual is on her Walrasian demand,

\[
z^* = \zeta (R_i + z_{-i}^*).
\]

Moreover, since the Walrasian demand \( \zeta (\cdot) \) is continuous (a general property of Walrasian demands) and strictly increasing (by assumption of normality), it is invertible. Applying the inverse we obtain

\[
R_i + z_{-i}^* = \zeta^{-1} (z^*)
\]
To interpret this, note that the inverse of the Walrasian demand, $\zeta^{-1}(z^*)$, gives the full wealth that at which the consumer demands $z^*$; the answer here is it must be $R_i + z^*_{-i}$. Now add the own contribution $z_i^*$ on each side and use that $z_i^* + z^*_{-i} = z^*$; rearranging give the following expression for the own contribution

$$z_i^* = R_i - \left[\zeta^{-1}(z^*) - z^*\right].$$

This is an intriguing equation: to see this note that the term in brackets is not particular to individual $i$. Hence we see that an increase in the individual’s own income $R_i$ increases her contribution $z_i^*$ one-for-one. To make it even more transparent we can denote the term in the bracket by $R^* = \zeta^{-1}(z^*) - z^*$. It is then clear that $R^*$ denotes the income level of a consumer who “just” contributes at the equilibrium.

$$R_i < R^* \Rightarrow z_i^* = 0 \quad (15)$$

In contrast, any consumer with income above $R^*$ contributes any income above $R^*$

$$R_i \geq R^* \Rightarrow z_i^* = R_i - R^* \quad (16)$$

FIG 3.7

So summarize:

**Lemma 2** If $z^*$ is a PPE, there exists a critical income level $R^* = \zeta^{-1}(z^*) - z^*$ such that that all consumers with $R_i \leq R^*$ contribute nothing, while any consumer with $R_i > R^*$ contributes $z_i^* = R_i - R^*$.

This result is even more remarkable than it first appears. To see this, consider two consumers $i$ and $j$ who are both contributing, $R_i, R_j > R^*$ and suppose that $R_i > R_j$ (say). Hence they have the same preferences but individual $i$ is richer than individual $j$. Consider then the consumption enjoyed by these to individuals. Consider first the consumption of the private good; from the individual’s budget constraint and using that, for any contributing individual $z_i^* = R_i - R^*$, it follows that

$$x_i^* = R_i - z_i^* = R_i - (R_i - R^*) = R^*. \quad (17)$$
Hence any individual who makes a positive contribution at the PPE will consume $x_i^* = R^*$ units of the private good. Thus our two contributing individuals will enjoy the same level of the private good. Moreover, since the public good is public they will also enjoy the same level $z^*$ of that good. Hence any two contributing individuals will enjoy exactly the same consumption of both goods; in particular, even though individual $i$ is richer than individual $j$, they both enjoy the same equilibrium utility!

This insight should lead us to suspect that it would have no impact on the outcome if we were to take £1 from individual $i$ and give it to individual $j$ (as long as they are still both contributing). Indeed, note that we can write the total contribution as

$$z^* = \sum_{i \in I} z_i^* = \sum_{i \in C^*} (R_i - R^*), \quad (18)$$

where $C^*$ is the same of individuals who contribute in the PPE. Hence the total contribution is exactly equal to the total amount of income, in excess of $R^*$, in the group of contributing individuals. Hence, indeed, if $i$ and $j$ are both contributing in the PPE, then transferring £1 from one to the other will have no effect on $z^*$ (nor on the consumptions of the private good and hence neither on the equilibrium utilities). Similarly, from (18) redistributing income within the group of non-contributers will have no impact on $z^*$. In this case, however, the redistribution will translate one-for-one into changes in private consumption.

Hence we have the following remarkable result:

**Proposition 2** Invariance to income redistributions. Let $z^*$ be the total contribution of a public good in a PPE. Then

- Any redistribution of income within the set of contributors will leave $z^*$ unchanged.
- Any redistribution of income within the set of non-contributors will leave $z^*$ unchanged.

Both these results consider redistribution of income within the groups of contributors and non-contributers respectively. It can also be shown that:

- A redistribution of income from a contributor to a non-contributor reduces $z^*$. 

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Example: Intra-Household Redistribution

One intriguing application of this result in the context of public policy is in the context of intra-household allocations. Suppose e.g. that we are concerned about the equality between genders and that we want to improve the well-being of wives relative to their husbands. In general we believe that there are many goods that are effectively “public” within households, e.g. housing costs, expenditures on children, etc. Suppose then that are contemplating changing the payment of e.g. child benefit so that it is paid out to the main carer (most often the female) rather than to the main earner (most often the male). If the intra-household allocation is one where there is voluntary contributions by the two spouses to (at least one) household public good, then the redistribution of income will have no effect on the allocation of consumption within the household – it will be neutralized by a corresponding change in contributions to the household public good by the two spouses (Browning, Chiappori, Lechene, 2005??).

4.4 Large Economies*

A feature of the private provision equilibrium is that the individuals are free-riding on the contributions of other individuals, and some individuals may even choose not to contribute. Will this be a bigger problem in large economies? Indeed that would seem most reasonable – as the economy grows, since the public good is non-rivalrous, if no one changed his or her behavior (in terms of contributions) the total public good provision would increase. But if the total public good provision increases, then we would expect a consumer to less willing to make contributions. One way to formalize this is to consider what happens to the set of contributors as we replicate an economy. What we will see is that replicating an economy will lead to an equilibrium where a smaller fraction of the population contributes (Andreoni, 1988).

Hence the question that we considering here is the following: What happens as an economy grows, in particular, will “free-riding” increase?

To answer this question consider an initial economy described by a set of consumers \( I = \{1, ..., n\} \) with identical preferences \( u(x_i, z) \) but with heterogenous incomes \( (R_i)_{i \in I} \). Refer to this as the initial economy, denote \( \mathcal{E}^0 \). From our previous analysis we have that
the PPE has the following features. There exists a critical income level $R_0^*$ such that, for all $i \in I$, the contribution of individual $i$ is $\max \{0, R_i - R_0^*\}$ whereby the total private provision in $E^0$ is thus $z_0^* = \sum_{i \in I} \max \{0, R_i - R_0^*\}$. Moreover,

$$R_0^* = \zeta^{-1}(z_0^*) - z_0^*.$$  \hfill (19)

Note that there is a positive relationship between $z^*$ and $R^*$.

**Lemma 3** $\zeta^{-1}(z^*) - z^*$ is a strictly increasing function of $z^*$.

**Proof.** Differentiating yields

$$\frac{d \left( \zeta^{-1}(z^*) - z^* \right)}{dz^*} = \frac{d\zeta^{-1}(z^*)}{dz} - 1 = \frac{1}{\zeta'} - 1.$$

Since $0 < \zeta' < 1$ (normality), $1 < 1/\zeta' < \infty$ whereby $\phi'(z) \in (0, \infty)$. ■

Now “replicate” the economy one; for every initial consumer $i \in I$ there is now an “identical twin”. Let $E^1$ to denote “once replicated economy” with consumers $2I$. Repeating the steps for the initial economy, in $E^1$ there is a PPE with a critical income level $R_1^*$ such that the contribution of individual $i$ is $\max \{0, R_i - R_1^*\}$ and the aggregate contribution will be $z_1^* = \sum_{i \in I} \max \{0, R_i - R_1^*\}$.

The key result now it that the critical income $R^*$ will have increased when we go to the larger economy. Formally:

**Proposition 3** The critical income is larger in the replicated economy, $R_1^* > R_0^*$.

**Proof.** Make the contradicting hypothesis, i.e. suppose that $R_1^* \leq R_0^*$. Then it immediately follows that

$$z_1^* \geq 2z_0^* > z_0^*,$$

(where $z_1^* = 2z_0^*$ if $R_1^* = R_0^*$). But then, since $\phi(.,)$ is increasing (Lemma 3)

$$R_1^* = \zeta^{-1}(z_1^*) - z_1^* > \zeta^{-1}(z_0^*) - z_0^* = R_0^*,$$

a contradiction. ■

Since the distribution of income is the same in $E^1$ as in $E^0$ it follows that the fraction of individuals who are making positive contributions will have gone down. A slightly more
difficult result to demonstrate is that, if we keep replicating the economy, at some point, only the richest consumers will contribute. Also when we keep replicating the economy total donations will converge to a finite positive number as the economy grows, but each consumer will be contributing less and less.

4.5 Empirical and Experimental Evidence on Private Provision of Public Goods


5 Voting over the Provision of Public Goods

So far we have seen that a private provision equilibrium will lead to underprovision of a public good. Indeed, this problem is often taken as a justification for public intervention. This requires collective action and thus some mechanism for collective decision making, typically voting. Hence we would like to know whether a democratic voting mechanism will lead to an efficient level of the public good being provided. There are many political models available which intend to capture different aspects of political processes. However, we will stick to the most basic model of a political process – we will consider a simple majority rule model. Hence the question we will be asking here is: Will voting lead to an efficient level of public good provision?

Before we can provide an answer to this question we will first need to consider how the public good will be financed. In particular, when voting over the level of the public good, there must also exist a rule for how the public good is to be financed – a cost-sharing rule. The cost-sharing rule will have an important role to play in shaping the individuals preferences over the level of the public good.

We will give two examples. The first example that we will consider is one where there the individuals in the economy have identical preferences but different incomes and where the public good is finance through linear taxation. The second example will have individuals with heterogenous preferences, equal income and equal cost-shares.
5.1 Example 1: Implicit Redistribution

As before suppose there is a public good $z$ and a private good $x$ each with unit prices. The individuals are assumed to have identical preferences. To make the problem particularly transparent we will assume that preferences are quasi-linear,

$$u(x_i, z) = \alpha x_i + H(z), \quad (20)$$

where $H(\cdot)$ is twice continuously differentiable, strictly increasing and strictly concave. Incomes, $R_i$, differ across the consumers.

Efficiency

To establish a benchmark we consider first the Pareto efficient level of the public good in this particular setup. Hence we make use of the Samuelson rule that states that the sum of the individual’s marginal rates of substitution should equal the marginal rate of transformation. Note that the marginal rate of substitution takes a particularly simple form due to the assumption of quasi-linear preferences; in particular:

$$\frac{\partial u_i}{\partial z} \frac{\partial u_i}{\partial x_i} = H'(z) \frac{\alpha}{n} \quad \text{for all } i \in I$$

Thus, from the Samuelson rule, we have that Pareto efficiency requires

$$\sum_{i \in I} \frac{\partial u_i}{\partial z} = 1 \iff H'(z) = \frac{\alpha}{n}.$$ 

where we used that the marginal rate of transformation between $z$ and $x$ is unity.

