THE
ECONOMIC
JOURNAL

The Economic Journal, **130** (*May*), 1140–1160 DOI: 10.1093/ej/ueaa008
Advance Access Publication Date: 14 February 2020

DISTRIBUTIVE JUSTICE FOR BEHAVIOURAL WELFARE ECONOMICS*

Michael Mandler

The incompleteness of behavioural preferences can lead many or even all allocations to qualify as Pareto optimal. But the incompleteness does not undercut the precision of utilitarian policy recommendations. Utilitarian methods can be applied to groups of goods or to the multiple social welfare functions that arise when individual preferences are incomplete, and policymakers do not need to provide the preference comparisons that individuals are unable to make for themselves. The utilitarian orderings that result, although also incomplete, can generate a unique optimum. Non-separabilities in consumption reduce this precision but in all cases the dimension of the utilitarian optima drops substantially relative to the Pareto optima.

Economic models of irrational decision-making increasingly interpret an individual who fails to choose consistently as a set of agents acting at different frames. An individual whose value for a good depends on his endowment is seen as a set of preference relations, one for each endowment (Tversky and Kahneman, 1991). A hyperbolic discounter becomes a set of agents that apply discount rates that depend on the date they choose (Laibson, 1997). But when individuals are viewed as sets of agents, it is no longer clear how to define their welfare. The most popular approach has been to narrow the set of welfare judgements for an individual to a behavioural preference that all versions of the individual agree on (Bernheim and Rangel, 2007; 2009, Mandler, 2004; 2005, Salant and Rubinstein, 2008). When the frame-based versions of individual i disagree about how to order x and y then i is declared to not have a preference between x and y: the behavioural preference for individual i is incomplete. This incompleteness can lead Pareto optimality to become highly indecisive. In an economy of individuals with incomplete behavioural preferences, the Pareto optima can form a vast set of the same dimension as the entire set of allocations (Mandler, 2014). Every allocation in the neighbourhood of an optimum will be another optimum and the characteristic lessons of policy analysis break down: if an economy's initial allocation is Pareto optimal and a small externality is introduced the allocation will remain optimal. In some cases every allocation will be Pareto optimal.

This article argues that maximising the sum of utilities—the second-most popular welfare criterion—can close the decisiveness gap. Standard utilitarian models cannot make this case, since they require completeness, but the construction offered here shows how to make interpersonal comparisons of utility when preferences are incomplete. Utilitarianism's dependence on complete preferences has in any event left it in a precarious position: preferences presumably display incompleteness in at least a corner of their domain.

Individuals will have utilities for groups of goods though these utilities can take the consumption of goods in other groups as arguments. Individuals however do not take a decisive stand on how to aggregate these group utilities: their attitudes to aggregation can depend on their decision-making frames, leaving their behavioural preferences incomplete. For example, an

* Corresponding author: Michael Mandler, Department of Economics, Royal Holloway College, University of London, Egham, Surrey, TW20 0EX, UK. Email: m.mandler@rhul.ac.uk

This paper was received on 19 December 2017 and accepted on 7 January 2019. The Editor was Martin Cripps.

Let me thank Martin Cripps and the four thoughtful referees who led me to rethink the foundations of this article.

individual might have von Neumann-Morgenstern utilities for the goods delivered at particular states but use different probabilities to weight these utilities as the decision-making frame varies.

The planner meanwhile follows classical utilitarian guidelines: for each group of goods, the planner judges how the utility functions of different individuals for that group should be aggregated. The planner can, but does not have to, fill in individuals' incomplete preferences: a solution to the decisiveness problem would be vacuous if a planner has to impose the preference judgements individuals themselves are unable to supply.

If the planner comes equipped with a full set of judgements, both across individuals and across groups for single individuals, then as in classical utilitarianism a single objective function will rule. Less dictatorial planners can make interpersonal comparisons of utility for each group of goods via a utilitarian objective function and judge allocation x to be superior to y only if all of these objective functions recommend x over y. The resulting 'utilitarian superiority' ordering will be incomplete but it does not lead to a large set of optima. I first show that when the individuals' utilities for any group depend only on the consumption of the goods in that group —'separability'—and are strictly concave, there is a unique utilitarian optimal allocation. In practice, advocates and policymakers presume separability when they debate policy questions; arguments about, say, public health expenditures are rarely conditioned on individuals' consumption of other goods.

When separability is not satisfied, there can be multiple utilitarian optima but the set of optima has measure 0 and a dimension that never rises above the number of goods minus 1. Utilitarianism thus escapes the extreme dimensional expansion of the Pareto optima that behavioural preferences induce. The dimension of the utilitarian optima in fact compares favourably with the dimension of the Pareto optima in a complete preference model.

These results illustrate the broader principle that Pareto optimality delivers sharp advice only in artificial settings—such as a general equilibrium model where the policymaker has certain knowledge of the economy's primitives—while utilitarian methods remain decisive in a wide array of modelling environments. Even a little preference incompleteness is enough of a wedge for this usefulness disparity to appear.

The utilitarian project is hardly trouble-free. A utilitarian planner must compare the utilities of different individuals and come to a normative judgement as to who gains the most from an extra increment of a good. But the difficulty of making these comparisons is made neither harder nor easier by preference incompleteness.

Though formal models of utilitarianism assume complete preferences, the classical utilitarians from the beginning recognised the difficulty individuals face in comparing different types of satisfaction. John Stuart Mill (1863) famously acknowledged the diversity of kinds of pleasure and the early neoclassical economists recognised that individuals waver in their appraisal of present versus future consumption and the costs of uncertainty. The presence of conflict within individuals did not however undermine the utilitarians' confidence that marginal utility diminishes and is interpersonally comparable. This article models exactly this combination of positions: individuals who do not weight groups of goods consistently and planners that can nevertheless judge how goods should be distributed across individuals.

The present reformulation of utilitarianism clarifies the Sen (1979; 1986) charges against 'welfarism', the doctrine that social decisions should be a function only of individual welfare levels rather than the social and psychological content of the policy options. For a welfarist, the details of content matter only insofar as they feed into individual utility levels. Incomplete preferences fit with Sen's view since then there are no utility numbers that represent the whole of an individual's welfare: the raw material of welfarism is missing. The present version of utilitarianism lets deliberation about interpersonal comparisons proceed autonomously in different domains; decisions about, say, education can draw on the particulars of how schooling affects individual lives while decisions about the environment can depend on the specifics of that sphere. Though Sen suggests that utilitarianism must rely on welfarist foundations, the present version does not.

The final part of the article addresses the fact that utilitarian optimal allocations will sometimes fail to be Pareto optimal. From a strict utilitarian perspective, Pareto suboptimality does not present a problem: utilitarian judgements should trump individual judgements. More helpfully, I will show that as the extent of individual preference incompleteness increases, the potential for conflict between utilitarian and Pareto optimality disappears. For sufficiently incomplete preferences, utilitarian optima are sure to be Pareto optimal.

When utility is non-separable—the utility for one group of goods is potentially affected by consumption of all of society's goods—calculation of the dimension of the optima presents technical challenges. By interpreting each of the sums-of-utilities that the planner takes as goals as the utility function of a hypothetical agent, the utilitarian optima can be understood as the Pareto efficient allocations of an economy composed of these hypothetical agents. Smale's (1974) concept of an isolated community, which I will adapt to groups of goods rather than individuals, then provides just the right mathematical tool. Though some hurdles must be cleared, due to the twist that each of the hypothetical agents can experience an externality from the consumption levels of the other hypothetical agents, one message of this article is that this old general equilibrium machinery is fruitful for a topic as far-flung as utilitarianism with behavioural preferences.

Several papers address the problem of how to make welfare decisions in the presence of behavioural preferences. Like this article, Kahneman *et al.* (1997) applies a utilitarian rather than a Paretian approach. Fleurbaey and Schokkaert ([FB] 2013) and Apesteguia and Ballester ([AB] 2015) both recognise the danger of indecisiveness that accompanies a Paretian ban on any policy that overrules the observable choice of any individual. FS offers methods for distributive decision-making compatible with incomplete preferences while AB proposes brokering among the conflicting preferences that individual behaviour reveals. List (2004) provides a general social-choice setting that incorporates individuals with incomplete ('multidimensional') preferences. Danan *et al.* (2015) and Argenziano and Gilboa (2019) propose versions of utilitarianism when preferences are incomplete. In most of the above work, social welfare judgements are posited, whereas in this article they are built from comparisons of individual satisfaction, as in Edgeworth and Marshall. Moreover our main target is the size of the set of optima, which has not been the focus of preceding research.

1. Behavioural Preferences and Pareto Optima

An individual has preferences that depend on how decision-making is framed, for example by the date the individual chooses or the individual's endowment. Let f denote a frame, drawn from a set of frames $\mathcal{F}$, let X be the domain of alternatives, and let $\succsim_f$ be the preference that rules at f which we assume is transitive and weakly increasing.[1]

DEFINITION 1. *The* ***behavioural preference*** $\succsim$*, a binary relation on X, is the unanimity ordering of the frame-based preferences:* $x \succsim y$ *if and only if* $x \succsim_f y$ *for all* $f \in \mathcal{F}$.

[1] A preference $\succsim$ is **weakly increasing** if $x \gg y$ implies $x \succsim y$ and not $y \succsim x$.

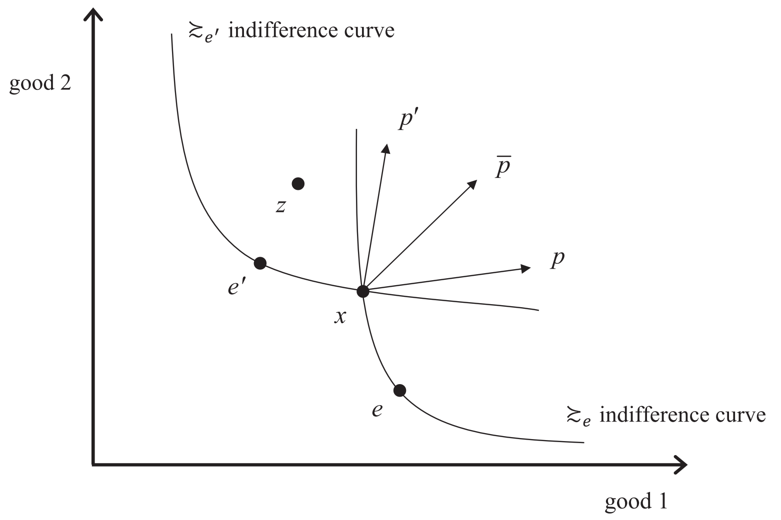

Fig. 1. *A Behavioural Preference with Multiple Supporting Prices.*

As usual strict preferences $\succ$ are defined by $x \succ y \Leftrightarrow (x \succsim y$ and not $y \succsim x)$. So $x \succ y$ obtains if at every frame x is weakly preferred to y and for at least one frame x is strictly preferred to y: x is a Pareto improvement over y for the individual's frame-based selves.

The status quo, the most common frame, can induce an endowment effect or status quo bias where individuals agree to move away from their *ex ante* allocation only if offered a substantial reward. An individual that begins with the status quo endowment e in Figure 1 has a frame-based preference $\succsim_e$ that judges a drop in good 1 consumption to the level at x to require a large increase in good 2 as compensation. If instead the individual has the status quo endowment e' then a drop in good 2 consumption to the level at x requires a large increase in good 1.[2] As the figure illustrates, the individual will then exhibit indifference curves at different frames that cross.

