1.1, 1.4, 1.7

Rule of 70

Output	Y	$Y(1+\frac{g}{100})$	$Y(1+\frac{g}{100})^2$	...
Year	1	2	3	

$$\left(1+\frac{g}{100}\right)^n = 2$$

$$n\left[\ln\left(1+\frac{g}{100}\right)\right) = \ln 2 = .7$$

$$n\frac{g}{100} \approx .7$$

$$n \approx 70/g$$

Solow Model

Y - output

K - Capital

L - Labor

A - Labor effectiveness (Knowledge, technology)

t - time

$$Y(t) = F(K(t), A(t)L(t))$$

AL - Harrod neutral

AK - Capital augmenting

AF - Hicks neutral

Big simplifying assumption

Cobb-Douglass Technology

$$F(K, AL) = K^{\alpha} (AL)^{1-\alpha}$$

Notice

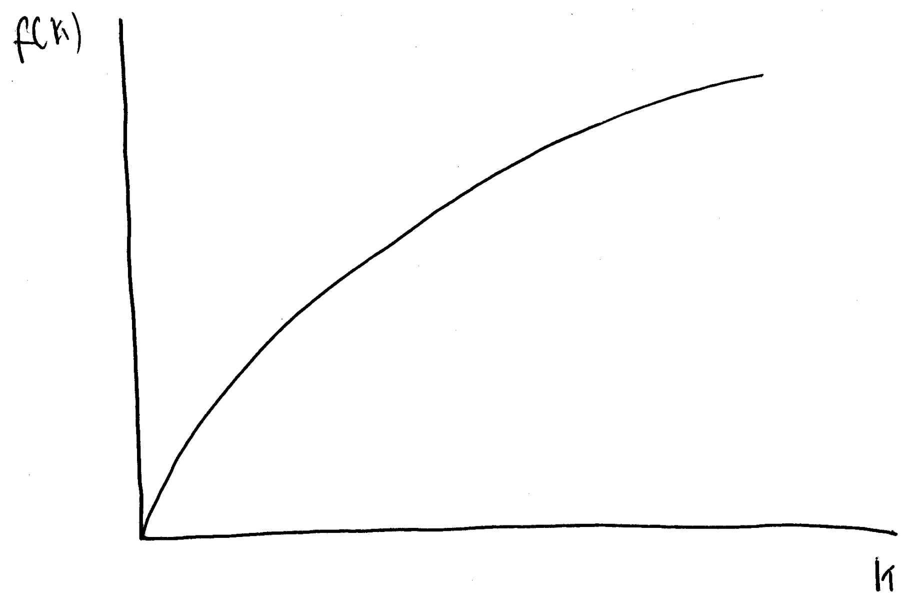
1. Three types of neutrality coincide
2. CRS

Divide by AL

$$\frac{F(K, AL)}{AL} = \left(\frac{K}{AL}\right)^{\alpha}$$

Define

$$f(k) = k^{\alpha} \qquad k = \frac{K}{AL}$$



$$\dot{L}(t) = n\,L(t)$$
$$\dot{A}(t) = g\,A(t)$$

Assume constant exogenous growth rates

$$L(t) = L(0)\,e^{nt}$$
$$A(t) = A(0)\,e^{gt}$$

Prove

$$Y(t) = \underbrace{C(t)}_{\text{consumption}} + \underbrace{I(t)}_{\text{investment}}$$

