## Tournaments and Relative Compensation

## Using promotions to motivate effort

## Wage is (partially) fixed to a position (level)

## When an opening occurs, workers at a lower level compete for promotion

## The prize for the winner is the wage spread between the two levels

**A Model of Promotion Tournaments**

**Suppose there are two employees: j, k**
**Two positions: boss, underling**

**The workers output is given by: $q_j = \mu_j + \varepsilon_j$ ; $q_k = \mu_k + \varepsilon_k$**
**Where $\mu$ is effort and $\varepsilon$ is "luck"**
**Q is the value of output**

**The worker's problem is:**

**$\text{Max}_{\mu j}\ W_B P + W_U(1-P) - C(\mu_j)$**
**P is the probability that j wins**
**$P=P(\mu_j, \varepsilon_j, q_k, \mu_k, \varepsilon_k)$**

**The first order conditions are:**

**$(W_U - W_B)\ \partial P/\partial \mu_j - C'(\mu_j) = 0$**

**or the marginal returns to effort (the spread times the marginal effect of effort on the probability of winning) are equal to the marginal cost of effort**

**The probability that worker j beats worker k is**

$$P = \text{Prob}(\mu_j + \varepsilon_j > \mu_k + \varepsilon_k) = \text{Prob}(\mu_j - \mu_k > \varepsilon_k - \varepsilon_j)$$

**The firm's problem is to set $W_U$ and $W_B$ to elicit optimal effort**

$$\text{Max}_{WB,\,WU} \{ Q(\mu) - (W_B + W_U)/2\}$$
$$\text{subject to } (W_B + W_U) \geq C(\mu)$$

$$\text{or Max}_{WB,\,WU} \{Q(\mu) - C(\mu)\}$$

**f.o.c.**

$$\partial\{\}/\partial W_B = (1 - C'(\mu))\, \partial Q(\mu)/\partial W_B = 0$$
$$\partial\{\}/\partial W_U = (1 - C'(\mu))\, \partial Q(\mu)/\partial W_U = 0$$

**Solving the first order conditions we get $C'(\mu) = 1$ or set $W_B$ and $W_U$ such that the marginal benefit to the firm of one more unit of effort £1 is equal to the marginal cost of effort to the worker**

## Implications of the Model

## 1) Winner's prize increases further up the hierarchy

**Magnification effect – performance of boss has an effect on output of underlings**

**Incentive effect – the prize for a promotion from level 1 to level 2 has incentives for workers at level 1, the prize for promotion from level 2 to level 3 has incentives for workers at levels 1 and 2, etc.**

**Option values – part of the prize for winning promotion from level 1 to level 2 is a higher probability of promotion to level 3. This declines with each promotion as there are fewer further promotions possible.**

**Marginal utility effect – as workers get richer (i.e. move up the hierarchy) it takes a larger prize to elicit the same effort levels**

**2) Effect of luck – the larger the influence of luck the smaller the effect of effort on P. Thus a larger spread is needed to have the same effect on effort.**

**Implies that growth in CEO salaries over time might be due to increasing economic uncertainty.**

**Advantages of incentives through tournaments**

**1) Lower information requirement – easier to judge which worker is better than to measure the output of each worker**

**2) Incentive compatibility – since the prize must be awarded, the firm has an incentive to reward the best worker. With absolute compensation the firm might claim that performance wasn't "good enough"**

**3) Common risk does not effect compensation**

**Recall $q_j = \mu_j + \varepsilon_j$ and $q_k = \mu_k + \varepsilon_k$**

**Suppose $\varepsilon_j = \varepsilon_c + \varepsilon_{js}$ and $\varepsilon_k = \varepsilon_c + \varepsilon_{ks}$**

**In other words luck has a common element (firm loses a contract) and an individual-specific element (person catches a virus)**

**With absolute compensation:**
**$W = bq$ thus $VAR(W) = VAR(\varepsilon_j) = VAR(\varepsilon_c) + VAR(\varepsilon_{js})$**

**With relative compensation**

$$P = \text{Prob}(\mu_j - \mu_k > \varepsilon_k - \varepsilon_j) =$$

$$\text{Prob}(\mu_j - \mu_k > (\varepsilon_c - \varepsilon_c) + (\varepsilon_{k\,s} - \varepsilon_{js})) =$$

$$\text{Prob}(\mu_j - \mu_k > \varepsilon_{k\,s} - \varepsilon_{js})$$

**Thus** $\text{Var}(COMP) \neq f(\text{VAR}(\varepsilon_c))$

**Problems with Tournaments**

**1) Cost – Loser's prize may have to be large to encourage participation (ex: gladiators)**

**2) If loss looks likely, participants may have little further to lose by taking excessive risk or by withholding further effort**

**Countermeasure – successive tournaments**

**3) Sabotage – j can improve his chances of winning by either increasing his productivity or reducing k's productivity**

**Ex: two solicitors competing for a partnership – one may give information to the opposing legal team on a case handled by his competitor**

**Countermeasures**

1) **Very high penalties for sabotage**
2) **Separation of participants in a given promotion tournament**
3) **Wage compression at lower ranks**

**Testing the tournament model**

**1) Are wages attached to rank in a firm?**

$$\text{Log } W = a + BX + \Sigma b_i pos_i + e$$

**Issue of omitted ability – better workers paid more and assigned higher positions**

**2) Within-rank and between-rank variability of wages**
**If within rank variability is low, it suggests that wages are attached to rank.**

**3) Salary increments – Do wages increase more at the time of promotion than at other times?**

**4) Prize structure and effort levels in sports tournaments – ex: as the prize spread increases in a car race do average times go down, all else equal? Do crashes increase?**