However the diversity of frames behind Figure 1 arises, the behavioural preference $\succsim$ labels the bundles to the northeast of both indifference curves to be $\succsim$-superior to x and the bundles to the southwest of both indifference curves to be $\succsim$-inferior to x. The remaining bundles such as z are unranked relative to x: the preference is incomplete. The kink in the set of $\succsim$-superior bundles that results is characteristic.

Beyond the Pareto rationale that no frame-based preference should be overruled, behavioural preferences can be given positive explanations. If the state of nature determines the benefits delivered by goods the individual is not sure how to value then the individual's decision-making frame can be viewed as a probability distribution and the Aumann (1962) and Bewley (1986) theory of multiple priors becomes a model of multiple frames. The operative prior can vary with the individual's endowment or mood or other factors, and the preference judgements backed by all of an individual's priors determine a $\succsim$ that can display phenomena like status quo bias.

[2] See Kahneman *et al.* (1990), Knetsch (1989), Samuelson and Zeckhauser (1988), and Thaler (1980).

A multiple priors model of behavioural preferences. *An individual consumes a bundle of goods $x_s \geq 0$ at state s and $x = (x_s)_{s\in\mathcal{S}}$ overall, where $\mathcal{S}$ is a finite set of states. Each x_s is evaluated by the von Neumann-Morgenstern utility u_s and a frame $f \in \mathcal{F}$ is a probability distribution π^f over $\mathcal{S}$. The agent at frame π^f therefore has a preference $\succsim_f$ defined by:*

$$x \succsim_f y \text{ if and only if } \sum_{s\in\mathcal{S}} \pi_s^f u_s(x_s) \geq \sum_{s\in\mathcal{S}} \pi_s^f u_s(y_s).$$

The behavioural preference $\succsim$ that results is then given by Definition 1 and its upper contours will be kinked.[3]

For status quo bias, let the first of two goods at state s have uncertain value and suppose utility at state s equals:

$$u_s(x_s(1), x_s(2)) = \gamma_s v_1(x_s(1)) + v_2(x_s(2)),$$

where γ_s is the realisation of the uncertain good's benefit at state s. If when the endowment of good 1 is large the agent's frame π^f assigns high probability to the states s where γ_s is large then the agent will display endowment-driven status quo bias. For the two-dimensional domain where consumption is state-independent, the kinked upper contours will resemble those in Figure 1. The overlap of the indifference curves that hold at different frames/priors indicates that $\succsim$ is incomplete.

In Aumann (1962) or Bewley (1986), the individual entertains all priors in $\mathcal{F}$ simultaneously and $x \succsim y$ holds only if x defeats y at every prior. Here each prior is interpreted as a decision-making frame. ■

With a judicious definition of states, the multiple priors model can cover many different behavioural influences on preferences, for example, mood-driven variations in the extent of diminishing marginal utility or risk aversion. These influences are embedded in an individual's cardinal vNM utilities while the frame affects only the probability weights on the utilities.

The weights that vary by frame need not be probabilities and individuals do not, even hypothetically, have to foresee the possible frames. With hyperbolic discounting, the utilities are defined on dated goods and the weights are the discount rates that agents apply to future utility.

Hyperbolic discounting. *An individual consumes at several time periods, dates 1 through 3 for concreteness. A consumption bundle is then a $x = (x_1, x_2, x_3) \geq 0$ where each x_i consists of ℓ goods. The frame is the date $t = 1, 2, 3$ at which decisions are made. Under hyperbolic discounting, for each t the preference $\succsim_t$ of the date-t decision-maker can be represented by the*

[3] So $x \succsim y$ if and only if:

$$\sum_{s\in\mathcal{S}} \pi_s^f u_s(x_s) \geq \sum_{s\in\mathcal{S}} \pi_s^f u_s(y_s)$$

for all $f \in \mathcal{F}$, or, equivalently,

$$\sum_{s\in\mathcal{S}} \widetilde{\pi}_s u_s(x_s) \geq \sum_{s\in\mathcal{S}} \widetilde{\pi}_s u_s(y_s)$$

for all $\widetilde{\pi} = \sum_{f\in\mathcal{F}} \alpha^f \pi^f$ with $(\alpha^f)_{f\in\mathcal{F}} \geq 0$ and $\sum_{f\in\mathcal{F}} \alpha^f = 1$ (assuming $\mathcal{F}$ is finite). We can therefore think of the individual as having a decision-making frame for each convex combination of the π^f.

utility function $u_t : \mathbb{R}_+^{(4-t)\ell} \to \mathbb{R}$ *defined by:*

$$u_1(x) = u(x_1) + \beta\left(\delta u(x_2) + \delta^2 u(x_3)\right),$$
$$u_2(x) = u(x_2) + \beta\delta u(x_3),$$
$$u_3(x) = u(x_3),$$

where $u : \mathbb{R}_+^{\ell} \to \mathbb{R}$ *is the agent's concave within-period utility and* β *and* δ *lie in* $[0, 1]$.

Consumption prior to t has already occurred for the date t agent and that agent consequently chooses only among bundles that specify the same consumption from 1 *to* $t-1$. *Since almost every pair of bundles specifies different values of* x_1 *and only the date* 1 *agent can reveal a preference between such pairs, there is virtually no conflict among the dated versions of the individual.*

To give the model some bite, we restrict the domain by letting one time period pass with $\overline{x}_1$ *consumed and consider the individual's preferences over the remaining two goods yet to be consumed, i.e., over the set* $\overline{X} = \{x : x_1 = \overline{x}_1\}$. *The dates* 1 *and* 2 *agents have complete preferences over* $\overline{X}$ *represented by the utilities* $u_1(\overline{x}_1, \cdot, \cdot)$ *and* u_2 *respectively. For x and y in* $\overline{X}$ *with* $x_2 \neq y_2$, *the preferences of the date* 3 *agent are irrelevant (since again that agent does not choose from such pairs). Following Bernheim and Rangel (2009), the behavioural preference* $\succsim$ *consists of the preference judgements that the individual's multiple selves agree on: set* $\mathcal{F} = \{1, 2\}$ *and, for the pairs* $x, y \in \overline{X}$ *such that* $x_2 \neq y_2$, *define* $\succsim$ *by Definition 1.*

As in the multiple priors model, $\succsim$ *will be incomplete and its upper contour sets will display kinks. Figure 1 pictures the case where* $\ell = 1$. ■

In a society where a typical individual i has one of the behavioural preference relations $\succsim^i$ we have considered, the set of Pareto optima will expand markedly relative to a classical economy. The kinks in the sets of bundles $\succsim^i$-superior to an arbitrary reference bundle imply that an interval of prices supports the reference bundle: for x in Figure 1, the boundaries of this interval are p and p'. Given a Pareto optimal allocation $(x^1, ..., x^I)$ for a society of I individuals, the second welfare theorem implies there will be a common supporting price vector: the individuals' price intervals intersect. If moreover the intersection is robust in that one of these supporting price vectors, for example $\overline{p}$ in Figure 1, does not lie on the boundary of any individual i's interval and if the boundaries of each i's interval varies continuously with x^i then, as $(x^1, ..., x^I)$ changes slightly, the intervals of supporting prices will continue to intersect. By the first welfare theorem, the new allocation must be Pareto optimal too. The Pareto optimal allocations with these properties therefore have the same dimension as the entire set of allocations and hence have positive (Lebesgue) measure.[4]

THEOREM 1 (MANDLER 2014). *If the Pareto optimum* $(x^1, ..., x^I)$ *has a common supporting price vector* $\overline{p}$ *such that, for each individual i,* $\overline{p}$ *lies in the interior of the set of prices that support* x^i *and i's set of supporting prices varies continuously in the allocation then any allocation sufficiently near x is Pareto optimal.*

The expansion of the Pareto optima is the problem this article addresses.

[4] To qualify as an allocation, there can be no disposal of goods. Throughout the article, 'dimension' refers to the dimension of C^0-manifolds with boundary.

2. Classical Utilitarianism

Classical utilitarianism, most importantly Edgeworth (1881), argued that the incremental or marginal value of resources to individuals should be equalised: a shift of a resource from agents with low marginal values for the resource to agents with high marginal values leads to an improvement. Though their discussions of measurement were casual, the classicals understood that a rescaling of units of pleasure or utility would not (and should not) change any ranking of allocations.

We use the following measurement terminology throughout the article.[5] If $u : Y \to \mathbb{R}$ is a function and a and b are real numbers, let $au + b$ denote the function $h : Y \to \mathbb{R}$ defined by $h(y) = au(y) + b$ for all $y \in Y$. Define $\widehat{\mathcal{U}}$ to be a **cardinal selection** from a Cartesian product of sets of real-valued functions $\mathcal{U} = U^1 \times \ldots \times U^n$ if there is a $u = (u^1, \ldots, u^n) \in \mathcal{U}$ such that, for all $\widehat{u} \in \mathcal{U}$,

$$\widehat{u} \in \widehat{\mathcal{U}} \iff \text{there exist real numbers } a > 0, b^1, \ldots, b^n \text{ such that } \widehat{u}^{\,i} = au^i + b^i \text{ for } i = 1, \ldots, n.$$

If both u and $\widehat{u}$ lie in the same cardinal selection then the 'units' of the functions in $\widehat{u}$ share a common rescaling relative to the functions in u: each $\widehat{u}^{\,i}$ is rescaled from u^i using the same constant a. Viewing each u^i as a utility, the ratios of utility increments (or marginal utilities) are consequently preserved across u and $\widehat{u}$ drawn from the same cardinal selection:

$$\frac{\widehat{u}^{\,i}(x) - \widehat{u}^{\,i}(y)}{\widehat{u}^{\,j}(w) - \widehat{u}^{\,j}(z)} = \frac{u^i(x) - u^i(y)}{u^j(w) - u^j(z)}$$

for all i and j and bundles $x,\ y,\ w$, and z. The constancy of this ratio implies two different types of decisiveness, both of which will be important. If i and j represent individuals, a utilitarian planner with a cardinal selection can decide which individual's utility change is greater. If i and j denote groups of consumption goods for a single individual then that individual or a planner can decide which of the group utility changes is greater. As we will see, a planner can be decisive in the first sense even when an individual or planner fails to be decisive in the second sense.

Individuals will be endowed with one or more sets of utilities. A set of utility functions U^i is **cardinal** if there is a $u^i : Y \to \mathbb{R}$ such that U^i equals u^i and its increasing affine transformations: for all $\widehat{u}^{\,i} : Y \to \mathbb{R}$,

$$\widehat{u}^{\,i} \in U^i \iff \text{there exist real numbers } a > 0 \text{ and } b \text{ such that } \widehat{u}^{\,i} = au^i + b.$$

Throughout the article, individuals will be indexed by the finite set $\mathcal{I} = \{1, \ldots, I\}$ and goods by the finite set $\mathcal{L} = \{1, \ldots, L\}$, a consumption for individual i is a $x^i = (x^i(1), \ldots, x^i(L))$, and an **allocation** is a profile $x = (x^1, \ldots, x^I) \in \mathbb{R}_+^{IL}$ of consumptions for each individual. A superscript attached to an object will now indicate the individual associated with that object, e.g., $\succsim^i$ for individual i's behavioural preference.

