Endogenous Growth 3.1, 3.7, 3.13

We model growth of A

For simplicity we ignore capital

Continuous time

$$Y(t) = A(t)\,(1-a_L)\,L(t)$$

$(1-a_L)$: fraction of labor committed to production

$$\dot{A}(t) = B\,[a_L\,L(t)]^{\gamma}\,A(t)^{\theta}$$

a_L: fraction of workers committed to R + D sector

$$\dot{L}(t) = n\,L(t)$$

g_A - Growth rate of A

$$g_A(t) \equiv \frac{\dot{A}(t)}{A(t)} = B\,[a_L L(t)]^{\gamma} A(t)^{\theta-1}$$

Growth rate of $\underline{g_A}$

$$\frac{\dot{g}_A(t)}{g_A(t)} = [\gamma n + (\theta-1)\, g_A(t)]$$

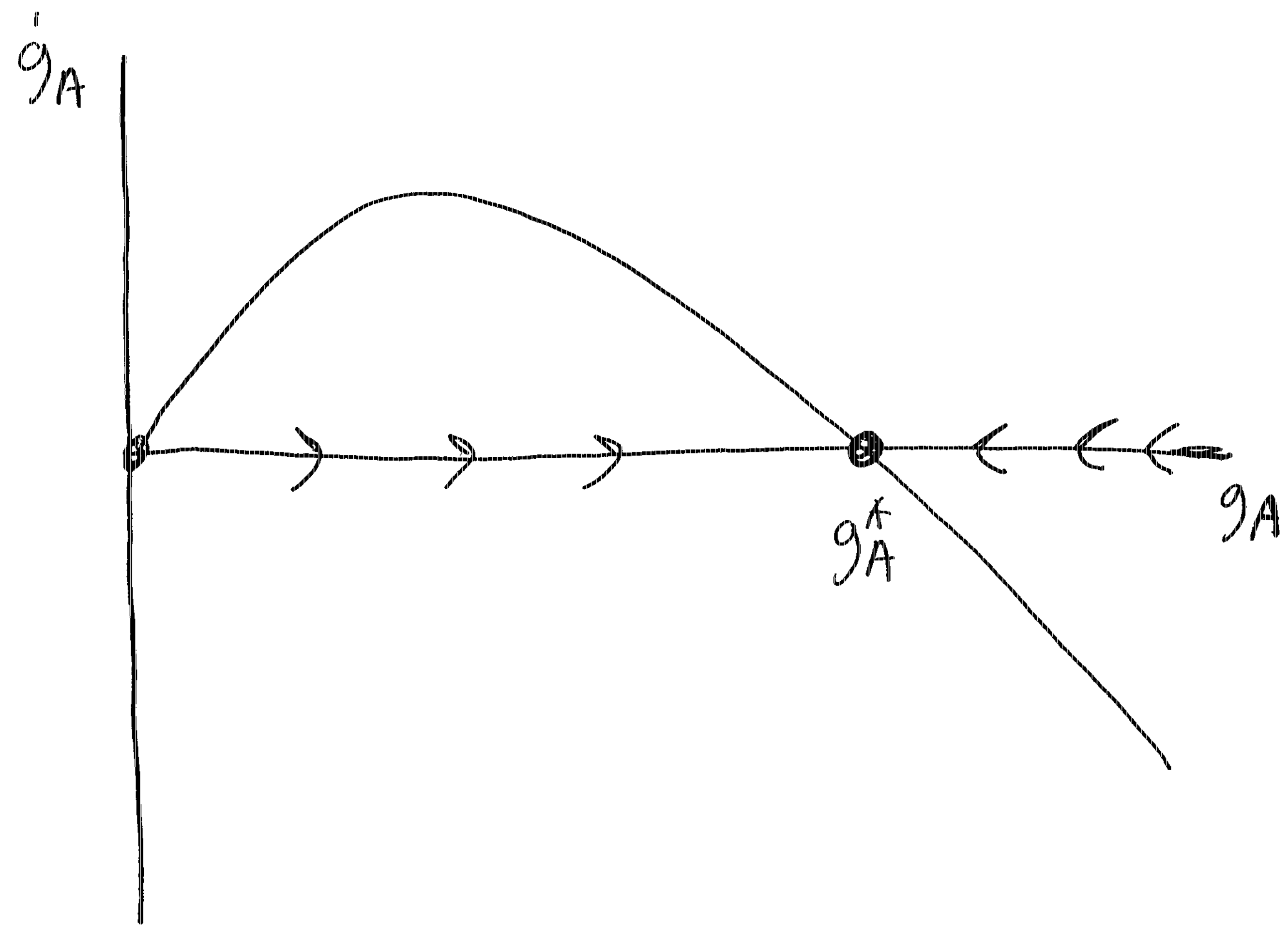
$$g_A(t) = \text{constant} \iff g_A = \frac{\gamma n}{1-\theta} \equiv g_A^{*}$$