Key Solow Assumption

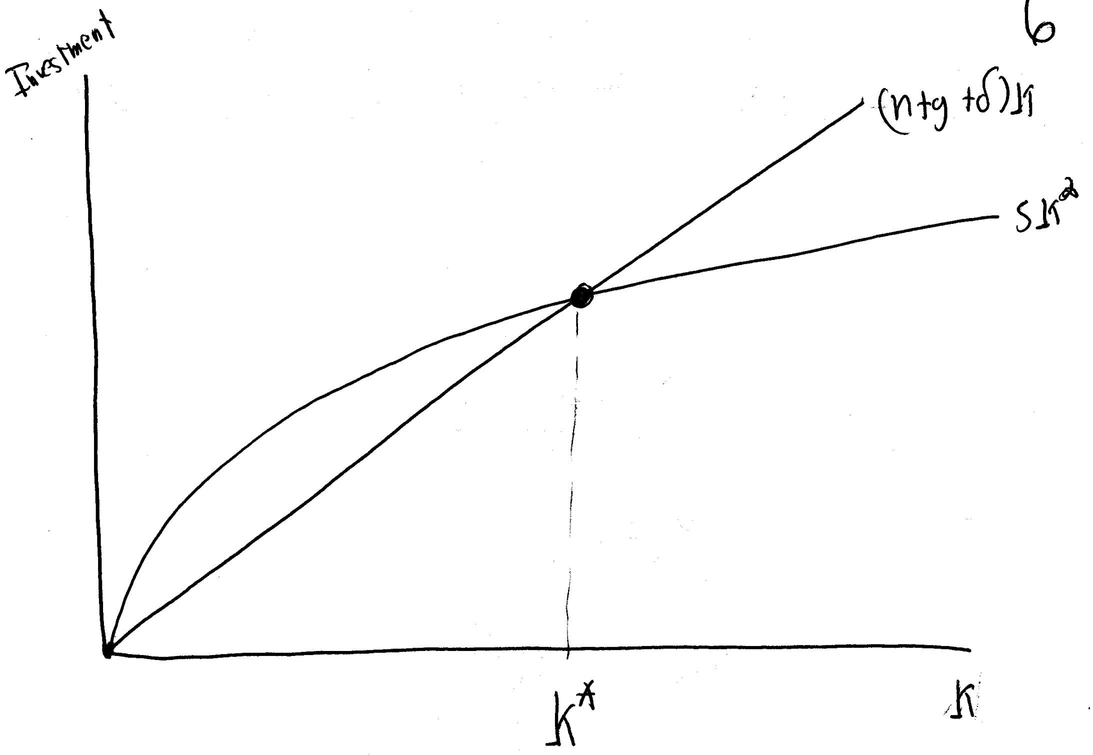
$$I(t) = s\,Y(t)$$

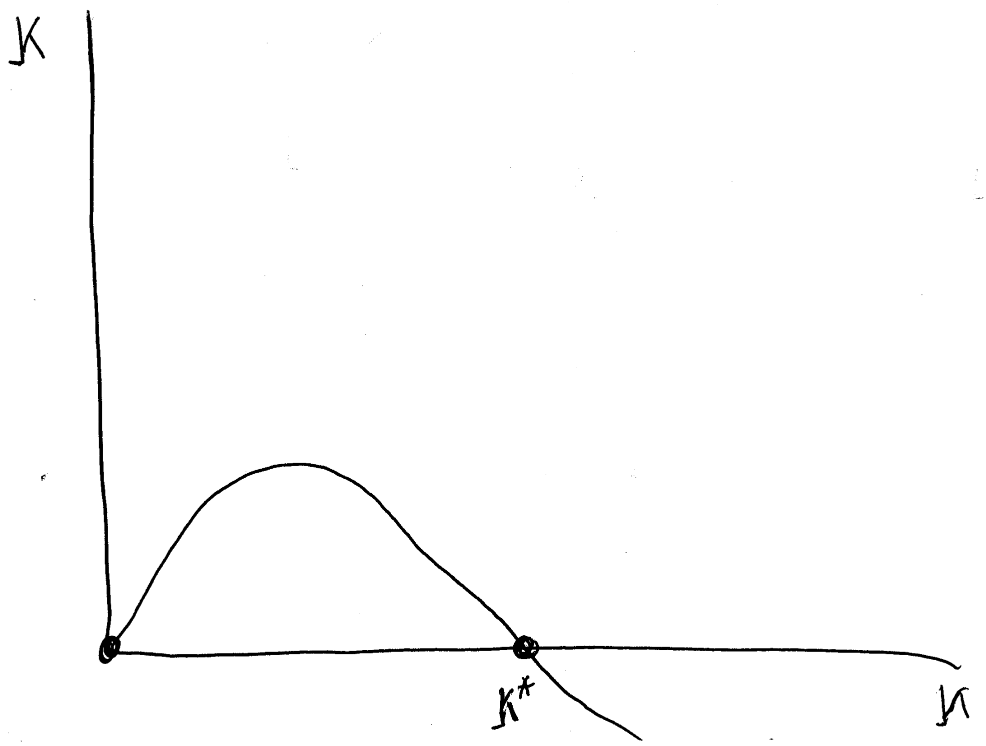
$$\dot{K} = \underbrace{s\,Y(t)}_{\text{investment}} - \underbrace{\delta K(t)}_{\text{depreciation}}$$

$$\dot{k} = \dot{\left(\frac{K}{AL}\right)} = \underbrace{s\,k(t)^{\alpha}}_{\text{gross investment}} - \underbrace{(n+g+\delta)\,k(t)}_{\text{replacement investment}}$$

$n+g$ - growth rate of effective labor

6





k^* - <u>Steady state</u> capital per effective unit of labor

<u>Balanced Growth Path</u> - All variables grow at constant rate

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Variable	Growth Rate
L	n
A	g
K	$n+g$
AL	$n+g$
Y	$n+g$
K/L	g
Y/L	g
K/AL	0
Y/AL	0

