Tournaments and Relative Compensation

Using promotions to motivate effort

Wage is (partially) fixed to a position (level)

When an opening occurs, workers at a lower level compete for promotion

The prize for the winner is the wage spread between the two levels
A Model of Promotion Tournaments

Suppose there are two employees: j, k
Two positions: boss, underling

The workers output is given by: \( q_j = \mu_j + \varepsilon_j; q_k = \mu_k + \varepsilon_k \)
Where \( \mu \) is effort and \( \varepsilon \) is “luck”
Q is the value of output

The worker’s problem is:

\[
\text{Max}_{\mu_j} \ W_B P + W_U (1-P) - C(\mu_j)
\]
P is the probability that j wins
P = P(\( \mu_j, \varepsilon_j, q_k, \mu_k, \varepsilon_k \))

The first order conditions are:

\[
(W_U - W_B) \frac{\partial P}{\partial \mu_j} - C'(\mu_j) = 0
\]

or the marginal returns to effort (the spread times the marginal effect of effort on the probability of winning) are equal to the marginal cost of effort
The probability that worker j beats worker k is

\[ P = \text{Prob} (\mu_j + \varepsilon_j > \mu_k + \varepsilon_k) = \text{Prob} (\mu_j - \mu_k > \varepsilon_k - \varepsilon_j) \]

The firm’s problem is to set \( W_U \) and \( W_B \) to elicit optimal effort

\[ \text{Max}_{W_B, W_U} \{ Q(\mu) - (W_B + W_U)/2 \} \]
subject to \( (W_B + W_U) \geq C(\mu) \)

or \( \text{Max}_{W_B, W_U} \{ Q(\mu) - C(\mu) \} \)

f.o.c.

\[ \frac{\partial \{\} }{\partial W_B} = (1 - C'(\mu)) \frac{\partial Q(\mu)}{\partial W_B} = 0 \]
\[ \frac{\partial \{\} }{\partial W_U} = (1 - C'(\mu)) \frac{\partial Q(\mu)}{\partial W_U} = 0 \]

Solving the first order conditions we get \( C'(\mu) = 1 \)
or set \( W_B \) and \( W_U \) such that the marginal benefit to the firm of one more unit of effort £1 is equal to the marginal cost of effort to the worker
Implications of the Model

1) Winner’s prize increases further up the hierarchy

   Magnification effect – performance of boss has an effect on output of underlings

   Incentive effect – the prize for a promotion from level 1 to level 2 has incentives for workers at level 1, the prize for promotion from level 2 to level 3 has incentives for workers at levels 1 and 2, etc.

   Option values – part of the prize for winning promotion from level 1 to level 2 is a higher probability of promotion to level 3. This declines with each promotion as there are fewer further promotions possible.

   Marginal utility effect – as workers get richer (i.e. move up the hierarchy) it takes a larger prize to elicit the same effort levels
2) Effect of luck – the larger the influence of luck the smaller the effect of effort on P. Thus a larger spread is needed to have the same effect on effort.

Implies that growth in CEO salaries over time might be due to increasing economic uncertainty.
Advantages of incentives through tournaments

1) Lower information requirement – easier to judge which worker is better than to measure the output of each worker

2) Incentive compatibility – since the prize must be awarded, the firm has an incentive to reward the best worker. With absolute compensation the firm might claim that performance wasn’t “good enough”

3) Common risk does not effect compensation

Recall \( q_j = \mu_j + \varepsilon_j \) and \( q_k = \mu_k + \varepsilon_k \)

Suppose \( \varepsilon_j = \varepsilon_c + \varepsilon_{js} \) and \( \varepsilon_k = \varepsilon_c + \varepsilon_{ks} \)

In other words luck has a common element (firm loses a contract) and an individual-specific element (person catches a virus)

With absolute compensation:
\[ W = bq \] thus \( \text{VAR} (W) = \text{VAR}(\varepsilon_j) = \text{VAR}(\varepsilon_c) + \text{VAR}(\varepsilon_{js}) \)
With relative compensation

\[ P = \text{Prob} \left( \mu_j - \mu_k > \varepsilon_k - \varepsilon_j \right) = \]
\[ \text{Prob} \left( \mu_j - \mu_k > (\varepsilon_c - \varepsilon_c) + (\varepsilon_k - \varepsilon_j) \right) = \]
\[ \text{Prob} \left( \mu_j - \mu_k > \varepsilon_k - \varepsilon_j \right) \]

Thus \( \text{Var} \text{(COMP)} \neq f \left( \text{VAR} \text{(} \varepsilon_c \text{)} \right) \)
Problems with Tournaments

1) Cost – Loser’s prize may have to be large to encourage participation (ex: gladiators)

2) If loss looks likely, participants may have little further to lose by taking excessive risk or by withholding further effort

Countermeasure – successive tournaments

3) Sabotage – j can improve his chances of winning by either increasing his productivity or reducing k’s productivity

Ex: two solicitors competing for a partnership – one may give information to the opposing legal team on a case handled by his competitor

Countermeasures

1) Very high penalties for sabotage
2) Separation of participants in a given promotion tournament
3) Wage compression at lower ranks
Testing the tournament model

1) Are wages attached to rank in a firm?

\[ \log W = a + BX + \sum b_i pos_i + e \]

Issue of omitted ability – better workers paid more and assigned higher positions

2) Within-rank and between-rank variability of wages

If within rank variability is low, it suggests that wages are attached to rank.

3) Salary increments – Do wages increase more at the time of promotion than at other times?

4) Prize structure and effort levels in sports tournaments – ex: as the prize spread increases in a car race do average times go down, all else equal? Do crashes increase?