Three cases

Case 1 : $\theta < 1$

$g_A \downarrow \iff g_A > g_A^*$

$g_A \uparrow \iff g_A < g_A^*$



g_A constant $\Rightarrow$ $\dfrac{\dot{Y}(t)}{Y(t)}$ constant

Long-run growth determined endogenously

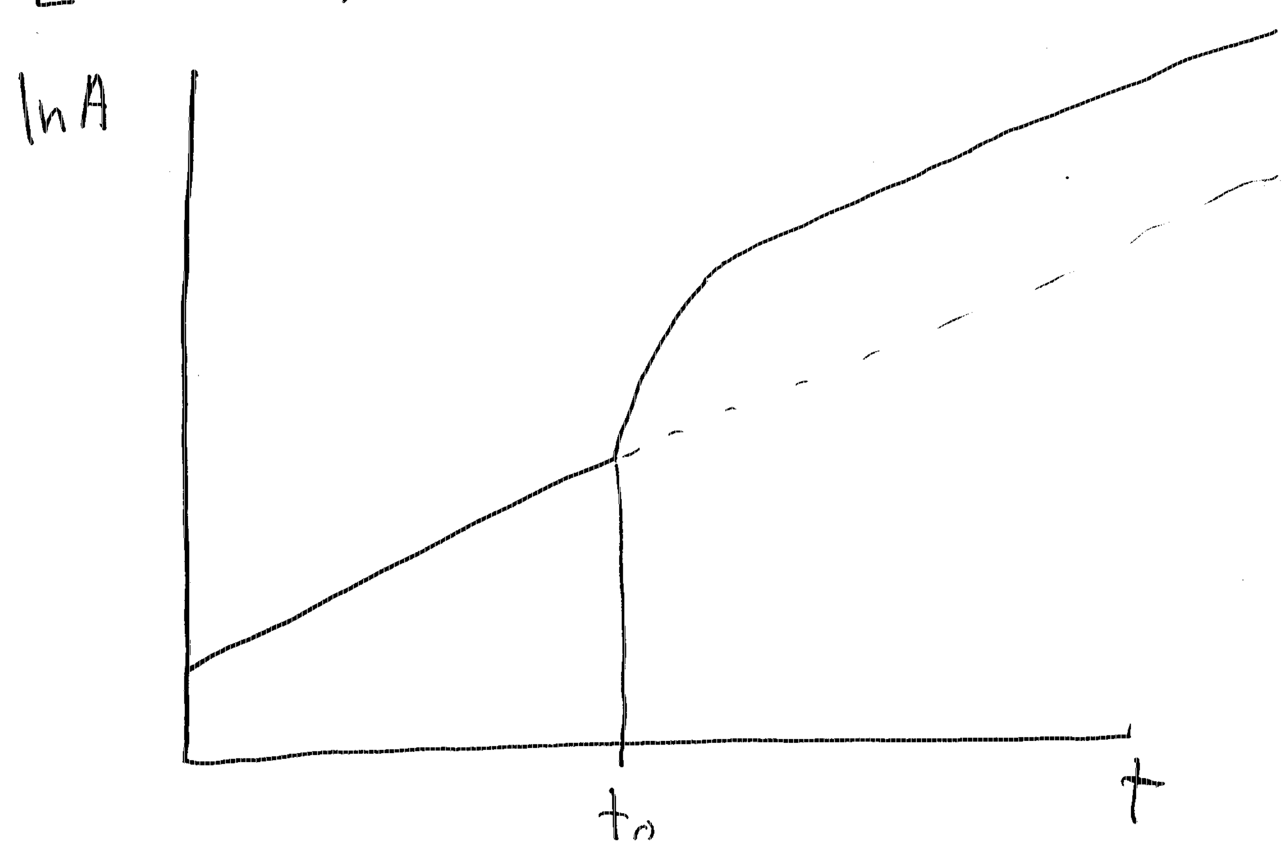
Kar - Diminishing returns to new knowledge in creating