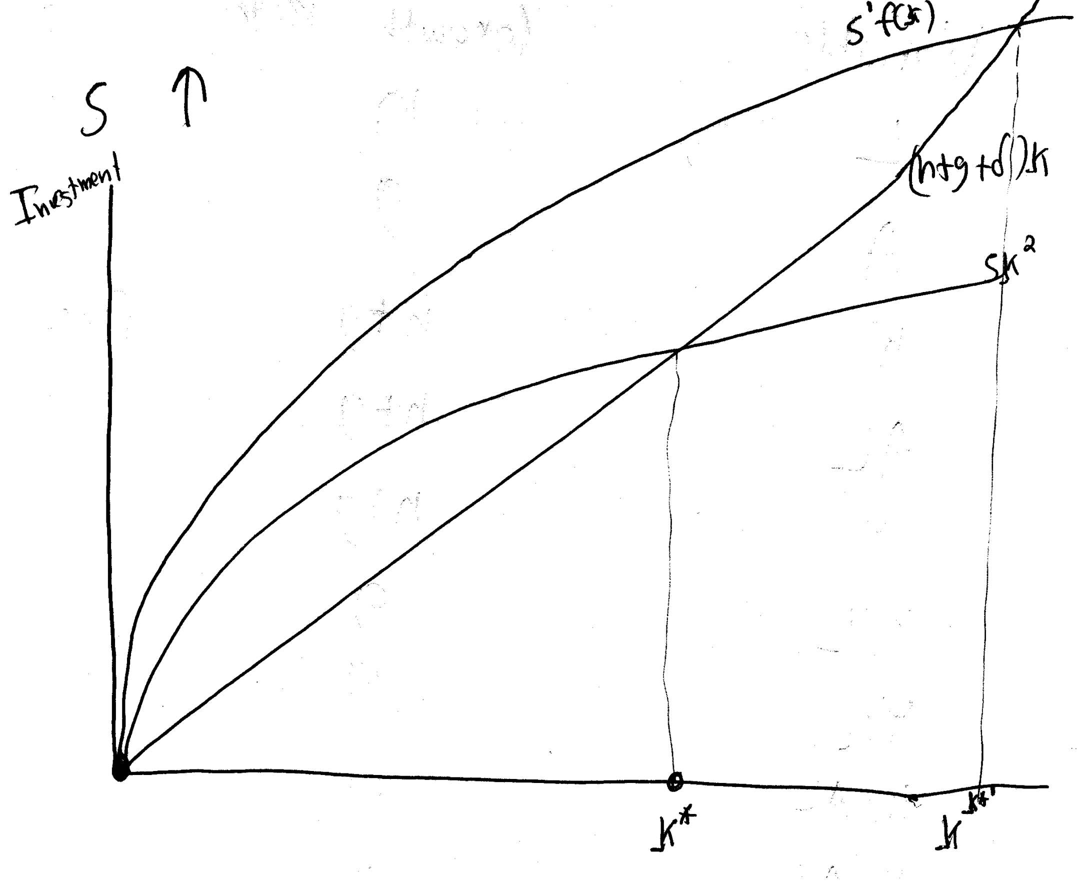
Prove

Kaldor (1961)

1. L, K & Y growth constant ✓

2. Y & K grow at same rate ✓

3. L grows more slowly ✓



1. Long-run growth rates don't change

2. $k^* \rightarrow k^{*'}$

3. Short run $\frac{\dot{Y}}{Y} > n+g$ $\frac{\dot{\left(\frac{Y}{L}\right)}}{\left(\frac{Y}{L}\right)} > g$

etc.

What is k^* ?

$$(n+g+\delta)\,k^* = s(k^*)^\alpha$$

$$k^* = \left(\frac{s}{n+g+\delta}\right)^{1/1-\alpha}$$ prove

Steady - state consumption ?

$$c^* = (1-s)(k^*)^\alpha = (1-s)\left(\frac{s}{n+g+\delta}\right)^{\alpha/1-\alpha}$$

Output ?

$$y^* = \left(\frac{s}{n+g+\delta}\right)^{\alpha/1-\alpha}$$

Golden Rule Consumption

$$\max_s \; c^*$$

equivalent to

$$\max_{k^*} \; k^{*\alpha} - (n+g+\delta)k^*$$

$$k^* = \left(\frac{\alpha}{n+g+\delta}\right)^{1/1-\alpha}$$

$$s^{gr} = \alpha \approx 1/3$$

Very high

$s \uparrow \Rightarrow y \uparrow$ by how much?

