2.1 1

Ramsey Model

Derive K^* Ramsey

Exogenous vs. Endogenous Variables

Solow model

Exogenous		Endogenous	
Labor	L, n	Capital	$K, \dot{K}$
Knowledge	A, g	Output	$Y, \dot{Y}$
Production Function - Cobb-Douglas		K^*	
Saving Rate - s		C^*	
Depreciation Rate - δ			

Ramsey Model & Diamond Model

s is endogenous

Both are micro-oriented stories based on optimization

$t = 0, 1, 2, \ldots$

1. Firms

Many identical firms with production functions $Y = A^{1-\alpha} K^{\alpha} L^{1-\alpha}$

maximize Profit

Perfect competition

A exogenous – grows at rate g

i.e. $A_{t+1} = A_t (1+g)$

In principle profit goes to household but profit turns out to equal 0.

2. Labor - $n = 0 \Rightarrow L_{t+1} = L_t$

L_0 given

3. Capital – $K(0)$ shared equally

i.e. each worker gets $\frac{K(0)}{L(0)}$

Full depreciation in every period

4. Household Income

$$I(t) = \underbrace{r(t)}_{\text{interest rate}} \frac{K(t)}{L} + \underbrace{w(t)}_{\text{wage rate}} + \underbrace{\frac{\Pi(t)}{L}}_{\text{profit share}}$$

$$\left.\begin{aligned} r(t) &= MPK_t = A_t^{1-\alpha}\,\alpha\, K(t)^{\alpha-1} L^{1-\alpha} \\ w(t) &= MPL_t = A_t^{1-\alpha}(1-\alpha)\, K(t)^{\alpha} L^{-\alpha} \end{aligned}\right\} \begin{array}{l}\text{Profit}\\ \text{maximization}\\ \text{plus}\\ \text{P.C.}\end{array}$$

$$K(t)\,r(t) + L\,w(t) = Y(t)$$

$$\Rightarrow \Pi(t) = 0 \quad \Rightarrow I(t) = A_t^{1-\alpha}\big(\underbrace{k}_{\text{small } k}(t)\big)^{\alpha}$$

5. Capital Dynamics

$$K(t+1) = Y(t) - \underbrace{C(t)}_{\text{total consumption } = L\,c(t)}$$

Full Depreciation of capital stock

6. $$U_i = \sum_{t=0}^{\overset{\text{or } T}{\infty}} \beta^t u(c_i(t))$$

$$0 < \beta < 1$$

$$u(\cdot) = \ln(\cdot)$$

7. Household Budget Constraint

$$C_i(t) \leq I(t) \quad \text{all } t$$

8. Household capital accumulation

$$\frac{K(t+1)}{L} = \frac{A_t^{1-\lambda} K(t)^{\alpha}}{L(t)^{\lambda}} - \frac{C(t)}{L(t)} \qquad K_i(t+1) = A_t^{1-\lambda} K_i(t)^{\lambda} - C_i(t)$$

$$\max \sum_{t=0}^{\overset{\text{or } T}{\infty}} \beta^t \ln(C_i(t))$$

s.t. i) $C_i(t) \leq I(t)$ all t.

ii) Household capital accumulation

Consider Finite Horizon Variant

$$L = \sum_{t=0}^{T} \beta^t \ln(C_i(t)) - \beta^{t+1} \lambda_{t+1} \left[K_i(t+1) - A_t^{1-\alpha} K_i(t)^{\alpha} + C_i(t) \right]$$

i) $\dfrac{1}{C_i(t)} = \beta \lambda_{t+1}$ $\quad t = 0, 1, \ldots, T$

ii) $\lambda_t = \beta \lambda_{t+1} \alpha K_i(t)^{\alpha-1} A_t^{1-\alpha}$ $\quad t = 1, \ldots, T$

iii) Household Accumulation Constraints

$K_i(T+1) = 0$ End of the world

$C_i(T) = K_i(T)^{\alpha} A_T^{1-\alpha}$ Accumulation Constraint

$\beta \lambda_{T+1} = \dfrac{1}{K_i(T)^{\alpha} A_T^{1-\alpha}}$ i)

$\lambda_T = \dfrac{\alpha}{K_i(T)}$ ii)

$$\frac{1}{C_i(T-1)} = \frac{B\alpha}{K_i(T)} = B\alpha\left[K_i(T-1)^{\alpha}A_{T-1}^{1-\alpha} - C_i(T-1)\right]^{-1}$$

i) + iii)

$$C_i(T-1) = \frac{1}{1+B\alpha} K_i(T-1)^{\alpha} A_{T-1}^{1-\alpha}$$ rearranging

$$\lambda_{T-1} = \frac{(1+B\alpha)\alpha}{K_i(T-1)}$$ ii)

Iterating we get

$$C_i(T-t) = \left[1 + B\alpha + (B\alpha)^2 + \ldots + (B\alpha)^t\right]^{-1} K_i(T-t)^{\alpha} A_{T-t}^{1-\alpha}$$

For large T + t this is

$$C_i(T) = (1-B\alpha)\, K_i(T)^{\alpha} A_T^{1-\alpha}$$

ALWAYS CONSUME A FIXED FRACTION

If every household does this then the model behaves like the Solow model with endogenous $s = \beta \alpha$

$\beta \uparrow \Rightarrow s \uparrow \quad \alpha \uparrow \Rightarrow s \uparrow$

We have assumed $\delta = 1$ & $n = 0$ but we could relax these

---

Golden Rule? NO

$K^*_{Ramsey} \leq K^*_{Golden\ Rule}$

Suppose not - Then cut in saving rate could lead to higher consumption in every period.

In fact, we can show that

$K^*_{Ramsey} < K^*_{Golden\ Rule}$

Temporary Increase in government expenditure captured as temporary decrease in income

$K \downarrow$ temporarily

$r \uparrow$ temporarily

$C_i \downarrow$ temporarily but not by full amount of income loss

Barro (1987) tests whether interest rates are high during wars and finds they are

---

Ricardian Equivalence

Government needs money now

Policy 1 - tax now

Policy 2 - a) Sell 10-year bonds now

b) tax in 10 years to pay back

Effect on consumption is theoretically the same.
Textbook goes through ways around this result.