In the usual complete preferences understanding of classical utilitarianism, an individual $i \in \mathcal{I}$ is described by a cardinal set of utilities U^i, where each $u^i \in U^i$ is a utility function on bundles of L goods. A classical utilitarian planner decides on a cardinal selection $\widehat{\mathcal{U}}$ from $U^1 \times \ldots \times U^I$ and judges allocation x to be weakly superior to y if, for any (and therefore all) $(u^1, \ldots, u^n) \in \widehat{\mathcal{U}}$, $\sum_{i\in\mathcal{I}} u^i(x^i) \geq \sum_{i\in\mathcal{I}} u^i(y^i)$. This 'weak superiority relation' is complete: for any pair $x, y \in \mathbb{R}_+^{IL}$ either x is weakly superior to y or y is weakly superior to x. If we specify a feasible set of

[5] The following account follows the tradition of modelling measurement via sets of utilities. See Sen (1970), d'Aspremont and Gevers (1977), Roberts (1980), Bossert and Weymark (2004).

allocations $\{x \in \mathbb{R}_+^{IL} : \sum_{i \in \mathcal{I}} x^i(k) \leq e(k) \text{ for } k \in \mathcal{L}\}$, where $e(k)$ is society's endowment of good k, and if, for each individual i, some and therefore every u^i in U^i is strictly concave and continuous there will be exactly one utilitarian optimum.

When individual preferences are even a 'little' incomplete, the starting point of this standard model of utilitarianism, the U^i, will be missing. One task ahead is to fill this void. As it happens, the classical utilitarians did not suppose that individuals had a global utility assessment of all possible alternatives and so the overhaul applies to their theories too.

3. Utilitarianism with Behavioural Preferences

To extend utilitarianism to behavioural or other forms of incomplete preferences, we begin with individuals who derive satisfaction separably from disjoint groups of goods. For the early neoclassical economists, each good formed a group while in expected utility theory and in most intertemporal models, the goods delivered at a particular state or date form separable groups.

Suppose each individual can specify a set of utility functions for each group—in most applications a cardinal set of utilities. In expected utility theory these functions would consist of the increasing affine transformations of the vNM utility u_s that holds at a particular state s. As in a complete preferences model, the planner takes these sets as data but now for every individual there is one set for each group of goods. If an individual i weights the utilities of groups differently at the various frames then the behavioural preference $\succsim^i$ that results will be incomplete: i cannot pin down a cardinal selection from the utilities of the different groups. But since individuals are endowed with utilities for any group g, a utilitarian planner can compare the utilities of different individuals for group g and make a cardinal selection from these utilities. If anything, it should be easier to compare the utilities of individuals for a group of goods than for all goods. Insofar as the individual utilities for groups are cardinal, the planner will be following the classical playbook.

The planner can in addition, but does not have to, make comparisons of an individual's utilities for different groups. With or without these further comparisons, there will be a unique utilitarian optimum. For dictatorial planners ready to make utility comparisons across all individuals and groups, this result reproduces Edgeworth's conclusions and his reasoning. For more cautious planners, the result shows that decisive policymaking does not require an imposition of the inter-group comparisons that individuals find problematic.

3.1. *Individual Utilities and Preferences*

Formally, a **group** g is a subset of the set of goods $\mathcal{L}$ and the set of groups $\mathcal{G}$ is a partition of $\mathcal{L}$ with G cells. Each individual i is endowed with a set of utility functions V_g^i for each group $g \in \mathcal{G}$, where each $v_g^i \in V_g^i$ maps $\mathbb{R}_+^L$ to $\mathbb{R}$ and indicates i's utility for goods in g. Expected utility theory furnishes the canonical example: if we associate a state s with the group g of contingent goods delivered at s then the functions in V_g^i are the von Neumann-Morgenstern cardinal utilities for these goods (or equivalently vNM utilities multiplied by a probability of state s). Let $x_g^i \in \mathbb{R}_+^{|g|}$ denote i's consumption of the goods in g and $x^i(k) \in \mathbb{R}_+$ denote i's consumption of good k.

The utilities of individual i for the groups are aggregated into a behavioural preference by a (not necessarily cardinal) selection $\mathcal{V}^i \subset V_1^i \times \dots \times V_G^i$ with typical element $v^i = (v_1^i, \dots, v_G^i)$.

Any $v^i \in \mathcal{V}^i$ defines a utility function $\sum_{g\in\mathcal{G}} v_g^i$ on $\mathbb{R}_+^L$ that weights i's utilities for groups and represents one of i's complete frame-based preferences $\succsim_f^i$.[6]

Each $\sum_{g\in\mathcal{G}} v_g^i$ formed from a $v^i \in \mathcal{V}^i$ or the $\succsim_f^i$ it represents is an equally legitimate way for i to evaluate bundles and i therefore prefers x^i to y^i only when all of these objective functions concur. Define $\mathcal{V}^i$ to **generate** the behavioural preference $\succsim^i$ on $\mathbb{R}_+^L$ when:

$$x^i \succsim^i y^i \iff \sum_{g\in\mathcal{G}} v_g^i(x^i) \geq \sum_{g\in\mathcal{G}} v_g^i(y^i) \text{ for all } v^i \in \mathcal{V}^i.$$

If $\mathcal{V}^i$ is a cardinal selection from $V_1^i \times \ldots \times V_G^i$, then $\{\sum_{g\in\mathcal{G}} v_g^i : v^i \in \mathcal{V}^i\}$ forms a cardinal set of functions and the $\succsim^i$ that $\mathcal{V}^i$ generates will be complete. But when $\mathcal{V}^i$ consists of a larger set of functions (in the extreme, all of $V_1^i \times \ldots \times V_G^i$), individual i backs conflicting ways of weighting the group utilities. The frame-based preferences then conflict and the $\succsim^i$ that $\mathcal{V}^i$ generates will be incomplete. For example, if $\succsim^i$ is a behavioural preference for the multiple priors model then, as $v^i \in \mathcal{V}^i$ varies, the probabilities implicit in the expected utility function $\sum_{g\in\mathcal{G}} v_g^i$ will vary as well. In all cases, $\succsim^i$ will be transitive.

Though it plays no formal role, it is natural to associate each decision-making frame f with a cardinal selection $\mathcal{V}_f^i \subset V_1^i \times \ldots \times V_G^i$ interpreted as a representation of the complete preferences $\succsim_f^i$ that hold for the individual at f: $x^i \succsim_f^i y^i \Leftrightarrow \sum_{g\in\mathcal{G}} v_g^i(x^i) \geq \sum_{g\in\mathcal{G}} v_g^i(y^i)$ for all $v^i \in \mathcal{V}_f^i$. Each $\mathcal{V}^i$ would then equal a union of cardinal selections from $V_1^i \times \ldots \times V_G^i$ and $x^i \succsim^i y^i$ obtains if and only if the utilities that can be constructed from the $\mathcal{V}_f^i$'s all judge x^i to be superior to y^i.[7]

Utilities satisfy **separability** relative to $\mathcal{G}$ when each $v_g^i \in V_g^i$ can vary only with respect to i's consumption of the goods in g: for any $g \in \mathcal{G}$ and $x_g^i \in \mathbb{R}_+^{|g|}$, $v_g^i \in V_g^i$ must be constant on $\{y^i \in \mathbb{R}_+^L : y_g^i = x_g^i\}$.[8] In the absence of separability, the consumption levels of the goods in non-g groups can be complements or substitutes for the g goods.

To illustrate how the V_g^i sets of utilities, the frame-based preferences $\succsim_f^i$, and $\succsim^i$ interrelate and to see how the V_g^i can be inferred from behaviour, we return to the multiple-priors and hyperbolic discounting models. The utilities in both satisfy separability.

Multiple priors redux. *An agent with von Neumann-Morgenstern utility $u_s^i(x_s)$ at state $s \in \mathcal{S} = \{1, \ldots, S\}$ chooses at various frames f where each $f \in \mathcal{F}^i$ is a probability π^f on $\mathcal{S}$.*

Each group $g \in \mathcal{G}$ consists of the goods delivered at some state $s \in \mathcal{S}$ and so we identify $\mathcal{G}$ with $\mathcal{S}$. To adjust the domain of the utilities to equal the entire set of goods, define $v_s^i : \mathbb{R}_+^L \to \mathbb{R}$ by $v_s^i(x) = u_s^i(x_s)$ for each s. A V_s^i then equals the cardinal set of functions that contains v_s^i. Separability is satisfied. Since a decision-making frame f is a probability π^f, the individual's

[6] I have assumed that the $\succsim_f^i$ are represented by sums of v_g^i's to fit with the expected utility and intertemporal preference models and with the Jevons-Marshall tradition. The assumption brings only a modest loss of generality in terms of the $\succsim^i$'s that can be generated: for example any $\succsim^i$ with a utility representation, whether additive or not, can be generated by some $\mathcal{V}^i$. See Appendix A.

[7] A $\mathcal{V}^i$ that does not equal a union of cardinal selections $\bigcup_f \mathcal{V}_f^i$ would weaken the frame interpretation but otherwise have no effect. Such a $\mathcal{V}^i$ deletes some v^i's from some $\mathcal{V}_f^i$'s without changing the $\succsim^i$ that $\bigcup_f \mathcal{V}_f^i$ represents.

[8] Although I call this assumption 'separability' for brevity, it is a restriction on the operative domain of each v_g^i. The link to standard usage is that the assumption implies that the $\sum_{g\in\mathcal{G}} v_g^i$ functions, which collectively represent $\succsim^i$, will satisfy 'additive separability' in Gorman (1959) or 'groupwise separability' in Jorgenson and Lau (1975). Since $\succsim^i$ might not have a utility representation, however, none of the preference definitions of separability apply to $\succsim^i$. I reserve 'additive' for functions equal to a sum of group utilities that do represent a preference.

frame-based preference $\succsim^i_f$ *is represented by the cardinal selection from* $V^i_1 \times ... \times V^i_S$ *given by:*

$$\mathcal{V}^i_{\pi^f} = \left\{ \left(a\pi^f_1 v^i_1 + b_1, ..., a\pi^f_S v^i_S + b_S \right) : a \in \mathbb{R}_{++} \text{ and } b_1, ..., b_S \in \mathbb{R} \right\}.$$

The multiple-priors behavioural preference $\succsim^i$ *is then generated by the selection* $\mathcal{V}^i = \bigcup_{\pi^f \in \mathcal{F}^i} \mathcal{V}^i_{\pi^f}$.

The V^i_s *can be inferred from individual i's preferences over lotteries with objective probabilities. Alternatively, each* v^i_s *and hence each* V^i_s *can be deduced from the expected utility representations of the* $\succsim^i_f$ *that holds at one of the frames* π^f. ■

Hyperbolic discounting redux. *At dates* 1 *and* 2, *which are the frames, an individual i has preferences* $\succsim^i_1$ *and* $\succsim^i_2$ *on the domain* $\overline{X}$ *(defined in Section 1) that are represented by the utilities* $u^i_1(x) = u^i(x_2) + \delta u^i(x_3)$ *and* $u^i_2(x) = u^i(x_2) + \beta\delta u^i(x_3)$. *The two groups are the goods that appear at dates* 2 *and* 3 *respectively.*

Let a group t utility for $t = 2, 3$, $v^i_t : \overline{X} \to \mathbb{R}$, *be defined by* $v^i_t(x^i) = u^i(x^i_t)$ *and set* V^i_t *to equal the cardinal set of functions that contains* v^i_t. *Separability is again satisfied. The frame-based preferences* $\succsim^i_1$ *and* $\succsim^i_2$ *are represented by the cardinal selections:*

$$\mathcal{V}^i_1 = \left\{ \left(av^i_2 + b_2, a\left(\delta v^i_3\right) + b_3 \right) : a \in \mathbb{R}_{++} \text{ and } b_2, b_3 \in \mathbb{R} \right\} \text{ and}$$
$$\mathcal{V}^i_2 = \left\{ \left(av^i_2 + b_2, a\left(\beta\delta v^i_3\right) + b_3 \right) : a \in \mathbb{R}_{++} \text{ and } b_2, b_3 \in \mathbb{R} \right\},$$

and $\mathcal{V}^i = \mathcal{V}^i_1 \cup \mathcal{V}^i_2$ *generates the hyperbolic behavioural preference* $\succsim^i$.

