

Ramsey Model

2.1
Derive k^* Ramsey

Exogenous vs. Endogenous Variables

Solow Model

Exogenous

Labor L, n

Knowledge A, g

Production Function - Cobb-Douglas

Saving Rate - s

Depreciation Rate - δ

Endogenous

Capital K, \dot{K}

Output Y, \dot{Y}

K^*

C^*

Ramsey Model & Diamond Model

s is endogenous

Both are micro-oriented stories based on optimization

$t = 0, 1, 2, \dots$

1. Firms

Many identical firms with production functions

$$Y = A^{1-\alpha} K^{\alpha} L^{1-\alpha}$$

Maximize Profit

Perfect ~~com~~ competition

A exogenous - grows at rate g

i.e. $A_{t+1} = A_t(1+g)$

In principle profit goes to household but profit turns out to equal 0.

2. Labor - $n=0 \Rightarrow L_{t+1} = L_t$

L_0 given

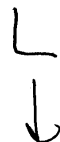
3. Capital - $K(0)$ shared equally

i.e. each worker gets $\frac{K(0)}{L(0)}$

Full depreciation in every period

4. Household Income

$$\underline{I}(t) = r(t) \frac{K(t)}{L} + w(t) + \frac{\pi(t)}{L}$$

interest
ratewage
rateprofit
share

$$r(t) = MPK_t = A_t^{1-\alpha} \alpha K(t)^{\alpha-1} L^{1-\alpha} \left. \begin{array}{l} \text{Profit} \\ \text{maximization} \end{array} \right\}$$

$$w(t) = MPL_t = A_t^{1-\alpha} (1-\alpha) K(t)^\alpha L^{-\alpha} \left. \begin{array}{l} \text{plus} \\ \text{P.C.} \end{array} \right\}$$

$$K(t)r(t) + Lw(t) = Y(t)$$

$$\Rightarrow \pi(t) = 0 \quad \Rightarrow \underline{I}(t) = A_t \underbrace{(K(t))^\alpha}_{\text{small } \pi}$$

5. Capital Dynamics

$$K(t+1) = Y(t) - C(t)$$

Full Depreciation of capital stock total consumption = $L C(t)$

$$6. U_i = \sum_{t=0}^{\text{or } T} \beta^t u(c_i(t))$$

$$0 < \beta < 1$$

$$u(\cdot) = \ln(\cdot)$$

7. Household Budget Constraint

$$c_i(t) \leq I(t) \quad \text{all } t$$

8. Household capital accumulation

$$\frac{K(t+1)}{L} = \frac{A_+^{1-\alpha} K(t)^\alpha}{L(t)^\alpha} - \frac{C(t)}{L(t)} \quad K_i(t+1) = A_+^{1-\alpha} K_i(t)^\alpha - C_i(t)$$

$$\max \sum_{t=0}^{\text{or } T} \beta^t \ln(c_i(t))$$

$$\text{s.t. i) } c_i(t) \leq I(t) \quad \text{all } t$$

ii) Household capital accumulation

Consider Finite Horizon Variant

$$L = \sum_{t=0}^T \beta^t \ln(c_t(t)) - \beta \lambda_{t+1} \left[k_t(t+1) - A_t^{1-\alpha} k_t(t)^\alpha + c_t(t) \right]$$

$$i) \frac{1}{c_t(t)} = \beta \lambda_{t+1} \quad t=0, 1, \dots, T$$

$$ii) \lambda_t = \beta \lambda_{t+1}^\alpha k_t(t)^{2-\alpha} A_t^{1-\alpha} \quad t=1, \dots, T$$

iii) Household Accumulation Constraints

$$k_t(T+1) = 0$$

End of the world

$$c_t(T) = k_t(T)^\alpha A_t^{1-\alpha}$$

Accumulation Constraint

$$\beta \lambda_{T+1} = \frac{1}{k_t(T)^\alpha A_t^{1-\alpha}}$$

i)

$$\lambda_T = \frac{2}{k_t(T)}$$

ii)

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$$\frac{1}{C_i(T-1)} = \frac{B\lambda}{K_i(T)} = B\lambda \left[K_i(T-1)^2 A_{T-1}^{1-\lambda} - (T-1) \right]^{-1}$$

i) + iii)

$$C_i(T-1) = \frac{1}{1+B\lambda} K_i(T-1)^2 A_{T-1}^{1-\lambda} \quad \text{rearranging}$$

$$\lambda_{T-1} = \frac{(1+B\lambda)\lambda}{K_i(T-1)} \quad \text{ii)}$$

Iterating we get

$$C_i(T-t) = \left[1 + B\lambda + (B\lambda)^2 + \dots + (B\lambda)^{t-1} \right]^{-1} K_i(T-t)^2 A_{T-t}^{1-\lambda}$$

For large $T-t$ this

is

$$C_i(T) = (1-B\lambda) K_i(T)^2 A_T^{1-\lambda}$$

ALWAYS CONSUME A FIXED FRACTION

If every household does this then ⁷
 the model behaves like the Solow
 model with endogenous $s = B\alpha$
 $B \uparrow \Rightarrow s \uparrow$ $\alpha \uparrow \Rightarrow s \uparrow$
 we have assumed $\delta = 1$ & $n = 0$
 but we could relax these

Golden Rule? NO

$$K^*_{\text{Ramsey}} \leq K^*_{\text{Golden Rule}}$$

Suppose not - Then cut in saving
 rate could lead to higher consumption
 in every period.

In fact, we can show that

$$K^*_{\text{Ramsey}} < K^*_{\text{Golden Rule}}$$

Temporary Increase in government expenditure
 Captured as temporary decrease in income

$K \downarrow$ temporarily

$r \uparrow$ temporarily

$C_i \downarrow$ temporarily but not by

full amount of income loss

Barro (1987) tests whether interest
 rates are high during wars and
 finds they are

Ricardian Equivalence

Government needs money now

Policy 1 - tax now

Policy 2 - a) sell 10-year bonds now

b) tax in 10 years to pay back

Effect on consumption is theoretically the same.

Textbook goes through ways around this result.