Since the marginal utility $H'(\cdot)$ is continuous and strictly decreasing it is invertible: let $\Psi(\cdot)$ be the inverse of $H'(\cdot)$, i.e. $\Psi = (H')^{-1}$ Since $H'(\cdot)$ is strictly decreasing, so is $\Psi(\cdot)$. The Pareto efficient level of the public good, which we will denote here by $z^*$, is uniquely determined by

$$z^* = \Psi\left(\frac{\alpha}{n}\right). \quad (21)$$

Why is the Pareto efficient level of the public good “unique” in the sense that it is independent of the distribution of utilities? This follows from the quasi-linearity of utility in income: to achieve a different utility distribution, only consumption of the private good should be shifted. The linearity implies that utility is effectively transferable. From (21)
we also see that the level of the public good should be smaller the stronger are the preferences for the private good as measured by $\alpha$ and the fewer individuals there are in the economy.

Since the Pareto efficient level of the public good, $z^*$, is unique in this case we have a good benchmark case. Hence we want to know whether voting over the public provision of $z$ will lead to a level of the public good that is efficient?

**Taxation**

Suppose that $z$ is finance by a linear income tax with tax rate $\tau$. Consumption of the private good is what remains of the individual’s income after the tax has been collected:

$$x_i = (1 - \tau) R_i \text{ for all } i \in I. \quad (22)$$

The government spends the tax revenues on the public good; hence

$$z = \sum_{i \in I} \tau R_i \Leftrightarrow z = \tau n \overline{R}$$

where $\overline{R}$ is the average income.

Using the government’s budget constraint to eliminating $\tau$ from (22) private consumption will be

$$x_i = \left(1 - \frac{z}{n \overline{R}}\right) R_i \text{ for all } i \in I. \quad (23)$$

Then, using (23) to replace $x_i$ in consumer $i$’s preferences (20), gives consumer $i$’s induced preferences over $z$, denoted $v_i$,

$$v_i (z) = \alpha \left(1 - \frac{z}{n \overline{R}}\right) R_i + H (z).$$

The “preferences” $v_i (z)$ describes the utility obtained by individual $i$ when the level of public provision is $z$. Hence $v_i (z)$ describes individual $i$’s preferences over policy. Note that $v_i (\cdot)$ is strictly concave in $z$; the first term is linear in $z$ while the second is strictly concave. Using the government’s budget balance condition, we have thus managed to reduce the problem to a single-dimensional policy problem – determining $z$ – and $v_i (z)$ states consumer $i$’s preferences over different levels of $z$.

We will be looking for a policy that is favoured by a majority over any other alternative.
Definition 7 A Condorcet winner is a policy that beats any other feasible policy in a pairwise comparison.

The first thing to figure out is what policy each consumer will like the most, i.e. her “ideal policy” or “bliss point”. This is the policy that maximizes $v_i(z)$. Since $v_i(z)$ is strictly concave individual’s ideal policy, which we will denote $z^i$, is unique. In particular, $z^i$ is characterized by

$$
\frac{dv_i}{dz} = -\frac{\alpha R_i}{nR} + H'(z^i) = 0.
$$

The next insight is that, since $v_i(\cdot)$ is concave in $z$, consumer $i$’s preferences over the level of public provision are single-peaked.

Definition 8 Policy preferences of consumer $i$ are single peaked if the following statement is true

If $z'' \leq z' \leq z^i$, or, if $z'' \geq z' \geq z^i$

then $v_i(z'') \leq v_i(z')$.

Using that $\Psi = (H')^{-1}$, we can “solve” for $z^i$,

$$
z^i = \Psi \left( \frac{\alpha R_i}{nR} \right).
$$

(24)

A well-known fact is that, if all voter have single peaked preferences, then a Condorcet winner exists.

Theorem 4 The Median Voter Theorem. If all voters have single-peaked preferences (over a given ordering of policy alternatives), a Condorcet winner exists and coincides with the median-ranked bliss-point.

This leaves the question: who has the median-ranked bliss-point? The answer is straightforward: it will be the individual with the median income. To see this, note that from (24) we see that the individual’s ideal policy $z^i$ is strictly decreasing in the individual income $R_i$. Hence if we rank the individuals in terms of their bliss points we obtain the reverse of the income ranking; this implies that the individual who is in “the middle” in terms of the income ranking is also in the middle in terms of the ranking of bliss points!
The reason why the bliss-point is decreasing in income is that the public good supply effectively redistributes income from high-income consumers to low-income consumers. Everyone obtains the same level of the public good (obviously), however, the high-income consumers pay more taxes to finance it. Hence they will oppose high levels of the public good. At the other extreme, a consumer with very little income effectively gets the public good for free.

Let \( R_m \) be the income of the median-income consumer. By the Median Voter Theorem we then know that the Condorcet winner is the bliss point for the individual with income \( R_m \). Hence the winning policy is the one where the level of public provision is

\[
z^m = \Psi \left( \frac{\alpha R_m}{n \overline{R}} \right). \tag{25}
\]

Most realistic income distributions are skewed so that the median is lower than the mean, \( R_m < \overline{R} \). Hence suppose that this is the case. We can then compare the winning policy to the efficient level of the public good.

**Proposition 5** Suppose that \( R_m < \overline{R} \). Then there is “over-provision” of the public good at the political equilibrium, \( z^m > z^* \).

The result follows from comparison of the characterization of the efficient level in (21) with the characterization of the efficient level in (25), and noting that

\[
\frac{\alpha R_m}{n \overline{R}} < \frac{\alpha}{n}
\]

and using that \( \Psi (\cdot) \) is strictly decreasing.

**FIG 3.9**

In the current model the public good presents a way to implicitly redistribute income. The since median consumer has a lower income than the average consumer, he will gain from public good provision in the sense that the cost of providing the good will fall predominantly on the high income consumers. In other words, the public good provision implicitly creates redistribution in favour of the median consumer. Hence he will want to push public good supply beyond the efficient level.
5.2 Example 2: Heterogenous Preferences

In the previous example, the cost-shares were determined by the assumption that the public good was financed through proportional taxation of incomes. The proportional taxation implied that consumers with below average incomes implicitly gained from public good provision – effectively if there were $n$ consumers, then any consumer with below average income will be paying less than $1/n$ of the total cost. With the majority of individuals having less than average income the equilibrium provision ended up being “too high”.

That voting does not generate a Pareto optimal level of a public good a very general phenomenon. To see this, let’s consider a second example. Whereas in the previous case we assumed that everyone had identical preferences, but that the implicit cost-shares differed across the consumers, we will now go in the opposite direction and assume that the cost-shares are identical, but that preferences are not.

Assume again $n$ consumers $i \in I = \{1, ..., n\}$. The individuals have the same incomes, $R_i = R$ for all $i \in I$ and we assume that there is an agreement that whatever level of $z$ will be provided, the cost will be shared equally: if $z$ is agreed, then consumer $i$ must pay $1/n$ of the cost, i.e. $z/n$.

Suppose however that tastes differ; consumer $i$’s utility is

$$u(x_i, z) = x_i + \beta_i H(z),$$  \hspace{1cm} (26)

where $\beta_i > 0$ measures the strength of individual $i$’s preferences for the public good and where $H(\cdot)$ twice continuously differentiable, strictly increasing and strictly concave.

Efficiency

As a benchmark, let us first characterize again the Pareto efficient level of the public good using the Samuelson rule. Note that the marginal rate of substitution for individual $i$ is

$$\frac{\partial u_i / \partial z}{\partial u_i / \partial x_i} = \frac{\beta_i H'(z)}{1} \quad \text{for all } i \in I.$$

Thus Pareto efficiency requires

$$\sum_{i \in I} \beta_i H'(z^*) = 1 \iff H'(z^*) n\bar{\beta} = 1$$
where $\overline{\beta}$ is the average $\beta_i$. Letting $\Psi(\cdot)$ be the inverse of $H'(\cdot)$ and solving for $z^*$ we have

$$z^* = \Psi \left( \frac{1}{n\overline{\beta}} \right)$$

which is unique since $\Psi(\cdot)$ is strictly decreasing.

**Policy Preferences**

As in the previous example we should try to determine the individuals’ preferences over the level of public provision. Note that if the level $z$ is provided, consumer $i$ must pay $z/n$. Hence her consumption of the private good will be

$$x_i = R - \frac{z}{n}.$$  \hfill (28)

Replacing $x_i$ in the preferences (26) we obtain the induced policy preferences

$$v_i(z) = R - \frac{z}{n} + \beta_i H(z).$$

Consumer $i$’s bliss-point, denoted $z^i$, maximizes $v_i(z)$ and thus satisfies

$$v_i'(z^i) = -\frac{1}{n} + \beta_i H'(z^i) = 0.$$  

Solving for $z^i$ we have

$$z^i = \Psi \left( \frac{1}{n\beta_i} \right).$$  \hfill (29)

Noting that the policy preferences $v_i(\cdot)$ are strictly concave in $z$ we conclude that all consumers have single-peaked preferences. Then, by the Median Voter Theorem, the median ranked bliss-point is the Condorcet winner. In this case there is heterogeneity among the voters in terms of the strength of their preferences for the public good. In particular, it individuals with stronger preferences for the public good will have larger bliss points: using that $\Psi(\cdot)$ is a strictly decreasing function it is easy to see that $\beta_i > \beta_j$ implies that $z^i > z^k$. Hence the individual with the median strength of preferences for the public good will also be the individual with the median bliss point.

Hence let $\beta_m$ be the preference-parameter of the consumer with the median strength preferences for the public good. Then the unique Condorcet winner is

$$z^m = \Psi \left( \frac{1}{n\beta_m} \right).$$  \hfill (30)
Suppose e.g. that this is a public good that is very much desired by a relatively small group; most people are not that bothered about the public good. In that case we would expect the average to exceed the median. Indeed comparing the characterization of the winning policy (30) to the efficient policy (27) we obtain the following result:

**Proposition 6** Suppose that $\bar{\beta} > \beta_m$; then there is “under-provision” at the political equilibrium $z^m < z^*$. 

In this case we have the “tyranny of the majority”: some group would value the public good a lot. However, the median consumer does not. The median voter is pivotal, and hence there will be under-provision at the political equilibrium.

### 6 Club Goods*

The basic public good model is interesting; however, are there really that many cases where the pure public good model fits as a description of the real world? In fact many public goods can be viewed as being

- congested (i.e. there is some rivalry)
- locally provided (in the sense of being enjoyed by a subset of the population).

These features makes the story quite a lot more intriguing. Suppose that we have a good from which people can be excluded. And suppose also that there is some congestion - e.g. a park. How many people should then share a common public good.

Back in the 1960s James Buchanan (??) put forward the idea of “club good” – a good that is enjoyed by a number of consumer, but which is not available to outsiders (see Cornes and Sandler (1996) for an introduction to club goods). So suppose we there is a public good which is congested in the sense that consumer i’s benefit from it depends negatively on the number of people also using it. What is then the optimal “club size”? On the one hand, the more people are using the public good, the smaller are the benefit to each consumer; but on the other hand, the cost of the public good is spread over a larger number of individuals.
Consider a public good with congestion. Since the public good is crowdable, there will be an incentive for groups of people to come together to enjoy the good, and to exclude others. In particular, when a public good is congested, the population may be partitioned into “clubs”, with each club consuming the locally supplied public good.