Both V^i_2 *and* V^i_3 *can be inferred from the additive representations of either* $\succsim^i_1$ *or* $\succsim^i_2$. ■

Finally, in the less formal early neoclassical model each group consists of a single good and utilities again satisfy separability: for each good k, the utilities in $V^i_{\{k\}}$ vary only with respect to $x^i_{\{k\}} = x^i(k)$. Individuals may not know how to weight the utilities of goods, a difficulty recognised by J.S. Mill, and may therefore adopt different weights on the $v^i_{\{k\}}$ and thus different cardinal selections as circumstances vary. Each selection in effect defines a frame-based preference.

3.2. *Social Welfare*

Our utilitarian planner takes as data a set of utilities V^i_g for each individual i and group g. Just as a classical utilitarian selects interpersonally comparable utilities for individuals, the planner selects interpersonally comparable utilities for individuals for each group g. These utilities form a cardinal selection from $V^1_g \times ... \times V^I_g$, denoted $\mathcal{W}_g$. A typical element of $\mathcal{W}_g$ is a vector $v_g = (v^1_g, ..., v^I_g)$, a group g utility for every individual i. Each $\mathcal{W}_g$ defines a **group** g **utilitarian ordering** that deems allocation x weakly superior to y if, for any and hence all $v_g \in \mathcal{W}_g$, $\sum_{i\in\mathcal{I}} v^i_g(x^i) \geq \sum_{i\in\mathcal{I}} v^i_g(y^i)$. Each $\sum_{i\in\mathcal{I}} v^i_g$ is substantively and mathematically a classical utilitarian objective function.

The planner may also compare an individual's utilities across groups. All of the planner's judgements taken together are given by a selection $\mathcal{W} \subset \prod_{(g,i)\in\mathcal{G}\times\mathcal{I}} V^i_g$ that places individual and group comparisons on the same footing: a vector of functions in $\mathcal{W}$, denoted $v = (v^i_g)_{(g,i)\in\mathcal{G}\times\mathcal{I}}$, lists a utility v^i_g for each group-individual pair (g, i). The selection $\mathcal{W}$ must be compatible with the $\mathcal{W}_g$: we require for each g that the projection of $\mathcal{W}$ onto $V^1_g \times ... \times V^I_g$ equals $\mathcal{W}_g$. Comparisons of individual utilities across groups thus do not dilute the planner's within-group comparisons of individual utilities, in line with the classical utilitarian position that individuals' vacillations about the worth of goods do not undermine interpersonal comparisons.

The model gives the planner wide latitude to embrace or abstain from comparisons of an individual's utilities for groups. At the abstention end of the spectrum, a planner refrains from all across-group comparisons when, for every $\overline{v} \in \mathcal{W}$ and $(\lambda_1, ..., \lambda_G) \gg 0$, there is a $v \in \mathcal{W}$ such that $v_g = \lambda_g \overline{v}_g$ for all g, and we then say the planner or $\mathcal{W}$ is **group agnostic**. At the opposite pole, a $\mathcal{W}$ is **dictatorial** if it is a cardinal selection: $\mathcal{W}$ then imposes a single relative weighting of each individual's utilities for groups. If there is just one group, planners are necessarily dictatorial. But when there is more than one group, planners have good reason not to go to the dictatorial extreme: they may be stymied by the same decisions that individuals cannot resolve consistently for themselves. A planner's judgement that the marginal utility of a group of goods is higher for the poor than for the rich need not make it any easier to decide which types of goods and pleasures deserve priority. Planners may also want to respect individuals' equivocations about how groups of goods should be weighed. [9]

Each $v \in \mathcal{W}$ defines a welfare function $\sum_{(g,i)\in\mathcal{G}\times\mathcal{I}} v^i_g$ which can also be written as $\sum_{i\in\mathcal{I}} \sum_{g\in\mathcal{G}} v^i_g$, the sum across individuals i of one of the utilities that $\mathcal{W}$ assigns to i.[10] For the early neoclassical economists, adding individual utilities to form social welfare functions was an assumption; for choice under uncertainty, the Harsanyi (1955) aggregation theorem and specifically Hammond (1981) provide rationales.[11]

Each of the $\sum_{(g,i)\in\mathcal{G}\times\mathcal{I}} v^i_g$ objective functions represents a complete ordering and thus can compare any pair of allocations. And each $\sum_{(g,i)\in\mathcal{G}\times\mathcal{I}} v^i_g$ gives classical utilitarian advice: welfare increases when the goods in some group g' are transferred from an agent i with low marginal utility for these goods to an agent j with high marginal utility. But when the planner's $\mathcal{W}$ inherits the inability of the $\succsim^i$ to rank alternatives the various welfare functions to which $\mathcal{W}$ leads can rank allocations differently. For example if x is an improvement over y with respect to the group g utilitarian ordering but a worsening with respect to the g' ordering and $\mathcal{W}$ is group agnostic then the welfare functions for $v \in \mathcal{W}$ that assign large weight to the group g utilities will approve a move from y to x while the welfare functions for $v \in \mathcal{W}$ that heavily weight g' will reject the move. Due to this diversity, we require unanimous consent before declaring an allocation to be an improvement. Letting $e = (e_1, ..., e_G) \geq 0$ be the economy's endowment of groups of goods, an allocation x is **feasible** if it lies in $\{x \in \mathbb{R}^{IL}_+ : \sum_{i\in\mathcal{I}} x^i \leq e\}$.

DEFINITION 2. *Allocation x is **utilitarian superior** to y if $\sum_{(g,i)\in\mathcal{G}\times\mathcal{I}} v^i_g(x^i) \geq \sum_{(g,i)\in\mathcal{G}\times\mathcal{I}} v^i_g(y^i)$ for all $v \in \mathcal{W}$ and strict inequality holds for at least one $v \in \mathcal{W}$, and is a **utilitarian optimum** if x is feasible and there is no feasible y that is utilitarian superior to x.*

[9] While I have described planners as beginning with individual welfare comparisons—the $\mathcal{W}_g$—they can also construct $\mathcal{W}$'s in one integrated step. The sequential view more closely resembles Edgeworth (1881) where the case for redistributing resources to high marginal utility individuals applies to any resource or 'means' that can generate pleasure. There is no fiction of an aggregate consumption good in Edgeworth nor any claim that a single all-encompassing utility function covers all sources of pleasure at all dates. Moreover the utilitarian optima are determined solely by the $\mathcal{W}_g$ when separability holds rather than by all of $\mathcal{W}$.

[10] As in individual decision-making, it is natural to associate each frame f with a cardinal selection $\mathcal{W}_f \subset \prod_{(g,i)\in\mathcal{G}\times\mathcal{I}} V^i_g$ and to assume that $\mathcal{W}$ equals the union $\bigcup_{f\in\mathcal{F}} \mathcal{W}_f$. Any two v's drawn from the same $\mathcal{W}_f$ will then lead to the same welfare function, up to affine transformation.

[11] The probabilities in the utility representations in Hammond coincide across individuals. Since individual utilities in this article are state-dependent (as in Hammond), we may let probabilities vary by individual, as we must when a change in the vector of utilities drawn from $\mathcal{W}$ indicates only a change in the probability frames of the individuals.

Though the caution of the unanimity requirement in Definition 2 will typically lead utilitarian superiority to be incomplete, utilitarian optimality discriminates with precision. We put aside optima that deliver the same utility levels by assuming that for each g the $v_g^i \in V_g^i$ are strictly concave on the goods in g.[12]

THEOREM 2. *If separability is satisfied and, for each group g and individual i, each* $v_g^i \in V_g^i$ *is strictly concave on g and continuous then there is a unique utilitarian optimum.*

PROOF. Fix some $g \in \mathcal{G}$, $y_{-g} = (y_{g'}^i)_{i \in \mathcal{I}, g' \in \mathcal{G}\setminus\{g\}}$, and $v \in \mathcal{W}$. For any i:

$$\sum_{g' \in \mathcal{G}} v_{g'}^i(x_g^i, y_{-g}^i) = v_g^i(x_g^i, y_{-g}^i) + \sum_{g' \neq g} v_{g'}^i(x_g^i, y_{-g}^i)$$

and, due to separability, $\sum_{g' \neq g} v_{g'}^i(x_g^i, y_{-g}^i)$ equals the same constant for all $x_g^i \geq 0$. Thus x' solves $\max_{x \geq 0} \sum_{i \in \mathcal{I}} \sum_{g' \in \mathcal{G}} v_{g'}^i(x_g^i, y_{-g}^i)$ s.t. $\sum_{i \in \mathcal{I}} x^i \leq e$ if and only if, for each g, x'_g solves $\max_{x_g \geq 0} \sum_{i \in \mathcal{I}} v_g^i(x_g^i, y_{-g}^i)$ s.t. $\sum_{i \in \mathcal{I}} x_g^i \leq e_g$.

Due to continuity, there is a $\overline{x}_g = (\overline{x}_g^1, \ldots, \overline{x}_g^I)$ that solves the latter problem and, due to strict concavity on g, this solution $\overline{x}_g$ is unique. Due to separability the solution does not depend on the choice of y_{-g} and, since the projection of $\mathcal{W}$ onto $V_g^1 \times \ldots \times V_g^I$ equals a cardinal selection, the solution does not depend on the choice of $v \in \mathcal{W}$. Hence $(\overline{x}_1, \ldots, \overline{x}_G)$ is the unique optimum. □

If the planner is dictatorial, the proof above repeats the uniqueness reasoning of a standard model of utilitarianism. Theorem 2 however places no restrictions on $\mathcal{W}$ beyond each $\mathcal{W}_g$ being a cardinal selection: the planner can fall anywhere between the group agnostic and dictatorial extremes. With group agnosticism, planners impose no comparisons of group utilities and utilitarian superiority reduces to the unanimity ordering of the group-by-group utilitarian orderings (whether or not separability holds). Define allocation x to be **group-unanimously superior** to y when $\sum_{i \in \mathcal{I}} v_g^i(x^i) \geq \sum_{i \in \mathcal{I}} v_g^i(y^i)$ for all $g \in \mathcal{G}$ and $v_g \in \mathcal{W}_g$ with at least one strict inequality and a **group-unanimity optimum** if x is feasible and no feasible y is group-unanimously superior to x.

