Real Business Cycles

Facts about Business Cycles

1. Economies have variations in aggregate output & employment.

2. These cycles are irregular
   a) Deep declines & shallow declines
   b) Some are long & some are short

   ⇒ good theories will have to have some random elements

3. Uneven distribution over types of output. Big items are:

   Consumer durables
   Investment (especially inventory investment)
   Net Exports

Very stable items are:

Services
Nondurables
Government purchases

4. Tend to be long periods when output is slightly above its long run path and short periods when it is far below

5. Cycles before WW II not hugely different from cycles after WW II

6. Great Depression was a unique event.

7. In a recession:

Employment ↓
Unemployment ↑
Hours worked per week ↓

Output ↓ (Much larger than)

Productivity (Output per worker) ↓

Okun's Law - Output ↓ 3% ⇒ Unemployment ↑ 1%

2% more accurate these days

Inflation usually ↓

Real wage ↓
interest rates ↓
money stock ↓

Types of Approaches

I. Competitive Equilibrium Model (Walrasian)

Here we use the Ramsey model but we must extend it because balanced growth is not cyclical

1. Add random shocks (productivity, government spending)
2. Put leisure in the utility function so labor supply can vary.

II. Keynesian approach

Departs from competitive model by introducing price stickiness.

Involuntary Unemployment

Very Simple RBC Model

Ramsey framework

Zero population
Infinitely lived agents
Discrete time

<u>Quadratic Utility</u> — $C_t - \theta C_t^2$ $\quad \theta > 0$

This is not a great utility function because marginal utility can go negative. But we will just assume that we are operating in a range where this does not happen.

Agents maximize $E\left[\sum_{t=0}^{\infty} B^t \left(C_t - \theta C_t^2\right)\right]$

expectation

$$Y_t = \underbrace{\frac{1-B}{B}}_{\text{Major Simplification}} K_t + \underbrace{e_t}_{\text{random}}$$

$$K_{t+1} = K_t + Y_t - C_t = \frac{K_t}{B} + e_t - C_t$$

$$e_t = \phi e_{t-1} + \epsilon_t$$

$-1 < \phi < 1$ ϵ_t's mean 0 i.i.d.

First-order autoregressive process

$$\max E\left[\sum_{t=0}^{\infty} B^t (C_t - \theta C_t^2)\right]$$

$$\text{s.t. } K_{t+1} = \frac{K_t}{B} + e_t - C_t$$

K_0 given

$$L = E\left\{\sum_{t=0}^{\infty}\left[B^t (C_t - \theta C_t^2) - B^{t+1} \lambda_{t+1} \left(K_{t+1} - \frac{K_t}{B} - e_t + C_t\right)\right]\right\}$$

F.O.C.

i) $\beta^{t}(1-2\theta C_t) - \beta^{t+1} E[r_{t+1}/t] = 0$ $t=0,1,\dots$

ii) $\beta^{t} r_t = \dfrac{\beta^{t+1}}{\beta} E[r_{t+1}/t]$ $t=1,2,\dots$

iii) Capital Accumulation Constraints

$$1-2\theta C_t = \beta E[r_{t+1}/t]$$

$$1-2\theta C_{t+1} = \beta r_{t+1}$$

$$E[1-2\theta C_{t+1}/t] = \beta E[r_{t+1}/t]$$

$$C_t = E[C_{t+1}/t]$$

i.e. consumption follows a random walk

Guess formula for consumption

$$C_t = a + bK_t + ce_t$$

Substitute into capital accumulation equation

$$K_{t+1} = \frac{K_t}{\beta} + e_t - a - bK_t - ce_t$$

$$= -a + \left(\frac{1}{\beta} - b\right)K_t + (1-c)e_t$$

$C_t = E[C_{t+1}]$ & $C_t = a + bK_t + ce_t$

$$\Rightarrow a + bK_t + ce_t = E[a + bK_{t+1} + ce_{t+1}]$$

$$a + bK_t + ce_t = a + b\underbrace{E\left[-a + \left(\frac{1}{\beta} - b\right)K_t + (1-c)e_t\right]}_{\text{not uncertain}} + c\underbrace{E[e_{t+1}]}_{c\phi e_t + 0}$$

$$a + bK_t + ce_t = a(1-b) + b\left(\frac{1}{\beta} - b\right)K_t + b(1-c)e_t + c\phi e_t$$

Must have

1. $b - \frac{1+B}{B} = 1 \Rightarrow b = \frac{1-B}{B}$

2. $c = b(1-c) - c\phi$

$c = \frac{b}{1+b+\phi} = \frac{1-B}{1-\phi B}$

3. $a = a(1-b) \Rightarrow a = 0$

$$C_t = \frac{1-B}{B} K_t + \frac{(1-B)e_t}{1-\phi B}$$

$$K_{t+1} = K_t + \frac{B-\phi B}{1-\phi B}$$

A sequence $\epsilon_0, \epsilon_1, \ldots$

will determine sequences

$C_0, C_1, \ldots$ &

$K_0, K_1, \ldots$

Consider two economies, A & B, that have the same sequence of shocks except at time s, $\epsilon_s^A - \epsilon_s^B = 1$

Without loss of generality we can assume the $\epsilon_t^B = 0$ for all t. Then ϵ_t^A also equals 0 at every time except time s when $\epsilon_s^A = 1$.

How will the consumption & capital trajectories differ?

For economy B

$$K_0^B = K_1^B = K_2^B = \ldots$$

$$C_0^B = \frac{1-B}{B} K_0^B = C_1^B = C_2^B = \ldots$$

For economy A for $t < s$, of course, everything is the same as in economy B.

At time s

$$K_s^A = K_s^B \quad \text{but} \quad C_s^A = C_s^B + \frac{1-B}{1-\phi B}$$

Continuing

$$K_{s+1}^A = K_s^B + \frac{B-\phi B}{1-\phi B} \qquad C_{s+1}^A = C_s^B + \frac{(B-\phi B)(1-B)}{(1-\phi B)(B)} + \frac{\phi(1-B)}{1-\phi B}$$

$$K_{s+2}^A = K_s^B + \frac{B-\phi B}{1-\phi B} + \frac{\phi(B-\phi B)}{1-\phi B}$$

$$C_{s+2}^A = C_s^B + \frac{(B-\phi B)(1+\phi)}{(1-\phi B)}\frac{(1-B)}{B} + \frac{\phi^2(1-B)}{1-\phi B}$$

After n periods the difference in the capital stocks is

$$\left(\frac{B - \phi B}{1 - \phi B}\right)\left(1 + \phi + \phi^2 + \dots + \phi^{n-1}\right)$$

& the difference in consumption levels is

$$\frac{(B - \phi B)(1 - B)}{(1 - \phi B)(B)}\left(1 + \phi + \phi^2 + \dots + \phi^{n-1}\right)$$

$$+ \left(\frac{1 - B}{1 - \phi B}\right)\phi^n$$

Thus transitory shocks have permanent effects, a key ingredient in business cycle models