**Definition 9** A public good is congested if the benefit to a consumer from consuming it depends negatively on the number of consumers consuming the same good.

To formalize this let \( z \) be a public good and \( x \) be a private good, each with unit prices. Let \( m \) be the number of consumers using \( z \) and let \( \alpha (\cdot) \) be a strictly increasing function with \( \alpha (1) = 1 \). Then define

\[
\hat{z} = \frac{z}{\alpha (m)}
\]

as the “effective” services of the public good \( z \) to a consumer when a total of \( m \) consumer is using the same good.

For simplicity assume identical consumers with preferences \( u(x, \hat{z}) \) and income \( R \). Note that the individuals derive utility from the “effective” services of the public good rather than from the level of the public good.

In this setting it is natural to consider the optimal “club size”: how many consumer’s should be sharing the same public good. Hence we want to know if there is an optimal “club size” \( m \). To check this we must first determine – for any given club size \( m \) – the optimal amount of \( z \).

Assuming equal cost-sharing, if the amount \( z \) is provided, the cost to consumer \( i \) is \( \frac{1}{m} \). Thus, consumption of the private good is (recall that all consumers are identical here)

\[
x = R - \frac{z}{m}.
\]

The utility of the representative consumer is then

\[
u \left( R - \frac{z}{m}, \frac{z}{\alpha (m)} \right).
\]

Maximizing \( u \) with respect to \( z \) yields the first order condition

\[
- \frac{\partial u}{\partial x} \frac{1}{m} + \frac{\partial u}{\partial \hat{z}} \frac{1}{\alpha (m)} = 0,
\]

25
or, by rearranging,

\[
m \frac{\partial u/\partial \hat{z}}{\partial u/\partial x} = \alpha (m).
\]  

(32)

This modified Samuelson rule states that the sum of the marginal willingness to pay for the effective public good \( \hat{z} \) (i.e. the left hand side) should equal \( \alpha (m) \).

Thus the sum of the marginal willingness to pay for the effective public good should equal \( \alpha (m) \). Why \( \alpha (m) \) and not unity? This is because – given \( m \) – giving up one unit of \( x \) will increase \( z \) by one unit; however, that will only increase the effective public good by \( d\hat{z} = 1/\alpha (m) \). Hence the congestion effectively increases the marginal cost of generating effective units of the public good.

Consider now the optimal club-size? First, why will there be an optimal club size? The reason why there will be an optimal club-size is that increasing \( m \) has two effects. One the one hand it reduces the cost to each consumer for any given supply of the public good \( z \); but on the other hand, it reduces the benefit to a consumer from any given amount \( z \) since it adds to the congestion. Hence, e.g. if there were no congestion, the optimal club size would be the whole population. If there is congestion it may be optimal to partition the population into smaller groups (the clubs) enjoying their own public goods.

Maximize utility in (31) with respect to \( m \). The first order condition is

\[
\frac{\partial u}{\partial x} \frac{z}{m^2} - \frac{\partial u}{\partial \hat{z}} \frac{z}{(\alpha (m))^2} \alpha' (m) = 0
\]

which, after using the modified Samuelson rule, reduces to

\[
1 = \frac{\alpha' (m) m}{\alpha (m)} \equiv \varepsilon_{am},
\]

where \( \varepsilon_{am} \) is the elasticity of \( \alpha (\cdot) \) with respect to \( m \).

**Proposition 7** The optimal club size, \( m^* \), satisfies \( \varepsilon_{am} = 1 \).

What is the logic behind the result that the optimal club size is characterized by the elasticity of \( \alpha (\cdot) \) with respect to \( m \) being equal to unity? The answer is that it minimizes the cost of the effective public good. To see this note that the marginal cost of the “effective” public good \( \hat{z} \) in terms the private good \( x \) is

\[
\frac{\alpha (m)}{m}.
\]
To see why, note that in order to obtain one unit of “effective” public good \( \hat{z} \), \( \alpha(m) \) units of the underlying public good \( z \) are required. But obtaining one unit of \( z \) requires giving up \( 1/m \) units of private consumption. Hence to obtain one unit of \( \hat{z} \), \( \alpha(m)/m \) units of private consumption need to be foregone.

Then note that if choosing \( m \) to minimize the marginal cost \( \alpha(m)/m \) we obtain exactly the optimal club size, \( m^* \).

7 Preference Revelation for Public Goods*

So far we have found that voting equilibria tend to be inefficient. The basic reason is that they are based on pre-assigned cost-shares as illustrated in the two examples above. Here is one more example.

Consider an economy with a set of individuals (flat mates) \( i \in \{1, \ldots, n\} \) who are contemplating buying a TV. The TV is a discrete public good so \( z \in \{0, 1\} \) and costs \( c \) pounds. Consumer \( i \)'s utility is \( u_i(x_i, z) \) and her initial income is \( R_i \). Consider \( i \)'s maximum-willingness-to-pay for \( z \). If they do not buy the TV individual \( i \) will have the utility \( u_i(R_i, 0) \). Hence the maximum-willingness-to-pay for the TV will be the amount \( r_i \) where the individual obtains the same utility level.

**Definition 10** Consumer \( i \)'s reservation price for \( z \) is implicitly defined through \( u_i(R_i - r_i, 1) = u_i(R_i, 0) \).

In this case the public good is discrete so we have to used a discrete analogue of the Samuelson rule to determine when it is efficient to buy the TV. From the Samuelson rule we would expect that it will be efficient to buy the TV if the sum of the individuals’ willingness-to-pay is enough to cover the cost. This intuition is indeed correct.

To see this, note that it is efficient to buy the TV if and only if by doing so some individual can be made better off while no one else is made worse off. The statement that no one is worse off simply says that no one will be asked to pay more than her reservation price. Formally, providing the public good will Pareto dominate not providing the public good if we can find contributions \( g_i \) to the cost of the public good such that everyone is better off with the good being provided than with the public good not being
provided. Hence there must be some set of contributions, \((g_i)_{i \in I}\) such that
\[ \sum_{i \in I} g_i = c \]
and
\[ u_i (R_i - g_i, 1) > u_i (R_i, 0). \]
But clearly, this will be the case if and only if
\[ \sum_{i} r_i > c. \]

**Proposition 8** Providing the public good Pareto dominates not providing the good if and only if
\[ \sum_{i \in I} r_i > c. \]

However, the consumers may not know each other’s preferences. So suppose then that they decide to vote in order to determine whether to provide the good or not. So suppose, e.g., that they decide to split the cost equally: if the good is provided each consumer would have to pay \(c/n\). The question is whether the public good will be provided whenever it is Pareto efficient to do so. The immediate answer is no. This can be proved by way of example: Suppose e.g. that \(n = 3, c = 90, r_1 = 60, r_2 = r_3 = 20\). Then since
\[ \frac{c}{n} = 30 > 20 = r_2 = r_3 \]
consumer 2 and consumer 3 will vote against, despite the fact that \(\sum_{i=1}^{3} r_i = 100 > 90 = c.\)

Indeed, the problem seems to be more pronounced when the distribution of reservation prices is more dispersed. Obviously, if everyone has the same reservation price, then voting will generate the Pareto optimal decision. Hence voting based on pre-assigned cost-shares cannot in general be expected to lead to Pareto efficiency. Ideally we would like the cost-shares to be related to the reservation prices. But the basic problem is that we may not know the reservation prices – that’s why we are voting in the first place – to figure out a “good collective decision”. This raises the question if it is possible to make the individuals reveal how much they value the public good and, if so, can we reach Pareto optimality? The answers to these questions turn out to be “yes” and “almost”.

### 7.1 The Groves-Clarke Mechanism

Consider again the case of a discrete public good \(z \in \{0, 1\}\) with cost \(c.\)

\[2\] The section is based on Varian (1992) [??]
Assume quasi-linear utility

\[ u_i(x_i, z) = x_i + \xi_i z \]  

(33)

In this formulation, individuals' reservation price is precisely \( \xi_i \). Define consumer \( i \)'s net value, \( v_i \), for the public good

\[ v_i = u_i(R_i - s_ic, 1) - u_i(R_i, 0) = (R_i - s_ic) + \xi_i - R_i = \xi_i - s_ic. \]

Note that if we sum up the individuals' net values we obtain

\[ \sum_{i \in I} v_i = \sum_{i \in I} (\xi_i - s_ic) = \sum_{i \in I} \xi_i - c \]  

(34)

since \( \sum_{i \in I} s_i = 1 \). Since \( \xi_i \) is individual \( i \)'s reservation price it is hence efficient to provide the public good if and only if \( \sum_{i \in I} v_i > 0 \) (independently of the distribution of the cost-shares \( (s_i)_{i \in I} \)).

The question is whether there is any way we can find out the net values. Suppose we simply ask the consumers to report the net values. This will not be such a good idea, since the consumers may then have a strong incentive to over- or under report their true net value. Can we design a scheme where the consumers report their net values truthfully?

Consider the following mechanism:

1. Each consumer reports a net value (a “bid”) \( b_i \) which may or may not coincide with her true net value \( v_i \).

2. The public good is provided if and only if \( \sum_{i \in I} b_i \geq 0 \).

3. Each consumer receives a side transfer equal to the sum of the other bids, \( \sum_{j \neq i} b_j \) if the public good is provided (negative if \( \sum_{j \neq i} b_j < 0 \)).

Given this scheme, the optimal strategy for a consumer is to report \( b_i = v_i \) (truthful reporting) independently of how the other consumers choose their bids, \( b_j \). To see this note that individual \( i \)'s payoff takes the form

\[
\text{payoff to } i = \begin{cases} 
v_i + \sum_{j \neq i} b_j & \text{if } b_i + \sum_{j \neq i} b_j \geq 0 \\
0 & \text{if } b_i + \sum_{j \neq i} b_j < 0.
\end{cases}
\]

(29)
In words, if the good gets provided, then individual $i$ gets to pay her share and enjoy the good; this gives the net utility $v_i$. In addition she obtains the side transfer $\sum_{j \neq i} b_j$ which, using that utility is linear in private consumption, simply adds to her utility.

Given these payoff, what is $i$’s best choice if $b_i$? There are two cases to consider:

**Case 1.** Suppose that $v_i + \sum_{j \neq i} b_j > 0$.

In this case the good will be provided if individual $i$ reports her true net value (or something higher). Hence if the individual reports $b_i = v_i$ the public good will be provided and her payoff to $i$ will be $v_i + \sum_{j \neq i} b_j$. Note that in this case the consumer will be better off if the good is provided since, by assumption, $v_i + \sum_{j \neq i} b_j > 0$. Now ask if the individual can do better by reporting anything else than $b_i = v_i$.

- Can reporting $b_i > v_i$ increase individual $i$’s utility?

The answer is no. With any $b_i > v_i$ the public good will still be provided and the individual’s payoff will be unaffected.