PROPOSITION 1. *If $\mathcal{W}$ is group agnostic then the group-unanimity and utilitarian superiority orderings coincide. For any $\mathcal{W}$, any utilitarian optimum is a group-unanimity optimum.*[13]

Proposition 1 underscores just how cautious the utilitarian superiority relation can be: when $\mathcal{W}$ is group agnostic, a change in allocations must be recommended by every group utilitarian ordering. In the multiple-priors model, for example, a change must deliver an improvement for the group g utilitarian ordering even when the group g goods are associated with a highly unlikely state. While cautious orderings ordinarily make it easier to declare allocations optimal and therefore invite indecisiveness, Theorem 2 shows that under separability there is a unique utilitarian optimum. Moreover any across-group comparisons that $\mathcal{W}$ does make end up being irrelevant: the utilitarian optimum is the unique group-unanimity optimum.

Hyperbolic discounting continued. A society of individuals with the hyperbolic preferences of Section 1 will have a large set of Pareto optima. Suppose for concreteness that there is one

[12] Formally, $v_g^i : \mathbb{R}_+^L \to \mathbb{R}$ is **strictly concave on** g if, for all $x^i \in \mathbb{R}_+^L$, v_g^i is strictly concave on $\{y^i \in \mathbb{R}_+^L : y^i(k) = x^i(k)$ for all $k \notin g\}$.

[13] Omitted proofs are in Appendix B.

good per time period, $\ell = 1$, which implies that each good is a group. If the feasible set $\overline{F}$ is $\{x \in \overline{X}^I : \sum_{i \in \mathcal{I}} x_t^i \leq e_t$ for $t = 2, 3\}$, the dimension of the Pareto optima will equal $2(I-1)$, which is also the dimension of the frontier of $\overline{F}$ (the non-disposal points where the inequalities that define $\overline{F}$ hold with equality). As explained in Section 1, each upper contour set for each $\succsim^i$ has a continuum of supporting prices, at a Pareto optimum these individual continua will intersect, and the intersection will typically persist following any small adjustment in the allocation. By the first welfare theorem, the new allocation must then also be Pareto optimal.

A utilitarian planner specifies for each $t = 2, 3$ a cardinal selection $\mathcal{W}_t$ from $V_t^1 \times ... \times V_t^I$, possibly supplemented by further intrapersonal comparisons of utility across goods. Assuming each u^i is strictly concave, there will be a unique distribution of each date-t good that is utilitarian optimal, found by maximising $\sum_{i \in \mathcal{I}} u^i(x_t^i)$ subject to $\sum_{i \in \mathcal{I}} x_t^i \leq e_t$ and $(x_t^1, ..., x_t^I) \geq 0$. Specifying the cardinal selection may present a normative challenge, but the difficulties should be no more formidable than with complete preferences. ■

3.3. *Social Welfare: Non-separable Utilities*

So far utilities have satisfied separability: each v_g^i has varied only with respect to goods in group g. Without this assumption there need not be a unique utilitarian optimum. For example, suppose each good is a singleton group and that the utility of good 1 depends on the consumption of good 2 and vice versa while the utilities for other goods are functions only of their own consumption levels. Assuming $\mathcal{W}$ is group agnostic, the utilitarian optima will coincide with the Pareto efficient allocations of a society of hypothetical agents with the group g utilitarian orderings, one for each $g \in \mathcal{G}$ (Proposition 1). Consequently there will typically be a one-dimensional set of utilitarian optima for the economy that consists of just goods 1 and 2: given a pair $\sum_{i \in \mathcal{I}} v_1^i$ and $\sum_{i \in \mathcal{I}} v_2^i$ that defines the groups 1 and 2 utilitarian orderings, the utility possibility frontier will normally be one-dimensional just as the utility possibility frontier of a standard two-agent economic model is one-dimensional. As there remains a unique optimum for the distribution of goods 3, ..., L, the utilitarian optima for the economy of all L goods will also be one-dimensional.

As we will see, however, even in worst cases the size of the set of utilitarian optima compares favourably with the size of the set of Pareto optima (as classically defined by the $\succsim^i$ not by the hypothetical agents above) both in terms of dimension and measure. Since the utilitarian optima form a subset of the group unanimity optima (Proposition 1), we can use the group-unanimity optima to bound the set of utilitarian optima. The above example lies far from the worst case: it displays the minimum extent of non-separability in consumption and accordingly the dimension of the group-unanimity optima expands modestly, from 0 to 1. The greatest expansion occurs when, for each group g, the $\sum_{i \in \mathcal{I}} v_g^i$ that represent the group g utilitarian ordering are non-trivial functions of all L goods in the model and each good is its own group: the dimension of the group-unanimity optima can then rise to $G - 1 = L - 1$. This case is similar but not identical to a standard general equilibrium model where the dimension of the Pareto efficient allocations would normally equal the number of individuals I minus 1. In our setting, the role of an individual is played by a group g with the associated 'utility' $\sum_{i \in \mathcal{I}} v_g^i$. The planner thus has G objective functions which suggests that the dimension of the group-unanimity optima will be $G - 1$. But while standard utilities are functions of different variables (the agents' private consumptions) each $\sum_{i \in \mathcal{I}} v_g^i$ is potentially a function of all LI of the model's consumption variables. Externalities in effect appear since the goods that affect the 'utility' $\sum_{i \in \mathcal{I}} v_g^i$ can also affect the 'utility' $\sum_{i \in \mathcal{I}} v_{g'}^i$, where $g' \neq g$. These externalities can reduce the dimension of the set of optima or even lead the set to not have a well-defined dimension (to not be a manifold). Consequently Theorem 3 below will provide only an upper bound on the dimension of the utilitarian optima.[14] 'Upper' is the bound of interest since we are interested in worst-case scenarios.

Dimension is neither the sole nor indisputable way to gauge the size of a set. Our result on dimension, Theorem 3, will imply that the utilitarian optima also form a 0 (Lebesgue) measure subset of society's allocations. The Pareto optima in contrast can have positive measure as we saw in Section 1. There are other yardsticks, however, for example the diameter of the set of optima, that Theorem 3 will not address.

To bound the dimension of the group-unanimity optima and hence the utilitarian optima as well, we will adapt the concept of an 'isolated community' from the general equilibrium theory of Pareto efficiency (Smale, 1974). A classical isolated community is a minimal subset of individuals that consumes only goods that individuals outside the community do not consume, for example when community members and non-members have utilities that are increasing on disjoint sets of goods. An 'isolated basket' will be a minimal set of groups of goods where for each group g in the basket the v_g^i can be non-trivial functions only of goods in the groups the basket contains. Define $h : \mathbb{R}_+^L \to \mathbb{R}$ to be **variable on good** k if there is a pair $x, y \in \mathbb{R}_+^L$ such that $x(l) = y(l)$ for each $l \neq k$ and $h(x) \neq h(y)$. To avoid trivialities, we assume in this section that for every good k there is an agent i and a group g such that each $v_g^i \in V_g^i$ is variable on k.

DEFINITION. *The non-empty pairwise-disjoint subsets of groups $\mathcal{B}_1, \ldots, \mathcal{B}_n \subset \mathcal{G}$ form **isolated baskets** if, for each $\mathcal{B}_j$, (i) any $v_g^i \in V_g^i$, where $g \in \mathcal{B}_j$ and $i \in \mathcal{I}$, is variable only on a non-empty set of goods in the groups in $\mathcal{B}_j$ and (ii) no non-trivial partition of $\mathcal{B}_j$ has cells that satisfy (i).*

A model $\left(V_g^i\right)_{(i,g)\in\mathcal{I}\times\mathcal{G}}$ has a unique family of isolated baskets, which may consist of just one basket. Each basket $\mathcal{B}_j$ in a family defines a corresponding set of objective functions, the $\sum_{i\in\mathcal{I}} v_g^i$ such that $g \in \mathcal{B}_j$ and $v_g \in \mathcal{W}_g$, that are variable only on goods in groups drawn from the same basket. Separability is the prominent example. Each $\sum_{i\in\mathcal{I}} v_g^i$ then varies only as a function of $x_g^1, \ldots, x_g^I$ while the remaining $\sum_{i\in\mathcal{I}} v_{g'}^i$, $g' \neq g$, are constant on these variables: each group g by itself forms an isolated basket. As the extent of non-separabilities in consumption increase from the floor given by separability, the sizes of the isolated baskets increase in tandem.

In general equilibrium theory, non-trivial isolated communities cause the dimension of the Pareto efficient allocations to fall below the number of individuals minus 1. The individuals in each isolated community form a free-standing economic model and the dimension of the Pareto efficient allocations for this community equals the number of individuals in the community minus 1. The dimension of the optima for the unified model then equals the sum of the community-specific dimensions, which must be less than the total number of individuals minus 1. For example, with two communities of I_1 and I_2 individuals, where $I_1 + I_2 = I$, the dimension of the optima will equal $I_1 - 1 + I_2 - 1 < I - 1$. In our model, where groups play the role of individuals, this accounting continues to hold: the dimension of the group-unanimity optima will fall below $G - 1$ as the number of isolated baskets increases. Theorem 2 is a case in point.

[14] For a simple case of the complications introduced by these externalities, notice that it is possible to have $\sum_{i\in\mathcal{I}} v_g^i = \sum_{i\in\mathcal{I}} v_{g'}^i$ for distinct groups g and g'. There are then effectively fewer than G objective functions in the model and the dimension of the group-unanimity optima accordingly falls.

Define $v^i_g : \mathbb{R}^L_+ \to \mathbb{R}$ to be **strictly concave and increasing on its variable goods** if, for all $x^i \in \mathbb{R}^L_+$, v^i_g is strictly concave and strictly increasing on $\{y^i \in \mathbb{R}^L_+ : y^i(k) = x^i(k) \text{ for } k \notin \mathcal{K}\}$ where $\mathcal{K}$ is the set of goods on which v^i_g is variable.[15]

THEOREM 3. *Suppose that, for any group g and individual i, each $v^i_g \in V^i_g$ is continuous and strictly concave and increasing on its variable goods, there are n isolated baskets with d_1, ..., d_n groups, and the set of group-unanimity optima has dimension d. Then $d \leq \sum_{j=1}^{n}(d_j - 1) \leq G - 1$.*

The dimensional expansion of the group-unanimity optima thus depends on the number of isolated baskets, which in turn depend on the extent of the non-separabilities in consumption, and on the number of groups. The worst case, where the dimension of the optima reaches $L-1$, occurs when the entire set of goods forms the sole isolated basket and each good is a singleton group. But even here the utilitarian optima have not undergone the $L(I-1)$ explosion of dimensionality that occurs for the Pareto optima when preferences are incomplete (see Section 1). If, as one presumes in market settings, the number of individuals is larger than the number of goods, $I > L$, then with incomplete preferences the dimension of the utilitarian optima will be less than the dimension of the Pareto optima that obtains when preferences are complete, $I - 1$. Moreover with incomplete preferences the measure of the Pareto optima will normally be positive while the measure of the utilitarian optima will typically be 0.

Theorem 3, unlike Theorem 2, does not offer a uniqueness result when each $d_j = 1$: it then states that the optima form a discrete set of points (a 0-dimensional set), not a single point.

