2.13, 2.14, 2.15

Diamond Model

Discrete time - $t = 0, 1, 2$

Individuals live two periods & overlap

L_t - # of agents born at time t

$L_t = (1+n) L_{t-1}$ - Constant population growth

Young agents have one unit of labor

C_{1t} - Consumption of young at time t

C_{2t} - Consumption of old at time t

Utility - $U(C_{1t}, C_{2,t+1}) = \ln(C_{1t}) + \dfrac{\ln(C_{2,t+1})}{1+\rho}$

$\rho > -1$

Production - $Y_t = A_t^{1-\alpha} K_t^{\alpha} L_t^{1-\alpha}$

$A_t = (1+g) A_{t-1}$ A_0 - given

Old Supply capital

$$r_t = \alpha A_t^{1-\alpha} K_t^{\alpha-1} L_t^{1-\alpha} = \alpha k_t^{\alpha-1}$$

Young Supply labor · $w_t = (1-\alpha) k_t^{\alpha} A_t$

Young divide w_t between consumption & saving (The book strangely calls this $w_t A_t$)

$$K_{t+1} = w_t L_t - C_{1t} L_t$$

(No depreciation. Old people consume all their capital after they rent it out & get it back)

$$(1+g)(1+n) k_{t+1} = \frac{w_t}{A_t} - \frac{C_{1t}}{A_t}$$

$$C_{2,t+1} = (1+r_{t+1})(w_t - C_{1t})$$ Budget Constraint

or

$$C_{1t} + \frac{1}{1+r_{t+1}} C_{2,t+1} = w_t$$ B.C.

$$L = \ln C_{1t} + \frac{1}{1+\rho} \ln C_{2t+1} +$$

$$\lambda \left[W_t - \left(C_{1t} + \frac{1}{1+r_{t+1}} C_{2t+1} \right) \right]$$

$$\frac{1}{C_{1t}} = \lambda$$

$$\frac{1}{1+\rho} \frac{1}{C_{2t+1}} = \frac{1}{1+r_{t+1}} \lambda$$

$$\frac{C_{2t+1}}{C_{1t}} = \frac{1+r_{t+1}}{1+\rho}$$

Now use the B.C.