- Can reporting $b_i < v_i$ increase $i$’s utility?

Again the answer is no. Reporting $b_i < v_i$ will either leave the payoff unaffected or it will lead to the public good not being provided, which would lower the individual’s utility.

Hence, if the consumer is better off with the public good being provided, she has no reason to either over- or under-report her net value. Consider now the opposite case where the consumer is better off with the public good not being provided.

**Case 2.** Suppose that $v_i + \sum_{j \neq i} b_j < 0$.

If the consumer reports truthfully $b_i = v_i$ the public good is not provided and her payoff is 0. Now ask if the individual can do better by reporting anything else than $b_i = v_i$.

- Can reporting $b_i > v_i$ increase $i$’s utility?

Again the answer is no. Reporting $b_i > v_i$ will either leave the payoff unaffected or it will lead to the public good being provided, which would lower the individual’s utility.

- Can reporting $b_i < v_i$ increase $i$’s utility?
The answer is again no. With any \( b_i < v_i \) the public good will still not be provided and the individual’s payoff will be unaffected.

Thus over- or under-reporting \( v_i \) can never increase consumer \( i \)'s utility, no matter how the other consumers choose their bids. In particular, the other consumers do not need to report truthfully.\(^3\)

So why does this mechanism work? It works effectively by making each consumer facing the social decision – if consumer \( i \) is pivotal in the sense that she alters the decision, then that will affect the net payment to her which equals the net impact of her decision on the other consumers in the economy. Hence with this mechanism, the public good is provided if and only if it is Pareto efficient to do so.

So are there any problems with the mechanism? Yes, the problem is that the side payment do not in general sum to zero – the sum may be either positive or negative. Hence, if we e.g. think about the mechanism being run by a government, the government cannot know a priori whether the policy will generate a surplus or a deficit. It is possible to redesign the mechanism so as to guarantee non-positive transfers (“Clarke taxes”), but this may lead to “wasted tax revenue” (which is not Pareto optimal). The problem is deep: in fact there is no way to design a mechanism that induces truthtelling and which is always budget balanced. However, if the economy is large, then the budget can be “almost balanced”.\(^4,5\)

8 **Externalities: An Overview**

We now turn to consider externalities. In fact public goods in a very particular form of externality: When someone voluntarily provides a public good, other people will benefit from it too. Hence, what follows is a more general analysis of externalities.

\(^3\)Truthful reporting is hence a “dominant strategy”.

\(^4\)The basic insight is that it is possible to affect the total transfers by introducing a second transfer to consumer \( i \); as long as it does not depend on consumer \( i \)'s reported benefit it will not affect the incentives to truthfully report the preferences.

\(^5\)The analysis above was for a discrete public good. Can it be applied to a continous public good? Yes it can. Assuming quasi-linear preferences the net pay off to consumer \( i \) is \( v_i (z) = \varphi_i (z) - s_i z \); the consumers can be induced to report the function \( \varphi_i (\cdot) \).
Our analysis will proceed in several steps. First we will define what we mean by externalities and introduce some useful distinctions.

**Part 1: Defining externalities and highlighting the market failure**

It turns out that it is not actually straightforward to define an externality. And moreover, allowing all possible effects within a general equilibrium model be very cumbersome; hence we will be looking at a mini-example with two producers and one consumer. That example will be sufficient for understanding what is required from a Pareto optimal allocation.

We will then go on to show why the decentralized equilibrium is inefficient; the decentralized equilibrium is also an important benchmark case, since several remedies to externality problems work by corrective mechanisms within the decentralized framework. One immediate upshot from that analysis is that one can think about the externality problem as a problem of missing markets. That in turn suggests that the way to “correct” externalities would be to establish property rights and to establish the relevant markets. This is an idea that we will come back to a bit later, but before that we will consider other types of “solutions”.

**Part 2: Solutions based on quotas, subsidies, and taxes**

The first obvious solution would be to impose quantity constraints: if a firm, say, generates a negative externality, then it produces too much output. In that case, imposing a quota should remedy the problem. However, quantity controls are quite demanding in terms of information requirement. If there is uncertainty, price controls may be more appropriate.

The traditional public finance solution to externalities is to impose taxes; corrective taxes are commonly known as Pigovian taxes (after Pigou, 1932). We will show how corrective taxes should be set equal to the marginal social damage.

However, the problem with Pigovian taxes is that they are equally information demanding as quantity controls. Both the quota-policy and a corrective tax-policy require that the regulator knows the extent of the externality. Suppose that that is not the case. We might then ask if it is sufficient that the parties involved both know the extent of the externality. Hence we will consider a mechanism that will work when the parties involved are well-informed about the externality, but where the regulator is not.
After going through those “solutions” we will turn to the idea that establishing property rights and establishing the relevant markets will solve the problem. This idea goes back to James Meade (1952); however, what Meade has in mind was the idea of prices and price-taking agents. But in many cases, there are only a few parties involved, in which case the idea of price-taking behavior becomes quite unrealistic. But of course, markets and price-taking behavior is generally not needed. Suppose e.g. that there are only two parties: a polluting firm and a negatively affected consumer (say). Then if the consumer was given the right to a clean environment, the two parties could bargain over the amount of pollution – most likely, the negative effect of a small amount of pollution on the consumer will be less than value to the firm in terms of increased profits. In that case, the Pareto optimal pollution level is strictly positive, and if the two parties bargain efficiently, they will achieve the Pareto optimal pollution level. This has become known as the Coase theorem (Coase, 1960).

It is questionable whether the Coase theorem is really a fundamental result – indeed, in many ways it is nothing but a tautology (there is no missing market problem is a market is established...) However, it is important in that it points out that the only government intervention that may be necessary is to establish property rights.

**Part 3: Establishing property rights**

However, there are several cases where we may be less inclined to believe that the Coasian approach will lead to an efficient allocation. The first such case is when an externality has a public good character; in that case exactly the same type of problem that we noted for privately provided public goods will reappear. If I buy clean air, that will benefit everyone else, an effect that I will not take properly into account. Hence I will contribute too little. Externalities with a public good character are known as non-depletable externalities. The case of non-depletable externality generates problem because it requires *multilateral* bargaining – there are numerous agents involved.

However, can we even expect the Coase claim to hold in the case of a simple *bilateral* externality? Maybe, but then again, maybe not! Another potential problem with the Coase theorem is that it assumes that the parties bargaining have perfect information about each others’ costs and benefits. This raises the question what happens when this is not the case – we will show that bilateral bargaining generally is inefficient when the
parties bargaining have private information about their costs and benefits.

9 The Nature of Externalities

Despite the fact that we all sort of know what an externality is, it turns out that it is not straightforward to define externalities precisely. Consider the following definition:

**Definition 11** There is an external effect, or an externality, when some agent’s actions directly influence either the production possibility set of a producer or the well-being of a consumer.

Though this sounds straightforward the word “directly” has generated some problems. In particular, it rules out what has become known as “pecuniary externalities” which are essentially effects through prices in the markets. (E.g. suppose I start up a firm which uses a lot of hamp; this brings up the price of hamp, which reduces the profitability of other firms using the same input. That would be a pecuniary externality — not an externality defined as above — and indeed, would not cause inefficiencies; at least not in an economy characterized by competition.)

**Definition 12** An externality that is favorable (unfavorable) to the recipient is referred to as a positive (negative) externality.

**Definition 13** An externality is said to be bilateral if there are only two parties involved, and multilateral if there more parties involved. Multilateral externalities are said to be depletable when they are “rivalrous” and non-depletable when they are “nonrivalrous”.

By nonrivalrous we mean that the externality has a public good (or public “bad”) character. This distinction will be important when we talk about the prospect of solving the externality problem by establishing property rights.

9.1 The Pareto Optimum

We don’t need to use the fully general case with $n$ consumers and $m$ firms in order to understand how externalities will affect the characterization of a Pareto optimum. Indeed,
the fully general case would generate more notation than insights. Hence we will consider
a simplified example with just two firms and one consumer.

Specifically, suppose there are two externalities affecting firm 2: one generated by the
consumer and one generated by firm 1.

\[ \text{FIG 3.10} \]

There are two goods in the economy and the consumer’s preferences are defined over
these two goods, \( u(x_1, x_2) \). The preferences are assumed to be strictly convex.

Firm 1 produces good 1 using good 2 as input. Denote the input \( \xi_2 \), and the output
\( y_1 \). Hence we assume that \( y_1 = f_1(\xi_2) \) where \( f_1 \) is increasing and concave and represents
firm 1’s technology.

Firm 2 produces good 2 using good 1, but is also affected by the consumer’s consump-
tion of good 1, i.e. \( x_1 \) and the output generated by firm 1 \( y_1 \). Hence \( y_2 = f_2(\xi_1, y_1, x_1) \),
where \( f_2 \) is increasing and concave in \( \xi_1 \) and represents firm 2’s technology and how
firm two is affected by \( y_1 \) and \( x_1 \). The externalities are both assumed to be “negative”:
\[ \frac{\partial f_2}{\partial y_1} < 0 \] and \[ \frac{\partial f_2}{\partial x_1} < 0. \]

The economy has an initial aggregate endowment of the two goods \( \overline{\omega} = (\overline{\omega}_1, \overline{\omega}_2) \).
Since we only have one consumer in the economy, the Pareto optimum can be found by
maximizing the utility of this one consumer. Hence consider the following problem:

\[
\max \ u(x_1, x_2) \\
\quad x_1 + \xi_1 \leq \overline{\omega}_1 + f_1(\xi_2) \\
\quad x_2 + \xi_2 \leq \overline{\omega}_2 + f_2(\xi_1, f_1(\xi_2), x_1)
\]

by choice of \((x_1, x_2, \xi_1, \xi_2)\). The first constraint says that the use (in consumption and as
input) of good 1 should not exceed what is available (as endowment and as output) of
that goods. The second constraint makes the same statement for good 2.

If \((x_1^*, x_2^*, \xi_1^*, \xi_2^*)\) is a solution, then by the Kuhn-Tucker theorem, there exists \((\lambda_1^*, \lambda_2^*)\)
such that the partial derivatives of the associated Lagrangian \( L \) is zero (at that point);
the partial derivatives reduce to

\[
\frac{\partial u}{\partial x_1} - \lambda_1^* + \lambda_2^* \frac{\partial f_2}{\partial x_1} = 0
\]

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\[
\frac{\partial u}{\partial x_2} = \lambda_2^* = 0
\]  
(37)

\[-\lambda_1^* + \lambda_2 \frac{\partial f_2}{\partial \xi_1} = 0 \]  
(38)

\[-\lambda_2^* + \lambda_1 \frac{\partial f_1}{\partial \xi_2} + \lambda_2 \frac{\partial f_2}{\partial y_1} \frac{\partial f_1}{\partial \xi_2} = 0 \]  
(39)

What is the interpretation of the multipliers? \(\lambda_h^*\) measures the social marginal valuation of good \(h\). In particular, the way we have set up the problem, \(\lambda_h^*\) measures the value – in terms of the objective function, i.e. the consumer’s utility – of having one more unity of good \(h\).