$$y = (k^*)^{\alpha} = \left(\frac{s}{n+g+d}\right)^{\alpha/1-\alpha}$$

$$\frac{dy}{ds} = \frac{\alpha}{1-\alpha}\left(\frac{s}{n+g+d}\right)^{\frac{2\alpha-1}{1-\alpha}} \frac{1}{n+g+d}$$

$$\frac{dy}{ds}\frac{s}{y} = \left[\frac{\alpha}{1-\alpha}\left(\frac{1}{n+g+d}\right)^{\alpha/1-\alpha} s^{\frac{2\alpha-1}{1-\alpha}}\right] s \frac{(n+g+d)^{\alpha/1-\alpha}}{s^{\alpha/1-\alpha}}$$

$$= \frac{\alpha}{1-\alpha}$$

$$\alpha = 1/3 \Rightarrow \frac{\% \Delta y}{\% \Delta s} \approx .5$$

<u>Not Huge</u>

How fast does $k \to k^*$

$$\dot{k}(k^*) = 0$$

$$\dot{k}(k) \approx \frac{\partial \dot{k}(k^*)}{\partial k}(k - k^*)$$

$$\frac{\partial \dot{k}(k^*)}{\partial k} = s\alpha (k^*)^{\alpha - 1} - n + g + \delta$$

$$= (\alpha - 1)(n + g + \delta)$$

$$\dot{k}(t) \approx (\alpha - 1)(n + g + \delta)(k(t) - k^*)$$

$$k(t) - k^* \approx e^{-(1-\alpha)(n+g+\delta)t}(k(0) - k^*)$$

$n + g + \delta \quad - .06$

$\alpha \quad = 1/3$

Catch up at 4% per year

Why are some countries rich and other countries poor?

Can capital explain it?

Suppose A_{US} & A_{India} are the same

Fact - US about ten times as rich as India

$\alpha \approx 1/3$

$(K_{US})^{1/3} = 10 (K_{India})^{1/3} \Rightarrow$

$K_{US} \approx 1000 K_{India}$

1. Reality - 2 or 3 times

2. Suppose anyway $K_{US} = 1000 K_{India}$

Marginal product of capital is

$\alpha K^{\alpha - 1} \equiv MP_K$

$\Rightarrow MP_{K\,India} = 100\, MP_{K\,US}$

Would be huge capital flows from US to India

Conclusion - Must look more at A, something that is very vague & poorly understood

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Growth Accounting

$$Y(t) = A(t)^{1-\alpha} K(t)^{\alpha} L(t)^{1-\alpha}$$

$$\dot{Y}(t) = \frac{(1-\alpha)\dot{A}(t)\,Y(t)}{A(t)} + \frac{\alpha\,\dot{K}(t)\,Y(t)}{K(t)} + \frac{(1-\alpha)\dot{L}(t)\,Y(t)}{L(t)}$$

$$\frac{\dot{Y}(t)}{Y(t)} = (1-\alpha)\frac{\dot{A}(t)}{A(t)} + \alpha\frac{\dot{K}(t)}{K(t)} + (1-\alpha)\frac{\dot{L}(t)}{L(t)}$$

$$\text{Output Growth} = \alpha\left(\text{Capital Growth}\right) + (1-\alpha)\left(\text{Labor Growth}\right) + \text{Solow Residual}$$

Alwyn Young (1995)

Growth Accounting ⇒ 4 Tigers growth due to factor accumulation

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Convergence

1. Do poor countries grow faster than rich countries ?

Solow model would suggest "yes" if poor countries are below steady state & rich countries are there

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Also, technological diffusion might cause $\frac{\dot{A}}{A}$ to converge across countries

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Capital might also flow to poor countries

Baumol (1986) - Convergence

16 rich countries

$$\ln\left(\frac{Y}{L}\right)_{1979} - \ln\left(\frac{Y}{L}\right)_{1870} = 8.457 - .995 \ln\left(\frac{Y}{L}\right)_{1870}$$

$(.094)$

$R^2 = .87$

Suggests convergence

DeLong (1988) - No Convergence

1. Sample Selection bias

Long data series $\Rightarrow$ rich today

Poor$_{1870}$ + Low Growth $\Rightarrow$ No data today

Poor$_{1870}$ + High Growth $\Rightarrow$ Data today

Rich$_{1870}$ $\Rightarrow$ Probably have Data today

2. Measurement error in 1870
   Country rich in 1870 $\Rightarrow$ probably overestimate
   " poor in 1870 $\Rightarrow$ probably underestimate

Mankiw, Romer & Weil (1992)

$$\ln y^* = \frac{\alpha}{1-\alpha} \ln s - \frac{\alpha}{1-\alpha} \ln (n+g+d)$$

$$\ln \frac{Y}{L} = \frac{\alpha}{1-\alpha} \ln s - \frac{\alpha}{1-\alpha} \ln (n+g+d) + \ln A$$

$$\ln \frac{Y}{L} = \underset{(.12)}{6.87} + \underset{(.12)}{1.48} \ln (s - \ln (n+0.05)) \qquad R^2 = .57$$

Problem $\hat{\alpha} = .6$

$1/3$ more reasonable