$$g_A^* = \frac{\gamma n}{1-\theta}$$

Increasing in n - Not realistic for countries - can be defended for worldwide growth

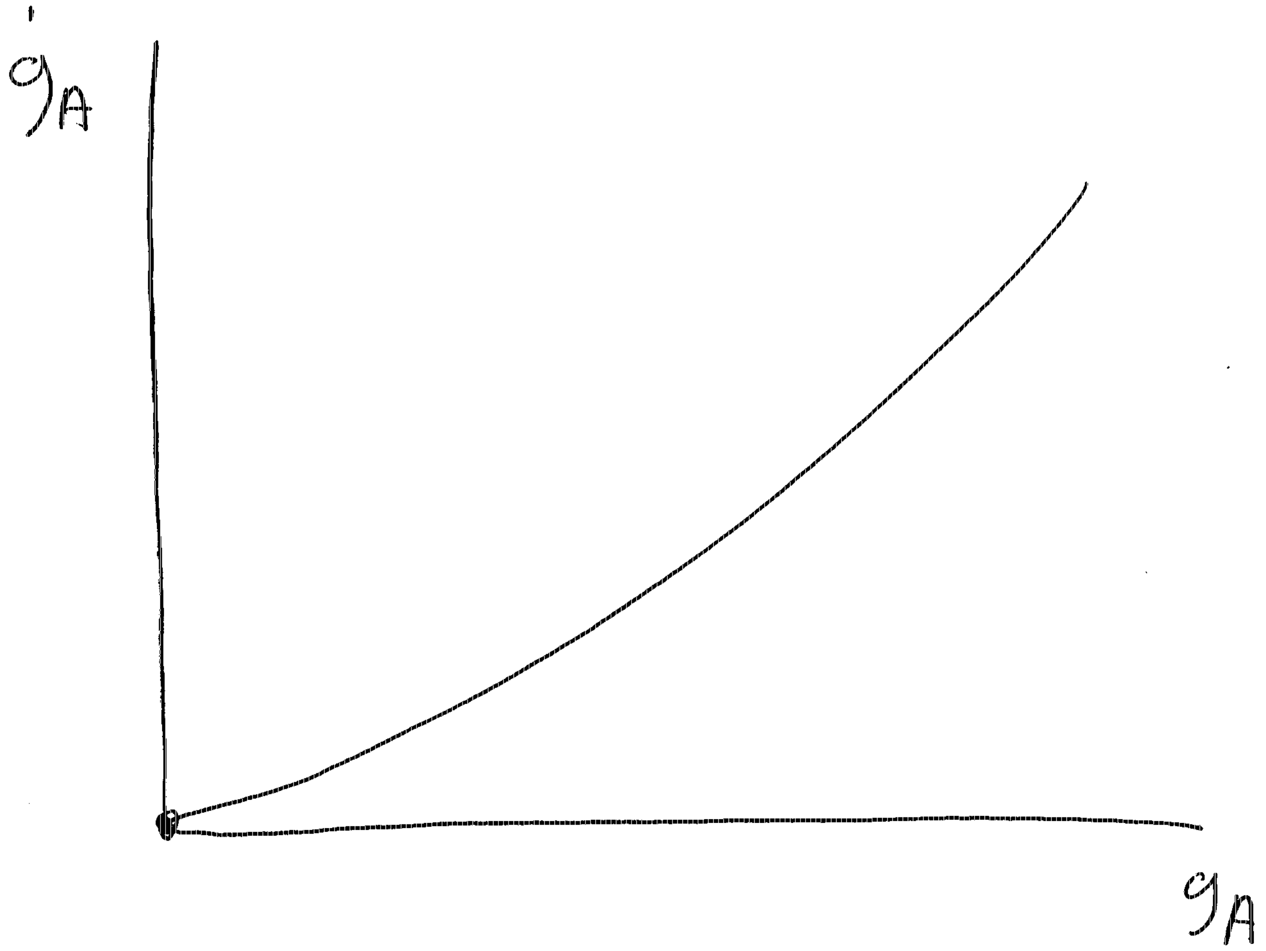
g_L does not affect g_A^*

$g_L \uparrow \Rightarrow$ temporary $g_A \uparrow$