From basic micro theory we know that Pareto efficiency requires that all consumers’ marginal rates of substitution should be the same and, moreover, should be equal to the firm’s marginal rates of transformation. Translated into the current example, if there were no externalities, the optimum would have \(MRS = MRT_1 = MRT_2\), i.e.

\[
\frac{\partial u}{\partial x_1} + \left(\frac{\partial u}{\partial x_2}\right) \frac{\partial f_2}{\partial x_1} = \frac{\lambda_1^*}{\lambda_2^*}.
\]  
(40)

Similarly, from (38) we obtain

\[
\frac{\partial f_2}{\partial \xi_1} = \frac{\lambda_1^*}{\lambda_2^*}.
\]  
(41)

Finally, from (39) we obtain

\[
1 - \frac{\left(\frac{\partial f_2}{\partial y_1}\right) \frac{\partial f_1}{\partial \xi_2}}{\left(\frac{\partial f_1}{\partial \xi_2}\right)} = \frac{1}{\frac{\partial f_2}{\partial y_1}} \frac{\partial f_2}{\partial \xi_2} = \frac{\lambda_1^*}{\lambda_2^*}.
\]  
(42)

The left hand side of (40) can be called the social marginal rate of substitution; it takes into account the externality caused by consumption of good 1. When the consumer consumes one more unit of good 1 it directly increases her utility by \(\partial u/\partial x_1\) but it also reduces the output of good two by \(\partial f_2/\partial x_1\) which affects the objective function – i.e. the consumer’s utility – by \((\partial u/\partial x_2) (\partial f_2/\partial x_1)\). In contrast, when the consumer consumes
one more unit of good two, this only has the direct effect $\partial u/\partial x_2$ since this is not causing any externality.

Firm 2 is not generating any externalities; hence at the optimum, its marginal rate of transformation should be equal to the relative social marginal values of the two goods, which is exactly what (41) tells us.

Firm 1’s production on the other hand reduces the production of good 2, its marginal rate of transformation at the Pareto optimum takes the negative effect into account as shown in (42). When firm 1 uses one additional unit of good 2, it produces $\partial f_1/\partial \xi_2$ extra units of good 1. However, by the externality, this reduces the output of good 2 by $(\partial f_2/\partial y_1)(\partial y_1/\partial \xi_2)$.

Thus, the first lesson is that, even when there are negative externalities involved, this does not require total elimination of their sources – only that the external effect are somehow internalized: it does not require that the consumer does not consume good 1 of that first 1 shuts down its production.

9.2 Inefficiency of the Competitive Equilibrium

We can now readily verify that a competitive equilibrium fails to be Pareto efficient. Hence suppose that this economy was at a competitive equilibrium with prices $p = (p_1^*, p_2^*)$. Of course, at the competitive equilibrium, the consumer sets her marginal rate of substitution equal to the price ratio; moreover, each firm sets its marginal product equal to the price ratio. E.g. firm 1 solves the following profit maximization problem,

$$\max_{\xi_2} \{ p_1 f_1 (\xi_2) - p_2 \xi_2 \}. $$

which generates the first order condition $p_1/p_2 = (\partial f_1/\partial \xi_2)^{-1}$. Utility- and profit maximization then implies that, at the decentralized competitive equilibrium,

$$\frac{\partial u/\partial x_1}{\partial u/\partial x_2} = \frac{\partial f_2}{\partial \xi_1} = \left( \frac{\partial f_1}{\partial \xi_2} \right)^{-1} = \frac{p_1^*}{p_2^*},$$

which, as just argued, would fail to be Pareto efficient. Normally we would expect that too much of good 1 will be produced and consumed.
10 Interventions for Externalities

10.1 Quota Policy

The simplest way to arrive at a Pareto optimum is of course for the government to set quotas specifying that the externality generating activities should be set at their Pareto optimal levels.

Hence suppose that the government designs a quota policy that imposes the quantity constraint that \( x_1 \leq x_1^* \) and \( y_1 \leq y_1^* \). Note that we are relying heavily on the convexity of the preferences, and on concavity of the production function for this policy to work. Given the quotas \((x_1^*, y_1^*)\) the Pareto optimal allocation is indeed an equilibrium allocation: consider prices \((p_1^*, p_2^*)\) that are proportional to relative social marginal values \((\lambda_1^*, \lambda_2^*)\).

We want to verify that, given the quota policy, the prices \((p_1^*, p_2^*)\) along with the Pareto optimal quantities constitute a competitive market equilibrium. Firm 2 does not face any quota on its output and hence maximizes it profits given the prices \((p_1^*, p_2^*)\) which leads to

\[
\frac{\partial f_2}{\partial \xi_1} = \frac{p_1^*}{p_2^*} = \frac{\lambda_1^*}{\lambda_2^*}
\]

(43)

which verifies that (41) is satisfied; the firm will produce the Pareto optimal output level \(y_2^*\).

Consider then firm 1. We know that its profit function is concave. Hence the firm would like to produce at the level where

\[
\frac{1}{\frac{\partial f_1}{\partial \xi_2}} = \frac{p_1^*}{p_2^*} = \frac{\lambda_1^*}{\lambda_2^*}
\]

(44)

However, that would violate the quota policy. Instead it will optimally produce the maximum amount allowed by the quota. Hence the firm will optimally produce the Pareto optimal amount \(y_1^*\).

Finally, by strong convexity of the preferences, \((x_1^*, x_2^*)\) is optimal for the consumer as illustrated by next figure.

FIG 3.11

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Moreover, since \((x_1^*, x_2^*, y_1^*, y_2^*)\) is just feasible, we also know that the markets will are clearing. Hence the quota policy works under the assumption of convexity of the firm’s production possibility sets and of the individuals’ preferences. However, it should also be noted that it is a very information-demanding policy. It requires precise information so that \(x_1^*\) and \(y_1^*\) can be calculated.

## 10.2 Corrective Taxes

The traditional public finance solution to externalities is “corrective taxation” or “Pigouvian taxation”. Hence, continuing with the previous example, suppose that the government can impose commodity taxes. In particular, suppose that the government imposes a tax \(t\) be a tax per unit of \(x_1\) purchased and another tax \(\tau\) be a tax per unit of \(y_1\) produced. Hence the taxes are levied on the externality-generating activities. Since the taxes will generate tax revenue, that revenue will somehow have to be returned to the economy. We assume that any collected tax revenue is given to the consumer as lump-sum transfer \(T\).

Consider then the competitive equilibrium with producer prices \((p_1^*, p_2^*)\). Since there is a tax on good 1 the consumer price of good 1 will be \(p_1^*\). Consider first the consumer. Facing the prices \((p_1^* + t, p_2^*)\) she solves the problem

\[
\max_x \{ u(x_1, x_2) \mid (p_1^* + t) x_1 + p_2^* x_2 = R^* \},
\]

where

\[
R^* = (p_1^* + t^*) \xi_1 + p_2^* \xi_2 + T^* + \pi_1^* + \pi_2^*
\]

is the consumer’s wealth. Note that the consumer owns the initial endowment (which is valued at the market prices \((p_1^* + t, p_2^*)\)), receives the transfer \(T^*\) and also own the profits from the firms, denoted \(\pi_1^*\) and \(\pi_2^*\). The lump-sum transfer equals

\[
T^* = t^* x_1^* + \tau^* y_1^*.
\]

Each firm maximizes its profits; hence the profits for firm

\[
\pi_1^* = \max_{\xi_2} \{ (p_1^* - \tau^*) f_1(\xi_2) - p_2^* \xi_2 \}, \quad (46)
\]

where we used that this firm faces a tax of \(\tau^*\) per unit of output. Similarly for firm 2 profits are

\[
\pi_2^* = \max_{\xi_1} \{ p_2^* f_2(\xi_1, y_1^*, x_1^*) - p_1^* \xi_1 \}. \quad (47)
\]
Now suppose that the taxes are set to equal the value of the marginal external effects (at the Pareto optimum)

\[ t^* = -p_2^* \frac{\partial f_2}{\partial x_1}, \quad \text{and} \quad \tau^* = -p_2^* \frac{\partial f_2}{\partial y_1}. \]  

(48)

At the competitive equilibrium (45) the consumer chooses to consume where the marginal rate of substitution is equal to the ratio of consumer prices; hence

\[ \frac{\partial u/\partial x_1}{\partial u/\partial x_2} = \frac{p_1^* + t^*}{p_2^*}; \]  

(49)

while profit maximization by firm 1 (46) implies that

\[ \frac{(p_1^* - \tau^*)}{p_2^*} = \left( \frac{\partial f_1}{\partial \xi_2} \right)^{-1}, \]  

(50)

and profit maximization by firm 2 (47) implies that

\[ \frac{\partial f_2}{\partial \xi_1} = \frac{p_1^*}{p_2^*}. \]  

(51)

But, using (48), (49) through (51) implies that the ratio of the equilibrium prices satisfy,

\[ \frac{p_1^*}{p_2^*} = \frac{\partial u/\partial x_1 + (\partial u/\partial x_2) (\partial f_2/\partial x_1)}{\partial u/\partial x_2} \]

\[ = \frac{\partial f_2}{\partial \xi_1} = \left[ \left( \frac{\partial f_1}{\partial \xi_2} \right)^{-1} - \frac{\partial f_2}{\partial y_1} \right]; \]

The equality of the last three terms was precisely the characterization of the Pareto efficient allocation \((x_1^*, x_2^*, \xi_1^*, \xi_2^*).\)\(^6\)

**Proposition 9** Given that the consumer’s preferences and the firms’ production possibility sets are convex, Pigouvian taxes can restore the Pareto optimum.

Convexity here plays very much the same role as it does in the second fundamental theorem of welfare economics – it ensures that the marginal condition indeed uniquely characterize the agents’ best choices.\(^7\)

---

\(^6\)When profits accrue to the consumer and she also receives the transfer \(T^* = t^* x_1^* + \tau^* f_1(\xi_2^*)\) she can just afford to buy \((x_1^*, x_2^*).\)

\(^7\)One should be a bit careful though – given the taxes \((t^*, \tau^*)\) there may be other competitive equilibria – in other words, we might worry about equilibrium multiplicity.
The corrective taxation solution also has the same problem as a quota policy: it requires detailed information about preferences and technologies.

10.3 A Compensation Mechanism*

Quotas and taxes thus require that the regulator knows the costs and benefits associated with an externality. This may not always be true; in particular, it is reasonable to think that the parties involved have a better idea. What we will show is that, if there is a bilateral externality between two firms which both parties have perfect information about, then one can design a mechanism that will make the both agree on the correct Pigouvian tax.8

Consider the same setup as before, only assume that consumption does not generate any externality. Hence focus on externalities in production (from firm 1 to firm 2):

\[ y_1 = f_1(\xi_2) \text{ and } y_2 = f_2(\xi_1, y_1). \]

As before, Pareto optimality requires

\[ \frac{\partial f_2}{\partial \xi_1} = \left( \frac{1}{\partial f_1/\partial \xi_2} - \frac{\partial f_2}{\partial y_1} \right) \]

Suppose that the government doesn’t know the size of the externality but that the firms both know the size of the externality. A Pigovian tax cannot then immediately be set – the government does not have the information required to do so. Hence we would like to know whether the firms can be induced to reveal the size of the externality?