4. Utilitarian Versus Pareto Optimality

Utilitarian optima can fail to be Pareto optimal when individuals are modelled by their $\succsim^i$ behavioural preferences. A utilitarian planner weights an individual i's utility for group g according to the planner's judgement about the satisfaction i derives from g and that weighting might not be compatible with the weightings implicit in $\succsim^i$. For example, suppose there are two goods, each of which forms a group, and two individuals a and b with identical v^i_k's that are separable, strictly concave, and differentiable. (When in this section a group consists of a single good, I ignore the notational difference between a good and a group.) Letting $\mathcal{W}_1$ place equal weight on v^a_1 and v^b_1 and $\mathcal{W}_2$ place equal weight on v^a_2 and v^b_2, the utilitarian optimum will satisfy $x^a = x^b$. Depending on the $\succsim^i$, Pareto improvements may be possible. If, at some frame f, $v^a \in \mathcal{V}^a_f$ assigns weights 2 and 1 respectively to v^a_1 and v^a_2 while $v^b \in \mathcal{V}^b_f$ assigns weights 1 and 2 to v^b_1 and v^b_2 then the marginal rates of substitution of the agents at f cannot align at the utilitarian optimum.[16] Some transfer between the individuals will therefore deliver a Pareto improvement for $\succsim^a_f$ and $\succsim^b_f$. Whether each individual i is $\succsim^i$-better off with this transfer will depend on the diversity the individuals display across frames. But if at every frame each individual places nearly the same relative weights on the utilities for goods then both individuals' behavioural preferences

[15] The function v^i_g is **strictly increasing** if $x^i \geq y^i$ and $x^i \neq y^i$ imply $v^i_g(x^i) > v^i_g(y^i)$.

[16] If $Dv^a_1(x^a_1) = Dv^b_1(x^b_1)$ and $Dv^a_2(x^a_2) = Dv^b_2(x^b_2)$ (letting D indicate derivatives w.r.t. variable goods) then $\frac{2Dv^a_1(x^a_1)}{Dv^a_2(x^a_2)} \neq \frac{Dv^b_1(x^b_1)}{2Dv^b_2(x^b_2)}$.

will back the transfer.[17] As this example suggests, both the compatibility and incompatibility of utilitarian and Pareto optimality are robust possibilities.

For an orthodox utilitarian, a failure of utilitarian optimality to achieve Pareto optimality is not a problem: the planner's judgements take precedence. For example, in the pre-WWII heyday of utilitarianism, it was common to hold that impatience and discounting amounted to failures of rationality—an offence 'against the rules of economic reason' in Schumpeter's words (1912, p. 35). A utilitarian planner therefore might well ignore individuals' time preferences altogether. In the hyperbolic discounting example in Section 3, such a planner would, at *every* date t, use the same cardinal selection $\mathcal{W}_t$ from $V_t^1 \times \ldots \times V_t^I$.

Whether or not utilitarian-Pareto conflicts are problematic, planners might want to avoid them. A liberal planner could deliberately choose weights on an individual's utility for groups to match the individual's weights. Or the planner could just choose $\mathcal{W}$ and a utilitarian optimum x such that x is Pareto optimal. Since this choice would impose an additional restriction beyond utilitarian optimality, it will not expand the set of optima.

More productively, I will show that the potential for a clash between utilitarian and Pareto optimality diminishes as preference incompleteness increases. We begin by defining what it means for a liberal planner to match an individual's weights on group utilities.

DEFINITION 3. *Given $\mathcal{V}^i$ for $i \in \mathcal{I}$,* ***preference compatibility*** *is satisfied for $\mathcal{W}$* if *there exists $v \in \mathcal{W}$ such that $v^i \in \mathcal{V}^i$ for $i \in \mathcal{I}$.*

Suppose that we now adjust the preferences, letting them get progressively more incomplete by expanding the selections $\mathcal{V}^i$ that define the $\succsim^i$ while keeping the sets V_g^i and the selection $\mathcal{W}$ fixed. If each selection $\mathcal{V}^i$ is large enough—there is enough incompleteness—then preference compatibility must be satisfied. In particular, when $\mathcal{V}^i = V_1^i \times \ldots \times V_L^i$ for each i, any $\mathcal{W}$ is preference compatible: with sufficient incompleteness, every planner becomes a liberal. As we will see, if incompleteness is in this sense sufficiently substantial and separability holds then any utilitarian optimum will be Pareto optimal.

It is worth pausing to consider the other extreme: if each $\mathcal{V}^i$ is a cardinal selection from $V_1^i \times \ldots \times V_L^i$, which implies that the $\succsim^i$ generated by $\mathcal{V}^i$ is complete, then preference compatibility effectively eliminates a planner's ability to specify the $\mathcal{W}_g$ independently. Given V_g^i for $i \in \mathcal{I}$ and $g \in \mathcal{G}$, define group g' to be **non-trivial** if each $V_{g'}^i$ contains non-constant functions.

PROPOSITION 2. *If each $\mathcal{V}^i$ is a cardinal selection then, given $\mathcal{W}_{g'}$ for some nontrivial group g', there is only one $\mathcal{W}_g$ for each $g \neq g'$ such that $\mathcal{W}$ can then satisfy preference compatibility.*

As one would expect, the combination of preference compatibility and complete preferences will stifle a planner's latitude to impose interpersonal comparisons.

The **Pareto optimality** of an allocation x has the standard definition: x must be feasible and there cannot exist a feasible y such that $y^i \succsim^i x^i$ for all $i \in \mathcal{I}$ and $y^j \succ^j x^j$ for some $j \in \mathcal{I}$, where each $\succsim^i$ is generated by $\mathcal{V}^i$.

Whether preference compatibility holds due to preferences being substantially incomplete or the careful choice of a liberal planner, the assumption is not quite enough to guarantee that utilitarian optima are Pareto optimal. The following example, driven by a failure of separability, illustrates.

[17] This argument bears some similarity to the difficulties of implementing Pareto efficient outcomes with separable preferences identified by Le Breton and Sen (1999).

Example. *Suppose there are two individuals a and b and two goods, each of which is a group, and define*

$$\overline{v}_1^a(x_1^a, x_2^a) = \ln x_1^a + 2\ln x_2^a, \overline{v}_2^a(x_1^a, x_2^a) = 2\ln x_1^a + \ln x_2^a,$$
$$\overline{v}_1^b(x_1^b, x_2^b) = 2\ln x_1^b + \ln x_2^b, \overline{v}_2^b(x_1^b, x_2^b) = \ln x_1^b + 2\ln x_2^b.$$

Let these four functions be the profile given by preference compatibility. For each i, one of the $v_1^i + v_2^i$ *formed by the* $v^i \in \mathcal{V}^i$ *is then* $3\ln x_1^i + 3\ln x_2^i$. *So, if* $\mathcal{V}^i$ *is a cardinal selection the* $\succsim^i$ *it generates is the complete preference represented by* $\ln x_1^i + \ln x_2^i$. *If* $e_1 = e_2$ *(the Edgeworth box is square), the Pareto optima satisfy* $x_1^i = x_2^i$ *for* $i = a, b$ *(the* 45° *line). On the other hand, the planner's good* 1 *objective function is* $\ln x_1^a + 2\ln x_2^a + 2\ln x_1^b + \ln x_2^b$ *which is maximised subject to the resource constraints at the allocation* $x^a = (\frac{1}{3}e_1, \frac{2}{3}e_2)$, $x^b = (\frac{2}{3}e_1, \frac{1}{3}e_2)$. *Since this allocation is the unique global optimum for the good* 1 *objective function, it must be a utilitarian optimum when* $\mathcal{W}$ *is group-agnostic.* ■

The difficulty in the Example is that each individual i's utility for one good is affected by i's consumption of the other good and though this 'side effect' is cancelled in the construction of $\succsim^i$ by i's utility for the other good, the cancellation does not enter into the planner's maximisation of the good 1 objective function. Separability blocks this path for trouble.

THEOREM 4. *If separability is satisfied,* $\mathcal{W}$ *is preference compatible, and each* $v_g^i \in V_g^i$ *is strictly concave on g for all g and i, then any utilitarian optimum is Pareto optimal.*

PROOF. Suppose x Pareto dominates y: $x^i \succsim^i y^i$ for all $i \in \mathcal{I}$ and $x^j \succ^j y^j$ for some $j \in \mathcal{I}$, where each $\succsim^i$ is generated by $\mathcal{V}^i$. Then for the $(\overline{v}_1^i, \ldots, \overline{v}_G^i) \in \mathcal{V}^i$, $i \in \mathcal{I}$, given by preference compatibility, $\sum_{g\in\mathcal{G}} \overline{v}_g^i(x^i) \geq \sum_{g\in\mathcal{G}} \overline{v}_g^i(y^i)$ for all $i \in \mathcal{I}$. Due to strict concavity, the z defined by $z^i = \frac{1}{2}x^i + \frac{1}{2}y^i$ for each $i \in \mathcal{I}$ satisfies $\sum_{g\in\mathcal{G}} \overline{v}_g^i(z^i) \geq \sum_{g\in\mathcal{G}} \overline{v}_g^i(y^i)$ for all i and $\sum_{g\in\mathcal{G}} \overline{v}_g^j(z^j) > \sum_{g\in\mathcal{G}} \overline{v}_g^j(y^j)$. Hence $\sum_{i\in\mathcal{I}} \sum_{g\in\mathcal{G}} \overline{v}_g^i(z^i) > \sum_{i\in\mathcal{I}} \sum_{g\in\mathcal{G}} \overline{v}_g^i(y^i)$. Now suppose x is feasible and that y is a utilitarian optimum. Then y and hence z would be feasible and therefore, by Proposition 1, separability would imply $\sum_{i\in\mathcal{I}} \overline{v}_g^i(y^i) \geq \sum_{i\in\mathcal{I}} \overline{v}_g^i(z^i)$ for each $g \in \mathcal{G}$. Hence $\sum_{g\in\mathcal{G}} \sum_{i\in\mathcal{I}} \overline{v}_g^i(y^i) \geq \sum_{g\in\mathcal{G}} \sum_{i\in\mathcal{I}} \overline{v}_g^i(z^i)$, a contradiction. Any utilitarian optimum is therefore Pareto optimal. □

In the absence of separability, utilitarian-Pareto disagreements will still typically disappear as preference incompleteness increases. Call y a **maximum for** $\widehat{v}$ if y is feasible and:

$$\sum_{(g,i)\in\mathcal{G}\times\mathcal{I}} \widehat{v}_g^i(y^i) \geq \sum_{(g,i)\in\mathcal{G}\times\mathcal{I}} \widehat{v}_g^i(x^i)$$

for any feasible x. Proposition 3 in Appendix A shows that it will normally be the case that a utilitarian optimum is a maximum for some $\widehat{v} \in \mathcal{W}$ if $\mathcal{W}$ is group agnostic. Comparably to utilitarian-Pareto clashes under separability, it may well be that some $\widehat{v}^i$ will fail to be in individual i's selection $\mathcal{V}^i$. But if we again let preferences become more incomplete by expanding the selection $\mathcal{V}^i$ then eventually $\mathcal{V}^i$ will contain $\widehat{v}^i$. The following theorem implies that there will then be no utilitarian-Pareto disagreements.