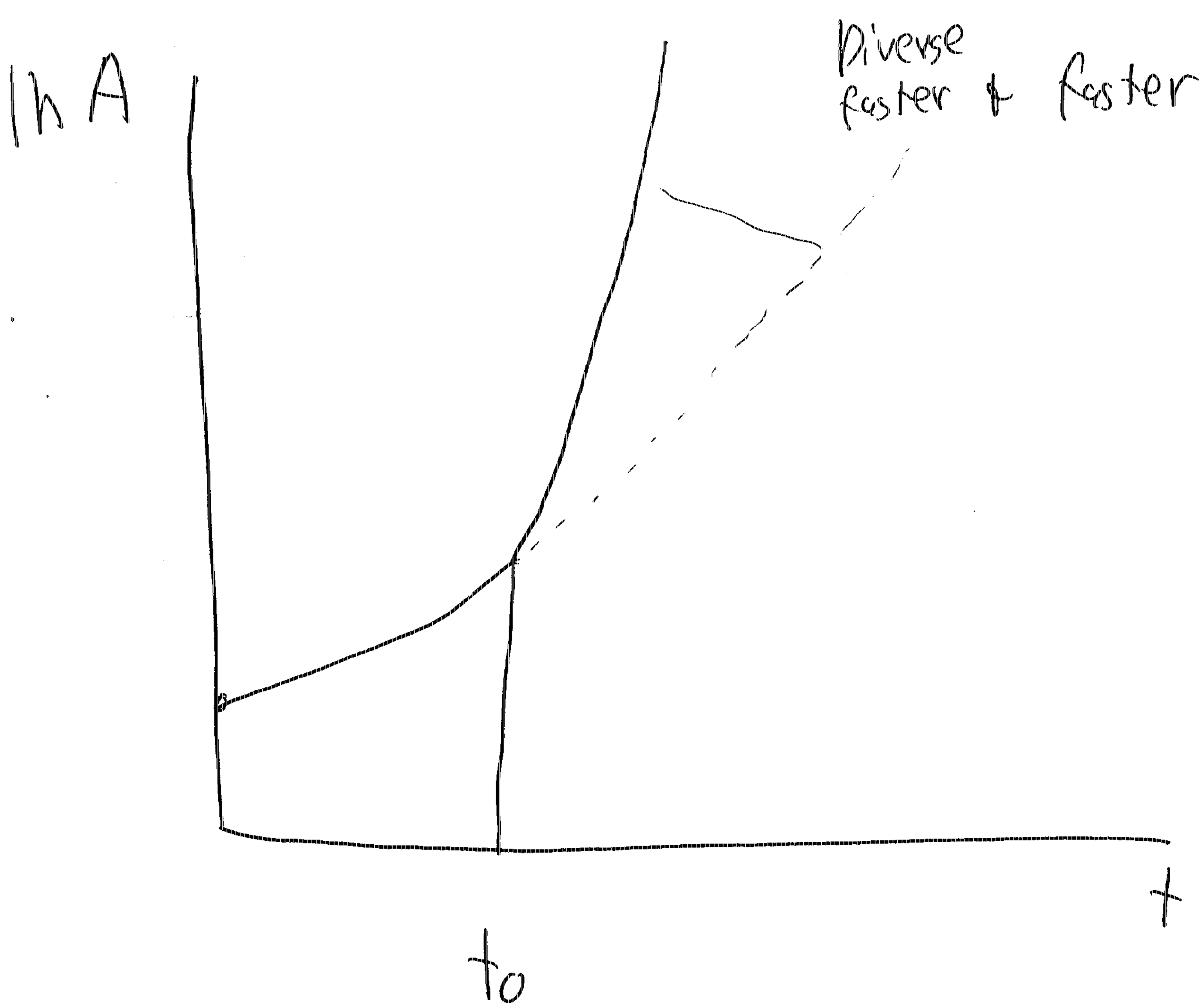


Case 2: $\theta > 1$

$g_A \uparrow \Rightarrow \dot{g}_A \uparrow$



$a_L \uparrow \Rightarrow g_A \uparrow \Rightarrow \frac{\dot{g}_A(t)}{g_A(t)} \uparrow$