Consider the following mechanism which has two stages:

**Stage 1:** Each firm \( j \) announces a tax level \( \tau_j \) which may or may not coincide with the Pigovian tax \(-p_2(\partial f_2/\partial y_1)\).

**Stage 2:** If firm 1 produces \( y_1 \), it has to pay a tax \( \tau_2 y_1 \) while firm 2 receives a compensation \( \tau_1 y_1 \). In addition, each firm pays a penalty if \( \tau_1 \neq \tau_2 \), e.g. \((\tau_1 - \tau_2)^2\).

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8This section is based on Varian (1992). [Should check original ref...]
The precise form of the penalty doesn’t matter as long as it is positive when \( \tau_1 \neq \tau_2 \) and zero when \( \tau_1 = \tau_2 \).

Note that each firm cannot directly affect the tax it pays/compensation it receives by its own announcement: firm 1’s tax payment is based on the tax announced by firm 2, and the compensation firm 2 receives is based on the tax that firm 1 announced. This breaks the link between the own tax announcement and the one payment/compensation; finally – the penalty implies, as we will see, that both parties will have an incentive to announce the same tax.

One can then show that it is optimal for each firm to announce the Pigovian tax. To do this, we will solve for a subgame perfect equilibrium (SPE) using backwards induction. Note that the game has two stages: first the firms announce tax rates, and then they choose output levels. We use an SPE to make sure that no equilibria are upheld by threats that are not credible.

**Solving stage 2 given \((\tau_1, \tau_2)\).**

After the taxes have been announced, each firm chooses its production; hence firm 1 chooses the level of its input \( \xi_2 \), and firm 2 chooses the level of its input \( \xi_1 \). Given prices \( p = (p_1, p_2) \) and tax announcements \((\tau_1, \tau_2)\), firm 1’s profits are

\[
\pi_1 = (p_1 - \tau_2) f_1 (\xi_2) - p_2 \xi_2 - (\tau_1 - \tau_2)^2,
\]

which are maximized when

\[
(p_1 - \tau_2) \frac{\partial f_1}{\partial \xi_2} = p_2.
\]

Note that firm 1’s output will depend on \( \tau_2 \), i.e. \( y_1 (\tau_2) \); by concavity of \( f_1, y'_1 (\tau_2) < 0 \). Intuitively, the smaller is the net-of-tax price for its output, the less firm 1 will produce.

Firm 2’s profits are

\[
\pi_2 = p_2 f_2 (\xi_1, y_1) - p_1 \xi_1 + \tau_2 y_1 - (\tau_1 - \tau_2)^2,
\]

which are maximized when

\[
p_2 \frac{\partial f_2}{\partial \xi_1} = p_1.
\]

Having characterize the output levels, we can now turn to the first stage – the announcement of taxes.
Solving stage 1

Consider first firm 1. In order to maximize $\pi_1$ firm 1 always wants to set

$$\tau_1 = \tau_2.$$  \hspace{1cm} (56)

This is immediately clear from (52): firm 1 is affected by its own announcement only through the penalty function; hence firm 1’s problem reduces to trying to predict what firm 2 will announce.

So what will firm 2 announce? Intuitively firm 2 has three goals – maximizing the transfer it receives, maximizing its own operational profits, and minimizing the penalty for differing announcements. However, it can effectively ignore the last goal since we know that firm 1 will have the incentive to exactly match firm 2’s announcement!

Since firm 1 has the incentive to match $\tau_2$, firm 2 will set $\tau_2$ to maximize the sum of its own operational profits plus the received transfer. Maximize $\pi_2$ by choosing $\tau_2$,

$$\frac{\partial \pi_2}{\partial \tau_2} = \left( p_2 \frac{\partial f_2}{\partial y_1} + \tau_2 \right) y_1' (\tau_2) + 2 (\tau_1 - \tau_2) = 0,$$

where, in light of (56), the last term vanishes at the SPE.

So what is the trade-off here? Firm 2 can effectively choose firm 1’s output by the tax is chooses to announce. The condition above implies that at the optimum, firm 1 must be indifferent to marginal variation in $y_1$. The only time he is indifferent about marginal variation in $y_1$ is when he is exactly compensated for the externality!

Since $y_1' (\tau_2) < 0$ firm 2 optimally announces

$$\tau_2 = -p_2 \frac{\partial f_2}{\partial y_1}.$$  \hspace{1cm} (57)

Combining (57) with (53) (and rewriting (55)) it follows that, in the SPE,

$$\left( \frac{\partial f_1}{\partial \xi_2} \right)^{-1} - \frac{\partial f_2}{\partial y_1} = \frac{\partial f_2}{\partial \xi_1} = \frac{p_1}{p_2}.$$

Thus, with prices $(p_1^*, p_2^*)$ proportional to $(\lambda_1^*, \lambda_2^*)$, the Pareto optimum is obtained in equilibrium.

**Proposition 10** The Compensation Mechanism implements the Pareto optimum, and in the unique subgame-perfect equilibrium of the tax-announcement game, both firms announce the Pigovian tax $\tau^* = -p_2^* (\partial f_2 / \partial y_1)$. 

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11 Establishing Property Rights

The Coase theorem is central to understanding the policy implications of externalities; it is important because it states the conditions under which we can expect the market to solve the externality problem with minimal intervention. The minimal intervention consists simply of establishing property rights. E.g. in the case of a firm polluting and reducing the profits of a second firm, establishing property rights simply involve specifying whether firm 1 has the right to pollute, or whether firm 2 has the right to a pollution-free environment.

Once one grasps the idea behind the Coase theorem, one can also perceive some of its weaknesses. E.g. in the case of two firms where one is polluting the environment for the second, it is not clear why bargaining should be necessary: anyone who has access to sufficient funds should be able to buy both firms and make a profit out of it! Another serious weakness is that it deals badly with multilateral non-depletable externalities (i.e. public bads) since there will be a free-rider problem.

The Coase theorem was never formally stated, but a generally agreed version might read as follows.

**Theorem 11** The Coase Theorem. *If property rights are clearly stated and transaction costs are zero, bargaining will lead to an efficient outcome no matter how the property rights are allocated.*

### 11.1 Some Words about Bargaining

In order to investigate the Coase theorem, we will need to use bargaining theory. This is a large literature on bargaining. However, one can quite broadly split the literature up into two parts.

First there is a “noncooperative bargaining approach” which models bargaining processes using noncooperative game theory. A commonly used version is the Rubinstein-Stahl model. In the basic Rubinstein-Stahl model, two parties bargain over how to split a given amount of money and the process goes as follows. First player 1 offers a split

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9See e.g. Mas-Colell, Whinston and Green (1995, Ch. 22).
which player 2 then can accept or reject; if player 2 accept the game ends, but if player
2 rejects, player 2 can suggest a split that player 1 can accept or reject etc. The game
can potentially go on forever, however, there is a delay cost as long as the game keeps
going. Both parties are rational and can see through what will happen if offers are re-
jected; hence by introspection of the game, they settle immediately at a split which is
more favorable to both than to start grinding their way through the time-consuming
process. The Rubinstein-Stahl model implies Pareto efficiency (in the case of splitting a
given amount of money this simply means that no money is thrown away and their is no
delays in settling). Pareto efficiency is a common feature of noncooperative bargaining
equilibria as long as there is no private information held by the parties.

The second branch of the bargaining literature is the “axiomatic bargaining ap-
proach”. This approach imposes conditions (axioms) that an outcome are expected to
satisfy (symmetry, individual rationality, etc.) which in turn generates a choice rule. A
commonly used version is the Nash Bargaining Solution (NBS). The NBS maximizes the
product of the agents’ utility-increases, i.e. \( \Pi = (u_1 - u_1^0)(u_2 - u_2^0) \) where \( u_i^0 \) is agent
\( i \)'s utility if there is no agreement. FIG 3.15 illustrates the case with \( u_i^0 = 0, i = 1, 2 \).
Note that with \( u_i^0 = 0 \) the objective function is just \( \Pi = u_1u_2 \); hence the level-curves (or
“indifference curves if you like) are just like standard Cobb-Douglas indifference curves.
The Nash bargaining solution is the only one that satisfies a certain set of axioms. How-
ever, since one of the axioms that is always adopted in the axiomatic bargaining approach
is Pareto optimality, the axiomatic bargaining solutions cannot be used to “prove” the
Coase theorem.

FIG 3.15

We will instead use the simplest version of a noncooperative bargaining solution: A
single take-it-or-leave-it offer. One agent can make an offer that the other agent can other
accept or reject (there can be no counter offers or any other further bargaining). In fact,
it can be viewed as the smallest version of a finite stage version of the Rubinstein-Stahl
model. The fact that there can only be one stage, implies that a lot of bargaining power
is given to the party making the offer, but that is not important for our purposes.
11.2 Bilateral Externalities

We will now use a slightly different model from that used above. Indeed, it is possibly the simplest model of an (abstract) externality involving just two individuals. Hence consider two consumers $i \in \{1, 2\}$. Consumer 1 chooses some action $h$ which has a private benefit to her, but which affects consumer 2 negatively. Each consumer has quasi-linear utility defined on $h$ and “income”.\textsuperscript{10}

$$u_i(h, x_i) = \phi_i(h) + x_i, \quad i = 1, 2.$$  

Assume that $\phi_i(0) = 0$, $i = 1, 2$ but $h > 0$ implies $\phi_1(h) > 0$ and that $\phi_2(h) < 0$. Initial endowments of income $(\omega_i)_{i=1,2}$.

Let’s start by characterizing Pareto optimality. We can do this by maximizing the utility of individual $i$ given a fixed utility level for individual 2,

$$\max_{h, x_1, x_2} \phi_1(h) + x_1$$

s.t

$$\pi_2 \leq \phi_2(h) + x_2$$

and

$$x_1 + x_2 \leq \omega_1 + \omega_2$$

The Lagrangean is

$$\mathcal{L} = \phi_1(h) + x_1 - \lambda(\pi_2 - \phi_2(h) - x_2)$$

$$-\mu(x_1 + x_2 - \omega_1 - \omega_2).$$

Simple calculus shows that, at the optimum $\lambda^* = \mu^* = 1$ and, importantly,

$$\phi_1'(h^*) + \phi_2'(h^*) = 0.$$  \hfill (58)

The reason why $\lambda = \mu = 1$ is because the consumers have quasi-linear utility. This implies that the social value of one extra unit of income is unity (whereby $\mu = 1$). Moreover, again due to quasi-linearity, the utility trade-off between consumer 1 and 2 is always one-for-one (whereby $\lambda = 1$) – the utility possibility frontier (UPF) has a slope of -1.