$$C_{1t} = \frac{W_t (1+\rho)}{2+\rho}$$

Fraction of income saved is

$\frac{1}{2+\rho}$ - Does not depend on r due to logrithmic utility

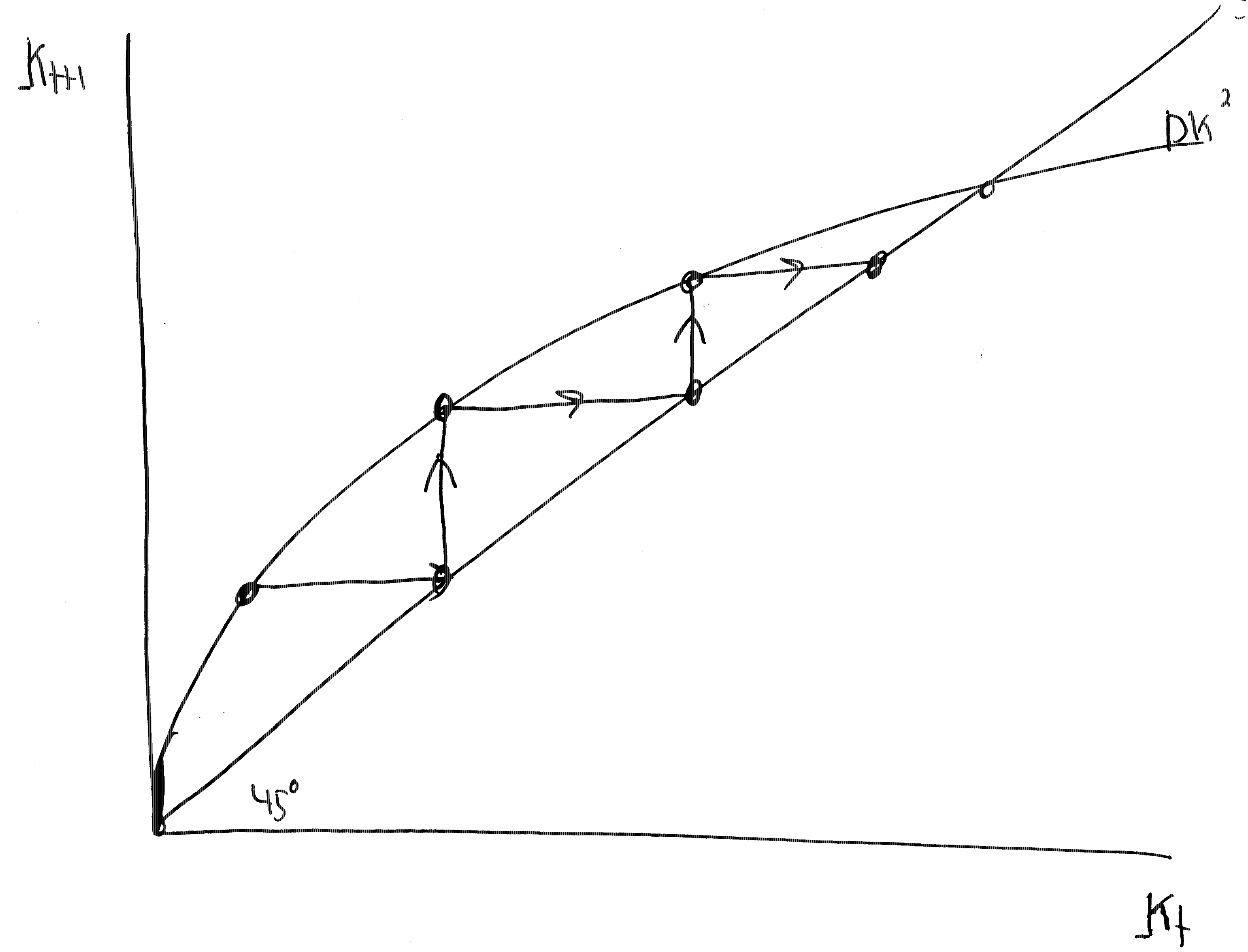
$$K_{t+1} = \frac{1}{2+\rho} L_t W_t$$

$$k_{t+1} = \frac{(1-\alpha)\, k_t^{\alpha}}{(2+\rho)(1+n)(1+g)} \equiv D\, k_t^{\alpha}$$

k^* - steady state $k^* = \left[\frac{1-\alpha}{(1+n)(1+g)(2+\rho)}\right]^{1/1-\alpha}$

Balanced Growth works just like

Ramsey - Solow



Golden Rule

steady state

$$K_{t+1} = K_t(1+n)(1+g) = A_t^{1-\alpha} K_t^{\alpha} L_t^{1-\alpha} - C_t$$

$$k(1+n)(1+g) = f(k) - c = k^{\alpha} - c$$ what consumpt is necessary to maintain steady state?

$$c = k^{\alpha} - k(1+n)(1+g)$$ – maximize consumption

$$\alpha k_{GR}^{\alpha-1} = (1+n)(1+g)$$ f.o.c.

$$\lambda (k^*)^{\lambda-1} = \frac{\lambda}{1-\lambda}(1+n)(1+g)(\lambda+e)$$

$\left\{\begin{array}{l}\text{Small } \lambda \\ \text{Small } e\end{array}\right. \Rightarrow k^* > k_{GR}$

$\Rightarrow$ C can go up today & in every future period

Dynamic Inefficiency (Relies on infinite number of agents)

Need Government intervention to break dynamic inefficiency

t - Step 1 - Todays young consume more & leave GR capital stock

t+1 - Step 2 - Compensate next period's old (same people) who otherwise would not agree to consume more t+1 young have less capital &

are taxed but consume a higher

fraction of their income & leave

GR capital stock

t+2- Step 3 - Again tax young to support old