Key — Knowledge is increasingly productive in producing more knowledge

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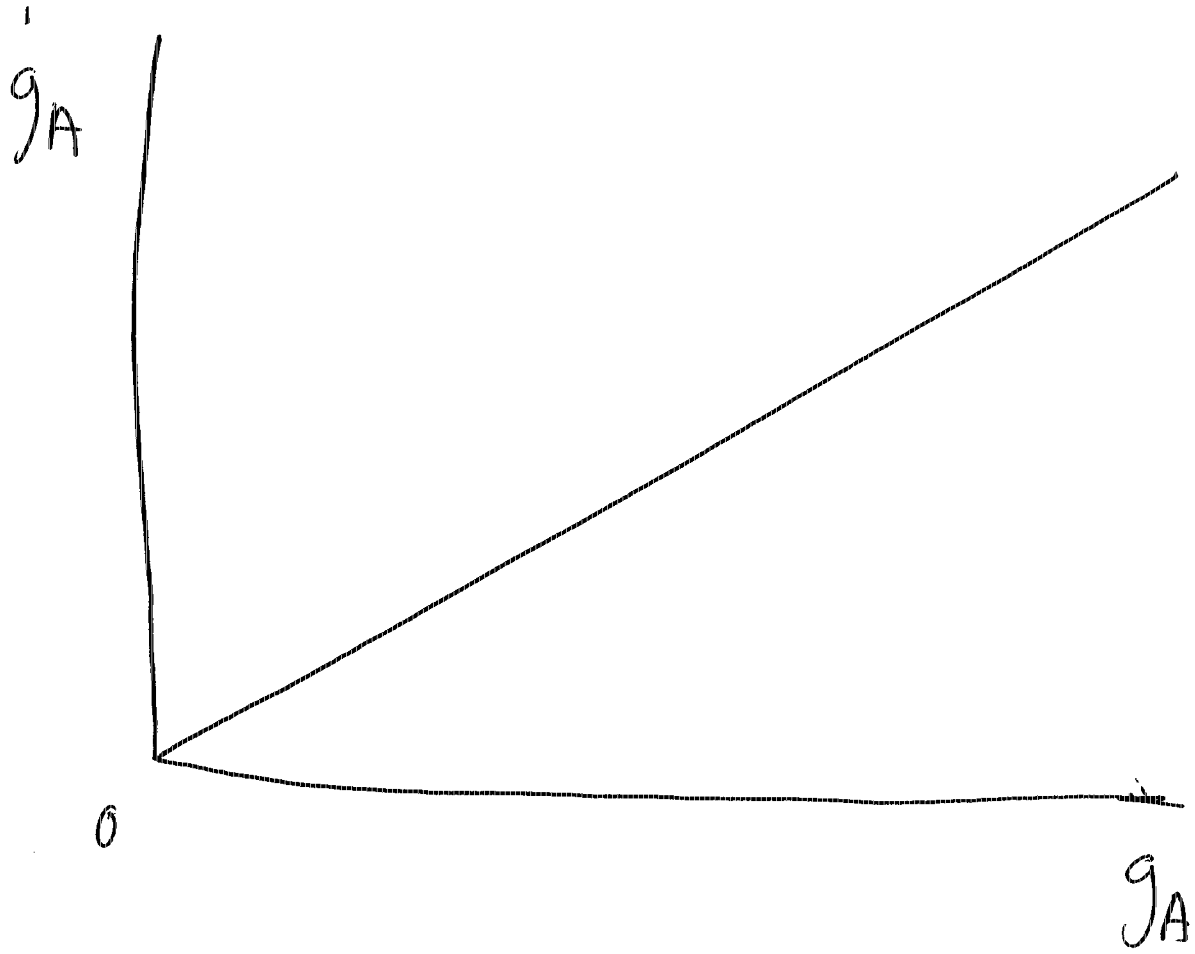
a_L like saving ⇒ saving affects even long-run growth