THEOREM 5. *If y is a maximum for* $\widehat{v}$, *where* $\widehat{v}^i \in \mathcal{V}^i$ *for each i, and every* $v_g^i \in V_g^i$ *is strictly concave on its variable goods for each g and i, then y is Pareto optimal.*

PROOF. Since y is a maximum for $\widehat{v}$, $\sum_{i\in\mathcal{I}}\sum_{g\in\mathcal{G}}\widehat{v}^{\,i}_g(y^i)\geq\sum_{i\in\mathcal{I}}\sum_{g\in\mathcal{G}}\widehat{v}^{\,i}_g(z^i)$ for any feasible z. But, as in the proof of Theorem 4, if the feasible x Pareto dominates y then, for the feasible z defined by $z^i=\frac{1}{2}x^i+\frac{1}{2}y^i$ for each i, $\sum_{i\in\mathcal{I}}\sum_{g\in\mathcal{G}}\widehat{v}^{\,i}_g(z^i)>\sum_{i\in\mathcal{I}}\sum_{g\in\mathcal{G}}\widehat{v}^{\,i}_g(y^i)$. $\square$

5. Conclusion

The métier of utilitarianism is the division of a resource among competing individual claims. It is accordingly the precision of the optimal distribution of goods that is compatible with the incompleteness of individual preferences. The theory provided here offers no help for non-distributive decisions that individuals themselves do not know how to make. If each individual cannot decide how much current output should be invested in the future—how much consumption should be sacrificed to lessen carbon pollution?—then a utilitarian advisor will have little to offer. Though the challenging decisions, such as carbon pollution, are inevitably accompanied by incomplete preferences, distributive decision-making can still proceed: utilitarianism can recommend how to divide up any given stock of goods.

Appendix A: Further Results

A.1. Generatable Preferences

Any $\succsim^i$ that has a utility representation u can be generated by some $\mathcal{V}^i$ regardless of the set of groups $\mathcal{G}$: set $V^i_g=\{au+b:a\in\mathbb{R}_{++},b\in\mathbb{R}\}$ for each $g\in\mathcal{G}$ and let $\mathcal{V}^i$ be any selection from $V^i_1\times\ldots\times V^i_G$. At the other end of spectrum, as long as $L>1$ we can admit the extreme incomplete preference relation $\succsim^i$ that orders no pair of distinct bundles: letting u satisfy $u(x^i)=u(y^i)$ if and only if $x^i=y^i$ (see Mandler, 2020), set $\mathcal{G}=\{\{1\},\{2,\ldots,L\}\}$, $V^i_{\{1\}}=\{au+b:a\in\mathbb{R}_{++},b\in\mathbb{R}\}$, $V^i_{\{2,\ldots,L\}}=\{-au+b:a\in\mathbb{R}_{++},b\in\mathbb{R}\}$ and $\mathcal{V}^i=V^i_{\{1\}}\times V^i_{\{2,\ldots,L\}}$. Less dramatically, we can admit the $\succsim^i$ with $x^i\succsim^i y^i$ if and only if $x^i\geq y^i$ by setting $\mathcal{G}=\{\{1\},\ldots,\{L\}\}$, $V^i_{\{k\}}=\{ah_k+b:a\in\mathbb{R}_{++},b\in\mathbb{R}\}$ for each good k, where $h_k:\mathbb{R}^L_+\to\mathbb{R}$ is defined by $h_k(x^i)=x^i_k$, and $\mathcal{V}^i=V^i_{\{1\}}\times\ldots\times V^i_{\{L\}}$.

A.2. Utilitarian Optima Maximise a Social Welfare Function

We show that the assumption that a utilitarian optimum is a maximum for some $\left(\widehat{v}^{\,i}_g\right)_{(g,i)\in\mathcal{G}\times\mathcal{I}}$ is mild. Call the utilitarian optimum x **interior** if $\sum_{i\in\mathcal{I}}v^i_g(x^i)>\sum_{i\in\mathcal{I}}v^i_g(0)$ for each $g\in\mathcal{G}$, where $(v^1_g,\ldots,v^I_g)\in\mathcal{W}_g$ for each g. The function v^i_g is **strictly increasing** if $x^i\geq y^i$ and $x^i\neq y^i$ imply $v^i_g(x^i)>v^i_g(y^i)$.

PROPOSITION 3. *If each $v^i_g\in V^i_g$ is concave and strictly increasing for all i and g and if x is an interior utilitarian optimum then there exists $\left(\widehat{v}^{\,i}_g\right)_{(g,i)\in\mathcal{G}\times\mathcal{I}}$, where each $\widehat{v}_g\in\mathcal{W}_g$, such that x is a maximum for $\left(\widehat{v}^{\,i}_g\right)_{(g,i)\in\mathcal{G}\times\mathcal{I}}$.*

PROOF. Let x be an interior utilitarian optimum and let $\left(v^i_g\right)_{(g,i)\in\mathcal{G}\times\mathcal{I}}\in\mathcal{W}$. Since Proposition 1 implies x is a group-unanimity optimum, for any $g\in\mathcal{G}$, x is a solution to $\max_y\sum_{i\in\mathcal{I}}v^i_g(y^i_g)$ s.t. $\sum_{i\in\mathcal{I}}v^i_{g'}(y^i_{g'})\geq\sum_{i\in\mathcal{I}}v^i_{g'}(x^i_{g'})$ for each $g'\in\mathcal{G}\backslash\{g\}$, $\sum_{i\in\mathcal{I}}y^i_{g'}\leq e_{g'}$ for each $g'\in\mathcal{G}$, and $y\geq0$. If we remove from the problem any good with a 0 endowment, then, due to the strict increasingness and interiority assumptions, there is a y' such that each inequality constraint is slack. Hence the Slater (1950) constraint qualification is satisfied (Boyd and Vandenberghe, 2004 [5.2.3 and 5.3.2]) and hence there exist $\lambda_{g'}^g \geq 0$ for $g' \in \mathcal{G}\backslash\{g\}$ such that

$$\sum_{i\in\mathcal{I}} v_g^i(x_g^i) + \sum_{g'\in\mathcal{G}\backslash\{g\}} \lambda_{g'}^g \sum_{i\in\mathcal{I}} v_{g'}^i(x_{g'}^i) \geq \sum_{i\in\mathcal{I}} v_g^i(y_g^i) + \sum_{g'\in\mathcal{G}\backslash\{g\}} \lambda_{g'}^g \sum_{i\in\mathcal{I}} v_{g'}^i(y_{g'}^i)$$

for all $y \geq 0$. Summing over g,

$$\sum_{g\in\mathcal{G}}\sum_{i\in\mathcal{I}} v_g^i(x_g^i) + \sum_{g\in\mathcal{G}}\sum_{g'\in\mathcal{G}\backslash\{g\}} \lambda_{g'}^g \sum_{i\in\mathcal{I}} v_{g'}^i(x_{g'}^i) \geq \sum_{g\in\mathcal{G}}\sum_{i\in\mathcal{I}} v_g^i(y_g^i) + \sum_{g\in\mathcal{G}}\sum_{g'\in\mathcal{G}\backslash\{g\}} \lambda_{g'}^g \sum_{i\in\mathcal{I}} v_{g'}^i(y_{g'}^i)$$

for all $y \geq 0$. For each $g \in \mathcal{G}$ and $i \in \mathcal{I}$, set $\widehat{v}_g^{\,i} = v_g^i + \sum_{g'\in\mathcal{G}\backslash\{g\}} \lambda_g^{g'} v_g^i = \left(1 + \sum_{g'\in\mathcal{G}\backslash\{g\}} \lambda_g^{g'}\right) v_g^i$. Since $1 + \sum_{g'\in\mathcal{G}\backslash\{g\}} \lambda_g^{g'}$ is a positive constant, $\left(\widehat{v}_g^{\,1}, \ldots, \widehat{v}_g^{\,I}\right) \in \mathcal{W}_g$. $\square$

Appendix B: Remaining Proofs

Proof of Proposition 1. Fix some $\mathcal{W}$ which need not be group agnostic, let x be group-unanimously superior to y, and set some $(v_g)_{g\in\mathcal{G}} \in \mathcal{W}$. Then $\sum_{i\in\mathcal{I}} v_g^i(x^i) \geq \sum_{i\in\mathcal{I}} v_g^i(y^i)$ for all g and with strict inequality for some g. Hence $\sum_{i\in\mathcal{I}}\sum_{g\in\mathcal{G}} v_g^i(x^i) > \sum_{i\in\mathcal{I}}\sum_{g\in\mathcal{G}} v_g^i(y^i)$: x is utilitarian superior to y. Any utilitarian optimum x is therefore a group-unanimity optimum. If not there would be a feasible y that is group-unanimously superior to x and hence utilitarian superior to x.

Conversely, let x be utilitarian superior to y and now assume in addition that $\mathcal{W}$ is group agnostic. Fix $v \in \mathcal{W}$. Suppose for some g' that $\sum_{i\in\mathcal{I}} v_{g'}^i(x^i) < \sum_{i\in\mathcal{I}} v_{g'}^i(y^i)$. Due to group agnosticism, for every $\lambda_{g'} > 0$ the $\widehat{v}$ that equals v except that $\widehat{v}_{g'}^i = \lambda_{g'} v_{g'}^i$ for each i is in $\mathcal{W}$. Since $\sum_{i\in\mathcal{I}}\sum_{g\in\mathcal{G}} \widehat{v}_g^i(x^i) < \sum_{i\in\mathcal{I}}\sum_{g\in\mathcal{G}} \widehat{v}_g^i(y^i)$ for $\lambda_{g'}$ sufficiently large, x would not be utilitarian superior to y. Hence $\sum_{i\in\mathcal{I}} v_g^i(x^i) \geq \sum_{i\in\mathcal{I}} v_g^i(y^i)$ for each g. We also cannot have $\sum_{i\in\mathcal{I}} v_g^i(x^i) = \sum_{i\in\mathcal{I}} v_g^i(y^i)$ for all g since then, by the assumption that each $\mathcal{W}_g$ is a cardinal selection, $\sum_{i\in\mathcal{I}}\sum_{g\in\mathcal{G}} \widetilde{v}_g^i(x^i) = \sum_{i\in\mathcal{I}}\sum_{g\in\mathcal{G}} \widetilde{v}_g^i(y^i)$ for all $(\widetilde{v}_g^i)_{(g,i)\in\mathcal{G}\times\mathcal{I}} \in \mathcal{W}$. Hence x is group-unanimously superior to y. $\square$

Proof of Theorem 3. Let $X_U \subset \mathbb{R}_+^{IL}$ denote the set of group-unanimity optima which, given our continuity assumptions, is non-empty. We first show that there cannot be distinct $x, x' \in X_U$ such that $\sum_{i\in\mathcal{I}} v_g^i(x^i) = \sum_{i\in\mathcal{I}} v_g^i(x^{i\prime})$ for $g \in \mathcal{G}$. Suppose to the contrary that such x and x' exist with $x^i(k) \neq x^{i\prime}(k)$ for some agent i and good k. If, for some group g, $v_g^i \in V_g^i$ is variable on k then the strict concavity assumption implies that $x, x' \notin X_U$. If, for all groups g, each $v_g^i \in V_g^i$ is not variable on k then there is an agent $\widehat{i}$ and group $\widehat{g}$ such that each $v_{\widehat{g}}^{\widehat{i}} \in V_{\widehat{g}}^{\widehat{i}}$ is variable on k. A transfer of $\max\,[x^i(k), x^{i\prime}(k)]$ from i to $\widehat{i}$ will then increase $\sum_{i'\in\mathcal{I}} v_{\widehat{g}}^{i'}$ and not reduce $\sum_{i'\in\mathcal{I}} v_g^{i'}$ for $g \neq \widehat{g}$. Hence either x or x' is not in X_U.

