Real Business Cycles
(The Role of Leisure)

$t = 1, 2$

Utility of representative consumer

$$U = \ln(C_1) + b\ln(1-\ell_1) + B\left[\ln(C_2) + b\ln(1-\ell_2)\right]$$

$0 < B < 1$

$0 \le \ell_t \le 1$

$b > 0$

C_t - consumption in period t

$1-\ell_t$ - "leisure" in period t

Budget Constraint

$$(w_1\ell_1 - C_1)(1+r_2) + w_2\ell_2 = C_2$$

l_t - "labor" in period t

w_t - wage in period t

r_2 - interest rate in period 2

(Agent born without capital)

Solving we get (among other things)

$$\frac{b}{1-l} = \lambda w_1 (1+r_2)$$

$$\frac{Bb}{1-l_2} = \lambda w_2$$

$$\frac{1-l_1}{1-l_2} = \frac{1}{B(1+r_2)} \frac{w_2}{w_1}$$

Intertemporal substitution

$w_2 \uparrow$ and/or $w_1 \downarrow$ $\Rightarrow$ agent takes relatively more leisure in period 1

$r_2 \downarrow$ also $\Rightarrow$ take more leisure in period 1 relative to period 2

Basic Story

1. Negative productivity shock
2. People quit jobs while wages are low anticipating higher wages in the future

Unemployment voluntary

Problems -

1. Technology shocks must be very large
2. Intertemporal substitution must be strong

Simple Stochastic Dynamic Optimization

$t = 1, 2$

I_1 - First period income

r_2 - second period interest rate

θ - mean 0 random shock

C_1, C_2 - consumption in first and second period

Budget constraint

$$C_2 = (I_1 - C_1)(1 + r_2) + \theta$$

Utility - $E[\ln(C_1) + \beta \ln(C_2)]$

$\downarrow$

mathematical expectation

$$L = E\Big\{ \ln(C_1) + \beta \ln(C_2) - \lambda_2 \beta [C_2 - (I - C_1)(1 + r_2) - \theta] \Big\}$$

i) $\frac{1}{C_1} = \beta(1+r_2)\,E[\lambda_2]$

ii) $\beta\,E\left[\frac{1}{C_2}\right] = \beta\,E[\lambda_2]$

iii) Budget constraint

$$\frac{1}{C_1} = \beta(1+r_2)\,E\left[\frac{1}{C_2}\right]$$

$$= \beta(1+r_2)\,E\left[\frac{1}{(I-C_1)(1+r_2)+\theta}\right]$$

Can be solved but not in closed form