Case 3 : $\theta = 1$

$$g_A(t) = B\, a_L^{\gamma} L(t)^{\gamma}$$

$$\dot{g}_A(t) = \gamma\, n\, g_A(t)$$



Growth rate always increasing if $n > 0$

$n = 0$ - constant growth

$a_L \approx$ saving affects growth

Learning by Doing

Knowledge accumulation a byproduct of other economic activity

1. Installing new capital * simplest
2. Spending time working with old capital

$$Y(t) = K(t)^{\alpha}\left[A(t)L(t)\right]^{1-\alpha}$$

$$A(t) = BK(t)^{\phi} \qquad B, \phi > 0$$

$$Y(t) = K(t)^{\alpha} B^{1-\alpha} K(t)^{\phi(1-\alpha)} L(t)^{1-\alpha}$$

$$\dot{K}(t) = sY(t) = sB^{1-\alpha} K(t)^{\alpha+\phi(1-\alpha)} L(t)^{1-\alpha}$$

Looks like

$$\dot{A}(t) = B\left[a_L L(t)\right]^{\gamma} A(t)^{\theta}$$

3 cases :

$$\lambda + \phi(1-\lambda) < 1$$

$$\lambda + \phi(1-\lambda) = 1$$

$$\lambda + \phi(1-\lambda) > 1$$

These theories give some insight into world growth rates

Less persuasive for why some countries are rich and others poor.

Why does not knowledge diffuse more rapidly?

Why do not firms with the knowledge set up in countries that do not have the knowledge but have low wages?

Kremer

$$Y(t) = R^{\lambda} [A(t) L(t)]^{1-\lambda}$$

R - Land

$$\frac{\dot{A}(t)}{A(t)} = B L(t)$$

$$\frac{Y(t)}{L(t)} = \bar{y} \quad - \quad \text{Malthusian}$$

$$\bar{y} L(t) = R^{\lambda} [A(t) L(t)]^{1-\lambda}$$

$$L(t) = \left(\frac{1}{\bar{y}}\right)^{1/\lambda} A(t)^{1-\lambda/\lambda} R$$

$$\frac{\dot{L}(t)}{L(t)} = \frac{1-\lambda}{\lambda}\frac{\dot{A}(t)}{A(t)} = \frac{1-\lambda}{\lambda} B L(t)$$

$$n_t = \underset{(.0355)}{-.0023} + \underset{.026}{.524} L_t \qquad R^2 = .92$$

Data from 1 million B.C.

Stops working when population passed 3 million