\textsuperscript{10}This formulation is not as restrictive as it may seem. $h$ might for example be a purchased good – we would then include the cost of $h$ in the definition of $\phi_1(h)$.

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The Pareto optimal \( h^* \) thus solves

\[
\max_h \{ \phi_1 (h) + \phi_2 (h) \} \tag{59}
\]

and we assume that \( \phi_1 (\cdot) + \phi_2 (\cdot) \) is strictly concave. Note that \( \phi_1 \) can be interpreted as the “total benefit” of \( h \) while and \(-\phi_2 (\cdot)\) can be interpreted as the “total damage”. Hence Pareto optimality simply requires that the marginal benefit (accruing to consumer 1) of the activity \( h \) should equal the marginal damage (on consumer 2).

FIG 3.16 illustrates the UPF given \( h^* \) and given \( h \neq h^* \). When \( h \neq h^* \) the utility possibilities frontier is not pushed to the limit: for any \( h \neq h^* \) the frontier lies within that for \( h^* \). The slope of minus unity reflects that fact that utility can effectively be transferred one-for-one between the two consumers by transfers of income, which is true whether of not \( h = h^* \).

FIG 3.16

**Take-it-or-leave-it bargaining**

The Coase theorem states that efficiency will obtain no matter how we allocate the initial property rights; as long as the property rights are specified and bargaining can occur. We will start by assuming that the property rights are provided to consumer 2; consumer 2 is given the right to a pollution-free environment.

**Case 1: Right to pollution-free environment.**

Start by determining the initial utilities (at \( h = 0 \))

\[
u_i^0 = \phi_i (0) + \omega_i, \quad i = 1, 2.
\]

The initial utilities \((u_1^0, u_2^0)\) are illustrated as point \( a \) in FIG 3.17.

FIG 3.17

Suppose now that consumer 1 can make a take-it-or-leave-it offer to consumer 2, offering to pay \( T \) for the permission to generate level \( h \). Consumer 2 will accept if and only if this makes her no worse off,

\[
\phi_2 (h) + \omega_2 + T \geq u_2^0. \tag{60}
\]
Given this acceptance rule for consumer 2, consider then how consumer 1 chooses the offer \((h, T)\) so as to maximize her own utility

\[
\max_{(h, T)} (\phi_1 (h) + \omega_1 - T) \quad \text{s.t. } (60).
\]

The acceptance-constraint will trivially bind – individual 1 will certainly not pay more than is required to make individual 2 accept the offer. Hence, substituting for \(T\), individual then chooses \(h\) to solve

\[
\max_h (\phi_1 (h) + \omega_1 + \phi_2 (h) + \omega_2 - u_2^0),
\]

which has the solution \(h^*\) (Pareto efficiency).

In this case we have given all the bargaining power to consumer 1 (by giving her the chance to make a take-it-or-leave-it offer). She used this to improve her utility as much as possible. Consumer 2 on the other hand will, by the binding acceptance constraint, remain at her initial utility level \(u_2^0\). In terms of the figure, only consumer 1’s utility has increased, and has moved the economy to the frontier.

According to the Coase theorem, we should obtain \(h = h^*\) also if we were to allocate the property right to consumer 1. Hence, consider giving consumer 1 the right to pollute as much as she wants.

**Case 2: Right to pollute**

Again, determine initial utilities. In this case, in the absence of any agreement individual 1 simply sets \(h\) unilaterally so as to maximize the own utility. Hence define \(h^0\) as the solution to \(\max_h \phi_1 (h)\). The initial utilities are then

\[
u^0_i = \phi_1 (h^0) + \omega_i, \quad i = 1, 2.
\]

The initial utilities \((u^0_1, u^0_2)\) are illustrated as point \(b\) in FIG 3.18.

**FIG 3.18**

Note that \(b\) is more favorable to consumer 1 than point \(a\) since she now has the right to pollute. Suppose that consumer 2 can make a take-it-or-leave-it offer to consumer 1 offering to pay \(T\) if consumer 1 reduces the action to \(h\). Consumer 1 accepts if it makes her no worse off,

\[
\phi_1 (h) + \omega_1 + T \geq u^0_1.
\]
Knowing individual 1’s acceptance rule, individual 2 choose the offer \((h, T)\) to maximize the own utility,
\[
\max_{(h,T)} (\phi_2(h) + \omega_2 - T) \quad \text{s.t.} \quad (61).
\]
But the constraint will bind; hence the chosen \(h\) solves
\[
\max_{(h,T)} (\phi_2(h) + \omega_2 + \phi_1(h) + \omega_1 - u_1^0),
\]
which again implies that \(h^*\) is offered.

In FIG 3.18 the trade moves the consumers from \(b\) to \(e\). In this case we have given all the bargaining power to consumer 2 (by giving her the chance to make a take-it-or-leave-it offer). She uses this to improve her utility as much as possible. Consumer 1 on the other hand will, by the binding acceptance constraint, remain at her initial utility level \(u_0\). In terms of the figure, only consumer 2’s utility has increased, and has moved the economy to the frontier.

In the two cases we gave the property rights to one consumer and the opportunity to make a take-it-or-leave-it offer to the other consumer. We could equally well have given both rights to the same consumer. Of course, the more rights and opportunities a consumer has the better off she will be in the final equilibrium. The main point is that any combination of property rights and power will lead to a Pareto efficient allocation as illustrated in FIG 3.19.

**Private Information***

So far we the Coase theorem has held up pretty well. So what might cause it to break down? One implicit assumption in the analysis above was that the two affected parties were assumed to have perfect information about each other. E.g. the individual making the take it or leave it offer knew exactly which offers the other individual would accept.

This assumption on information is required for the Coase theorem to go through. If the agents bargaining do not have perfect information about each others’ costs and benefits, bargaining will generally not be efficient. What do we mean by that? The idea is best illustrated by an example.

Hence consider again a simple example with take take-it-or-leave-it bargaining. As before there are two risk-neutral agents \(i \in I = \{1, 2\}\). This time, we make the setting
even simpler by assuming that the action taken by individual $i = 1$ is discrete, $h \in \{0, 1\}$; if individual 1 chooses $h = 1$ this generates a negative externality.

Engaging in the activity $h = 1$ generates some benefit (or “value”) to individual 1 which we can denote $v_1$. Individual 2 on the other hand has a willingness to pay for (or “value” of) $h = 0$ that is is $v_2$. Suppose that initially we give the property rights to individual 1 who then chooses $h = 1$. Each individual $i$ know his or her own value $v_i$ but not the value of the other individual. Hence from the point of view of individual 1, $v_2$ is a random variable and vice versa. Suppose each $v_i$ is drawn from a uniform distribution with support $[0, 1]$.

Pareto efficiency in this case is trivial: if $v_1 > v_2$ then $h = 1$ in any Pareto optimal allocation whereas if $v_2 > v_1$ then $h = 0$ in any Pareto optimal allocation. The question is whether bargaining will ensure that Pareto optimality obtains. To explore this, suppose that individual 2 makes a take-it-or-leave-it offer, $b$, to individual 1. As before we will solve the game using backwards induction.

**Stage 2:** Suppose that individual 2 has offered to pay $b$ in order for individual 1 not to engage in the activity, $h = 0$. Will individual 1 accept or reject this bid $b$? The answer is simple: individual 1 accepts if and only if $b \geq v_1$.

**Stage 1:** Consider now how individual 2 will choose what offer to make. Individual 2 is aware of the decision rule that individual 1 in the second stage. Moreover, we would expect individual 2 to make a larger bid the more she values $h = 0$. Since the individual is risk-neutral she will choose the offer $b$ in order to maximize her expected utility condition on her own valuation $v_2$. If her offer is turned down, there will be no payment and no change in $h$ (individual 1 will continue to engage in the activity, $h = 1$) so the change in individual 2’s net utility will be zero. On the other hand, if the offer is accepted, then the activity stops, which increases individual 2’s utility by $v_2$, but she must also pay the offer $b$. Hence the impact on her utility if the offer is accepted is $v_2 - b$. Hence we can write the change in individual 2’s expected utility as

$$E [u_2] = (v_2 - b) \Pr (\text{Individual 1 accepts}) + 0. \Pr (\text{Individual 1 accepts})$$

Note that individual 2 will not know in advance whether her offer will be accepted or not. However, she knows that individual 1 is rational (and she knows the distribution of
v_1 \). Hence she can determine, for any offer \( b \) that she might make, what is the probability of the offer being accepted. To see this, note that since \( v_1 \sim \text{Uni}[0,1] \) and using that the offer \( b \) is accepted if and only if \( v_1 \leq b \) we obtain that, for any \( b \in [0,1] \)

\[
\Pr(\text{Agent 1 accepts}) = \Pr(v_1 \leq b) = b
\]

where we used that, for the uniform distribution, the CDF evaluated at \( b \) is just \( b \). Hence agent 2 will, conditional on her own value \( v_2 \), solve

\[
\max E[u_2] = \max_b \{(v_2 - b) b\}.
\]

Solving this problem we see that she will make an offer that equal half of her own valuation,

\[
b = \frac{v_2}{2}.
\]

What is the interpretation of this? Clearly, individual 2 has an incentive to place a bid that is less than her value \( v_2 \); after all if she were to offer to pay \( b = v_2 \) she would never make any gain; her net utility would be zero whether she won the object or not! She can obtain a positive expected utility by shaving the offer a bit. There is of course a trade-off. By shaving the offer, she will make a larger gain if the offer is accepted; but on the other hand, she runs the risk of the offer being rejected!

We can now verify that there is a non-zero probability of the final outcome failing to be Pareto efficient. To do this, consider the ex ante likelihood of an inefficient outcome? Ex ante here means before the values \( v_1 \) and \( v_2 \) are fixed.

Note that an inefficient outcome obtain when \( v_2 > v_1 \) (so that the activity should be terminated) but \( b < v_1 \) (so that the offer to pay \( b \) for the activity to be terminated is rejected). Using that \( b = v_2/2 \) the probability of this happening is

\[
\Pr(v_1 < v_2 \text{ and } v_1 > b) = \Pr\left(\frac{v_2}{2} < v_1 < v_2\right) = \frac{1}{4}.
\]

Hence, there is in this case a 25 percent chance that the final outcome will be Pareto inefficient. In this case this is the only possible type of inefficiency – the activity should be terminated but fails to be terminated in equilibrium. Had we instead give the initial property right to individual 2 then there would have been a positive probability that
Pareto efficiency require that the activity should be take place but it fails to do so in equilibrium.

FIG 3.20

The fact that take-it-or-leave-it bargaining failed to be efficient in this case was no coincidence. This is a very general phenomenon. Indeed, there is a general result due to Myerson and Satterthwaite (1983). Very loosely stated Myerson and Satterthwaite proved the following:

- When there is two-sided uncertainty in a bilateral bargaining situation, then no way of bargaining will ever guarantee ex post efficient outcomes.