The map $f: X_U \to \mathbb{R}^G$ defined by $f(x) = (\sum_{i\in\mathcal{I}} v_1^i(x^i), \ldots, \sum_{i\in\mathcal{I}} v_G^i(x^i))$ is therefore injective. Since f in addition is continuous and X_U is compact, f is a homeomorphism between X_U and $f(X_U)$ and thus $\dim f(X_U) = d$. For an isolated basket $\mathcal{B}_j$ with d_j groups, we may define the group-unanimity optima for the model that consists of just these groups, $X_U^{\mathcal{B}_j} \subset \mathbb{R}_+^{I\ell}$ where $\ell = \sum_{g\in\mathcal{B}_j} |g|$, and the map $f^{\mathcal{B}_j}: X_U^{\mathcal{B}_j} \to \mathbb{R}^{d_j}$ given by $f^{\mathcal{B}_j}(x) = (\sum_{i\in\mathcal{I}} v_1^i(x^i), \ldots, \sum_{i\in\mathcal{I}} v_{d_j}^i(x^i))$. Then $\dim X_U^{\mathcal{B}_j} < d_j$: if to the contrary $\dim X_U^{\mathcal{B}_j} = d_j$ then $\dim f^{\mathcal{B}_j}(X_U^{\mathcal{B}_j}) = d_j$ and, for any w in the nonempty interior of $f^{\mathcal{B}_j}(X_U^{\mathcal{B}_j})$, $w + (\varepsilon, \ldots, \varepsilon) \in f^{\mathcal{B}_j}(X_U^{\mathcal{B}_j})$ for some $\varepsilon > 0$ which contradicts $w \in f^{\mathcal{B}_j}(X_U^{\mathcal{B}_j})$. Since, with n isolated baskets $\mathcal{B}_1, ..., \mathcal{B}_n$, X_U equals $X_U^{\mathcal{B}_1} \times ... \times X_U^{\mathcal{B}_n}$, $d \leq \sum_{j=1}^{n} \dim X_U^{\mathcal{B}_j}$ and hence $d \leq \sum_{j=1}^{n}(d_j - 1)$. $\square$

Proof of Proposition 2. Suppose $\mathcal{V}^i$ for $i \in \mathcal{I}$ are cardinal selections such that g' is non-trivial and let $\mathcal{W}$ and $\mathcal{W}'$ be preference-compatible welfare selections such that $\mathcal{W}_{g'} = \mathcal{W}'_{g'}$. Let $v_g = \left(v_g^1, ..., v_g^I\right) \in \mathcal{W}_g$ and $v'_g = \left(v_g^{1\prime}, ..., v_g^{I\prime}\right) \in \mathcal{W}'_g$ for $g \in \mathcal{G}$ denote the corresponding profiles given by preference compatibility. Since $v_{g'}$ and $v'_{g'}$ are both elements of $\mathcal{W}_{g'}$, there exist $a \in \mathbb{R}_{++}$ and $b \in \mathbb{R}^I$ such that $v_{g'} = a v'_{g'} + b$. Defining $\widehat{v}^{\,i} = a\left(v_1^{i\prime}, ..., v_G^{i\prime}\right) + (b^i, ..., b^i)$ for $i \in \mathcal{I}$, we have $\widehat{v}^{\,i} \in \mathcal{V}^i$ (since $\left(v_1^{i\prime}, ..., v_G^{i\prime}\right) \in \mathcal{V}^i$ by preference compatibility) and $\widehat{v}_g \equiv (\widehat{v}_g^{\,1}, ..., \widehat{v}_g^{\,I}) \in \mathcal{W}'_g$ for $g \in \mathcal{G}$.

Fix some $i \in \mathcal{I}$. Since $v^i \in \mathcal{V}^i$ and $\widehat{v}^{\,i} \in \mathcal{V}^i$ there exist $\alpha^i \in \mathbb{R}_{++}$ and $\beta^i \in \mathbb{R}^G$ such that $\widehat{v}^{\,i} = \alpha^i v^i + \beta^i$. Let $x, y \in \mathbb{R}_+^L$ satisfy $v_{g'}^i(x) \neq v_{g'}^i(y)$. Since $\widehat{v}_{g'}^{\,i} = v_{g'}^i$, we have $v_{g'}^i(x) = \alpha^i v_{g'}^i(x) + \beta_{g'}^i$ and $v_{g'}^i(y) = \alpha^i v_{g'}^i(y) + \beta_{g'}^i$ which implies $(1-\alpha^i)(v_{g'}^i(x) - v_{g'}^i(y)) = 0$ and hence $\alpha^i = 1$. As this argument applies to each $i \in \mathcal{I}$, $\widehat{v}_g = v_g + (\beta_g^1, ..., \beta_g^I)$ for each $g \in \mathcal{G}$. Given that $v_g \in \mathcal{W}_g$ for each $g \in \mathcal{G}$, we have $\widehat{v}_g \in \mathcal{W}_g$ for each $g \in \mathcal{G}$. Since for any $g \in \mathcal{G}$ and $v''_g \in \mathcal{W}'_g$, $\widehat{v}_g \in \mathcal{W}'_g$ implies there exist $a'' \in \mathbb{R}_{++}$ and $b'' \in \mathbb{R}^I$ such that $\widehat{v}_g = a'' v''_g + b''$, we conclude that $v''_g \in \mathcal{W}_g$. $\square$

Royal Holloway College, University of London

References

Apesteguia, J. and Ballester, M. (2015). 'A measure of rationality and welfare', *Journal of Political Economy*, vol. 123, pp. 1278–310.

Argenziano, R. and Gilboa, I. (2019). 'Perception-theoretic foundations of weighted utilitarianism', Economic Journal, vol. 129, pp. 1511–28.

Aumann, R. (1962). 'Utility theory without the completeness axiom', *Econometrica*, vol. 30, pp. 445–62.

Bernheim, B.D. and Rangel, A. (2007). 'Toward choice-theoretic foundations for behavioral welfare economics', *American Economic Review*, vol. 97, pp. 464–70.

Bernheim, B.D. and Rangel, A. (2009). 'Beyond revealed preference: choice theoretic foundations for behavioral welfare economics', *Quarterly Journal of Economics*, vol. 124, pp. 51–104.

Bewley, T. (1986). 'Knightian decision theory: Part I', Cowles Foundation Discussion Paper 807, Cowles Foundation: New Haven. Also published in *Decisions in Economics and Finance*, vol. 25, pp. 79–110, 2002.

Bossert, W. and Weymark, J. (2004). 'Utility in social choice', in (S. Barberà, P. Hammond, and C. Seidl, eds.), *Handbook of Utility Theory*, vol. 2, pp. 1099–177, Boston: Kluwer.

Boyd, S. and Vandenberghe, L. (2004). *Convex optimization*, Cambridge: Cambridge University Press.

d'Aspremont, C. and Gevers, L. (1977). 'Equity and the informational basis of collective choice', *Review of Economic Studies*, vol. 44, pp. 199–209.

Danan, E., Gajdos, T. and Tallon, J.-M. (2015). 'Harsanyi's aggregation theorem with incomplete preferences', *American Economic Journal: Microeconomics*, vol. 7, pp. 61–9.

Edgeworth, F. (1881). *Mathematical Psychics: An Essay on the Application of Mathematics to the Moral Sciences*, London: Kegan Paul.

Fleurbaey, M. and Schokkaert, E. (2013). 'Behavioral welfare economics and redistribution', *American Economic Journal: Microeconomics*, vol. 5, pp. 180–205.

Gorman, W. (1959). 'Separable utility and aggregation', *Econometrica*, vol. 27, pp. 469–81.

Hammond, P. (1981). 'Ex-ante and ex-post welfare optimality under uncertainty', *Economica*, vol. 48, pp. 235–50.

Harsanyi, J. (1955). 'Cardinal welfare, individualistic ethics, and interpersonal comparisons of utility', *Journal of Political Economy*, vol. 63, pp. 309–21.

Jorgenson, D. and Lau, L. (1975). 'The structure of consumer preferences', *Annals of Economic and Social Measurement*, vol. 4, pp. 49–101.

Kahneman, D., Knetsch, J. and Thaler, R. (1990). 'Experimental tests of the endowment effect and the Coase theorem', *Journal of Political Economy*, vol. 98, pp. 1325–48.

Kahneman, D., Wakker, P. and Sarin, R. (1997). 'Back to Bentham? Explorations of experienced utility', *Quarterly Journal of Economics*, vol. 112, pp. 375–406.
Knetsch, J. (1989). 'The endowment effect and evidence of nonreversible indifference curves', *American Economic Review*, vol. 79, pp. 1277–84.
Laibson, D. (1997). 'Golden eggs and hyperbolic discounting', *Quarterly Journal of Economics*, vol. 112, pp. 443–77.
Le Breton, M. and Sen, A. (1999). 'Separable preferences, strategyproofness, and decomposability', *Econometrica*, vol. 67, pp. 605–28.
List, C. (2004). 'Multidimensional welfare aggregation', *Public Choice*, vol. 119, pp. 119–42.
Mandler, M. (2004). 'Status quo maintenance reconsidered: changing or incomplete preferences ?', Economic Journal, vol. 114, pp. 518–35, .
Mandler, M. (2005). 'Incomplete preferences and rational transitivity of choice', *Games and Economic Behavior*, vol. 50, pp. 255–77.
Mandler, M. (2014). 'Indecisiveness in behavioral welfare economics', *Journal of Economic Behavior and Organization*, vol. 97, pp. 219–35.
Mandler, M. (2020). 'The lexicographic method in preference theory', *Economic Theory*, forthcoming, https://doi.org/10.1007/s00199-020-01256-2.
Mas-Colell, A. (1985). *The Theory of General Economic Equilibrium: A Differentiable Approach*, Cambridge: Cambridge University Press.
Mill, J.S. (1863). *Utilitarianism*, London: Parker, Son, & Bourn.
Roberts, K. (1980). 'Interpersonal comparability and social choice theory', *Review of Economic Studies*, vol. 47, pp. 421–39.
Samuelson, W. and Zeckhauser, R. (1988). 'Status quo bias in decision making', *Journal of Risk and Uncertainty*, vol. 1, pp. 7–59.
Salant, Y. and Rubinstein, A. (2008). '(A, f) choice with frames', *Review of Economic Studies*, vol. 75, pp. 1287–96.
Schumpeter, J. (1912). *The Theory of Economic Development*, New York: Oxford University Press, 1978.
Sen, A. (1970). 'Interpersonal aggregation and partial comparability', *Econometrica*, vol. 38, pp. 393–409.
Sen, A. (1979). 'Utilitarianism and welfarism', *Journal of Philosophy*, vol. 76, pp. 463–89.
Sen, A. (1986). *Welfare Economics and the Real World*, Memphis: Seidman Foundation.
Slater, M. (1950). 'Lagrange multipliers revisited', Cowles Commission Discussion Paper: Mathematics 403, Cowles Commission for Research in Economics, University of Chicago.
Smale, S. (1974). 'Global analysis and economics V', *Journal of Mathematical Economics*, vol. 1, pp. 213–21.
Thaler, R. (1980). 'Toward a positive theory of consumer choice', *Journal of Economic Behavior and Organization*, vol. 1, pp. 39–60.
Tversky, A. and Kahneman, D. (1991). 'Loss aversion in riskless choice: a reference-dependent model', *Quarterly Journal of Economics*, vol. 106, pp. 1039–61.