What Myerson and Satterthwaite proved was that, when both parties to a bargain hold private information about their valuation of the traded good, there is no way to design bargaining such that ex post efficiency is guaranteed.

Proving that theorem is understandably quite difficult: after all, if you want prove that there is no way to achieve efficiency, you have to rule out quite a lot of possible ways to bargain (flipping coins, “stone-paper-scissors”, sealed bids, ...). The way that Myerson and Satterthwaite proved their result was quite ingenious. What they showed was that any mechanism that one can come up with boils down to two objects: a probability of a trade being made, and the size of the transfer. Of course, the probability and the transfer will generally depend on the valuations $v_1$ and $v_2$. Once you realize that any method of bargaining is completely characterized by its induced probability- and transfer-function, one can equally well restrict one’s attention to mechanism where each agent truthfully announces her valuation $v_i$: rather than using the initial bargaining method, the parties can just as well simply use the induced probability- and transfer-functions directly. Now that insight was of course crucial: now the search could be narrowed down to direct mechanisms (direct in the sense that each agent is asked to announce her valuation) which induce truth telling (revealing) If there is no direct revealing mechanism that is efficient, there simply cannot exist any efficient mechanism. From that point onwards, the Myerson and Satterthwaite is really not such a big mystery and the proof is not that outrageously difficult. (For details, see MWG)
11.3 Multilateral Externalities*

So far we have considered bargaining involving only two parties: an externality generating party and an affected party. Bargaining becomes more problematic when many individuals are involved. This is particularly the case for “non-depletable” externalities. The problem here is that most “solutions” based on bargaining suffer from the same free-rider problems as voluntary public good provision equilibria.

A Public Bad Example

To illustrate the problem we can look at the case of a firm choosing some action \( h \) which negatively affects a set of individuals. Hence suppose that there is a firm that chooses an action \( h \) (“pollution”) generating net profits \( \pi(h) \).

We will assume that the firm initially has the right to pollute as much as it feel like it. It then chooses \( h \) to maximize \( \pi(h) \). We will then assume that a “market” for reduction in \( h \) is established where the consumers can contract with the firm for marginal reductions in \( h \). We will consider the competitive case and show that we get a case which is exactly parallel to the case of under-provision of a public good in the case of voluntary contributions. Hence initially the firm has the “right to pollute” and hence chooses \( h^0 \) that solves \( \max_h \pi(h) \). There is a set of individuals \( i \in I = \{1, 2, ..., n\} \) with identical quasi-linear preferences over \( h \) and “income” \( x \),

\[
\phi(h) + x.
\]

With quasi-linear utility it is valid to characterize Pareto optimality as maximizing the sum of utilities and profits.\(^{11}\) Pareto optimality thus requires that \( h = h^* \) satisfying

\[
\pi'(h^*) + \sum_{i \in I} \phi'(h^*) = 0. \tag{62}
\]

The key thing to note is that \( h \) should be set so that the marginal profits should equal the total marginal impact on the consumers’ utilities, i.e. the marginal benefit

\(^{11}\)Effectively, since the consumers have quasi-linear utility, social marginal value of a unit of income is equal to unity. Hence a one pound increase in profits increases total welfare by one unit. Hence the units are effectively aligned. See MWG, ch. 10.
should equal total marginal damage which is obtained by summing over the consumers. Effectively, this is the Samuelson rule for a public good. We can interpret \(- \sum_{i \in I} \phi'(h)\) as the sum of the consumers’ marginal willingness to pay for a reduction in \(h\). This should correspond to the marginal cost to the firm (in terms of profits) of a reduction in \(h\).

We now establish a “market” where a consumer can buy reductions in \(h\); let \(p\) be the price for a one unit reduction in \(h\). Consider the competitive equilibrium with a price \(p^*\). The firm (treating \(p^*\) as given) chooses how many units of pollution reduction so trade,

\[
\max_h \{ \pi(h^0 - h) + p^* h \}
\]

where \(h\) is the total reduction sold. Thus, at the competitive equilibrium

\[
\pi'(h^0 - h) = p^*.
\] (63)

Let \(h_i\) denote the number of unit of pollution reduction that individual \(i\)’s purchases reduction, and let \(h_{-i} = \sum_{j \neq i} h_i\) be the total pollution reduction bought by all other individual other than \(i\). Individual \(i\) treats \(h_{-i}\) as given (the Nash assumption) and chooses \(h_i\) to solve

\[
\max_{h_i} \{ \phi(h^0 - h_{-i} - h_i) - p^* h_i \}.
\]

Hence at the competitive equilibrium

\[
-\phi'(h^0 - h) = p^*, \quad i \in I
\] (64)

where we used that \(h = h_i + h_{-i}\) for any individual \(i\).

Combining (63) and (64), we see that at the competitive equilibrium

\[
\pi'(h^0 - h) + \phi'(h^0 - h) = 0,
\] (65)

which when compared to (62) indicates that \(h^0 - h > h^*\) due to free-riding. Hence, due to free riding, the pollution is not reduced enough to achieve efficiency.

The problem is not exclusive to the case of the competitive solution; it applies equally well to any case where the consumers are bargaining with the polluter. Each consumer, when bargaining, will not take into account the effect of the reduction in pollution that she buys. Hence each consumer will buy too little pollution reduction.
11.4 An Empirical Externality: Passive Smoking

As an example of a negative externality we will consider passive smoking. Passive smoking constitutes a negative externality in that it has been linked to several illnesses such as lung cancer or heart disease in the adult population. Of particular concern is that passive smoking affects the health of young children and babies, causing asthma, bronchitis or sudden infant death syndrome. As a consequence, government have introduced (and are planning further) policies to limit the exposure of non-smokers and generally to discourage smoking. A recent study by Adda and Cornaglia (2006) consider the impact of smoking bans and cigarette taxes in the US on passive smoking.

Measuring Passive Smoking and Data

The damage of smoking is caused by tar and carbon monoxide which is highly correlated with the nicotine yield of a cigarette. However, nicotine is difficult to measure since it degrades within few hours of being absorbed. Rather then measure nicotine focuses on the amount of cotinine, a metabolite of nicotine, which stays in the body longer and can be measured in an individual’s saliva.

The data used by AC comes from the National Health and Nutrition Examination Survey, a nationwide representative sample of the US civilian population. Data is from 1988-1994 and 1998-2002. The survey contains information on the cotinine concentration in both smokers and non smokers, along with information on the number of cigarettes smoked in the household. In addition the data contains information on the age, sex, race, health, education and occupation of the individual. Since AC consider the effect of policy on passive smoking they select as their sample the group of non-smokers (26,997 individuals). They also separate non smokers who share their household with smokers, from non smokers who live in “smoke free” households.

AC merge this data with information on the smoking regulation in place by year and by state in different locations. The smoking ban variable is coded on a scale 0-3 measuring its strictness.\footnote{Zero if no restriction, one if smoking is restricted to designated areas; two if smoking is restricted to separate areas; three if there is a total ban on smoking.} Moreover, the degree of regulation is measured by type of location: (i)
bars, restaurant and other recreational places, (ii) public transport, (iii) shopping malls, (iv) work place, and (v) in schools. In addition to measuring bans, AC use information on (excise) taxes on cigarettes by state and year.

The next figure (Table 1 in AC) provides basic descriptive statistics. It shows that the vast majority of non-smokers are affected to some extent by passive smoking, although the measured levels of cotinine is much higher among non-smokers who live is households with some smoker.

FIG 3.21

Empirical Methodology

The empirical methodology is an extended version of the difference-in-difference approach. It uses that regulation and taxes have varied across US states and across time. The main estimating equation can be written as

$$Cot_{ist} = \alpha_0 + \gamma R_{st} + \beta \tau_{st} + \alpha_1 X_{ist} + \delta_s + \lambda_t + u_{ist}$$  \hspace{1cm} (66)

where $R_{st}$ is the variable measuring the degree of regulation in state $s$ and year $t$; $\tau_{st}$ similarly is the log excise tax on cigarettes in states at time $t$. Individual characteristics of individual $i$ residing in state $s$ at time $t$ are collected in the vector $X_{ist}$. $\delta_s$ is a set of state dummy variables while $\lambda_t$ is a set of year dummy variables.

The inclusion of state- and year dummy variables imply that identification of the effect of regulation and taxes comes from variation across states and time, and not from cross-sectional differences in the level of state regulations or taxes.$^{13}$ The identification of the effect of policy thus relies on the exogeneity of changes in policy within states. Indeed, over the period studied regulation became more stringent. E.g. in 1991 only 27% of the states had at least a total ban on smoking in some public space, whereas the figure

$^{13}$To see how the estimating regression is a generalization of the difference-in-difference approach, suppose e.g. there only been two states and two years, with one state introducing some regulation between the two years. In that case the above equation reduces to

$$Cot_{ist} = \alpha_0 + \gamma R_{st} + \alpha_1 X_{ist} + \delta_{s=treated\_state} + \lambda_{t=after} + u_{ist}$$  \hspace{1cm} (67)

where the change in regulation $R_{st} = 1$ if and only if $s = treated\_state$ and $t = after$. 

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is 51% in 2001. Similarly, excise taxes on increase over the period in question (by on average increased by 2 cents per year). Hence there is indeed policy variation to allow identification.

**Trends in Passive Smoking**

AC first consider the trends in passive smoking. The following two figure (Figure 3 and 4 in AC) graph the trends in cotinine levels for each group of non smokers who live in “smoke free” households and non-smokers in smoking households, respectively. The figures are striking. During the period in question tougher policy against smoking was introduced. This trend in regulation coincides with a downward trend in the level of cotinine observed in non smokers who live in “smoke free” households. However, this is not the case for non smokers who share their household with smokers. Hence, despite the increasing level of severity in regulations and higher excise taxes, it would seem that tobacco exposure of non smokers living in smoking households did not decrease.

![FIG 3.22](#)

![FIG 3.23](#)

The authors hypothesize that this may be due to a “displacement” effect whereby, as a response to bans in certain public places, smokers tend to smoke more inside the household. This provides a strong justification for exploring the impact of bans in different locations.

**Main Results**

Some of the authors main results are highlighted in the following figure (Table 4 in AC). The table gives the $\gamma$ coefficient and the $\beta$ coefficient from running the main estimating regression. In order to highlight in particular the effect on children, the sample is split by the age of the individual, grouping together children aged under eight, children aged 8-12, children aged 13-20, and those over 20.

![FIG 3.24](#)
The results are striking: the effect of bans on smoking in bars, restaurant and other recreational places (“going out”) significantly increases the cotinine level in children up to the age of 12 (the estimated effect of 0.65 for children under the age of 8 corresponds to smoking 1/20 of a cigarette), clearly suggesting a displacement effect whereby smokers smoke more inside the household. In contrast banning smoking in shopping malls have the (intended) effect of reducing passive smoking by children. The table also shows that increasing the tax on cigarettes reduces passive smoking by children.

References


