Human Capital

Mankiw Romer + Weil (1992)

$$Y(t) = K(t)^{\alpha} H(t)^{\beta} [A(t)L(t)]^{1-\alpha-\beta}$$

$\alpha > 0$, $\beta > 0$, $\alpha + \beta < 1$

$$\dot{K}(t) = s_K Y(t)$$

$$\dot{L}(t) = n L(t)$$

$$\dot{A}(t) = g A(t)$$

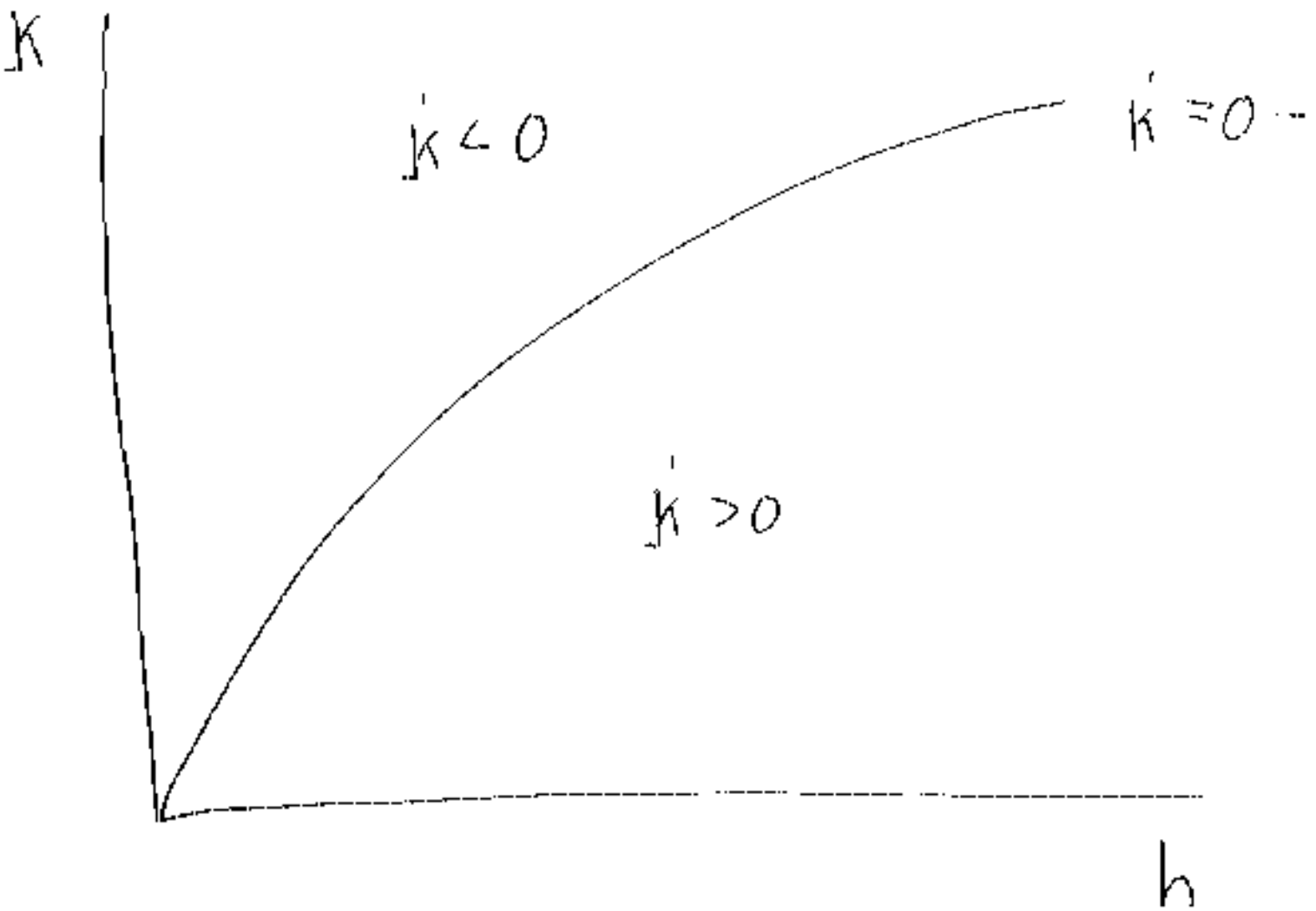
$$\dot{H}(t) = s_H Y(t)$$

$$k = \frac{K}{AL} \qquad h = \frac{H}{AL} \qquad y = \frac{Y}{AL}$$

$$y(t) = k(t)^{\alpha} h(t)^{\beta}$$

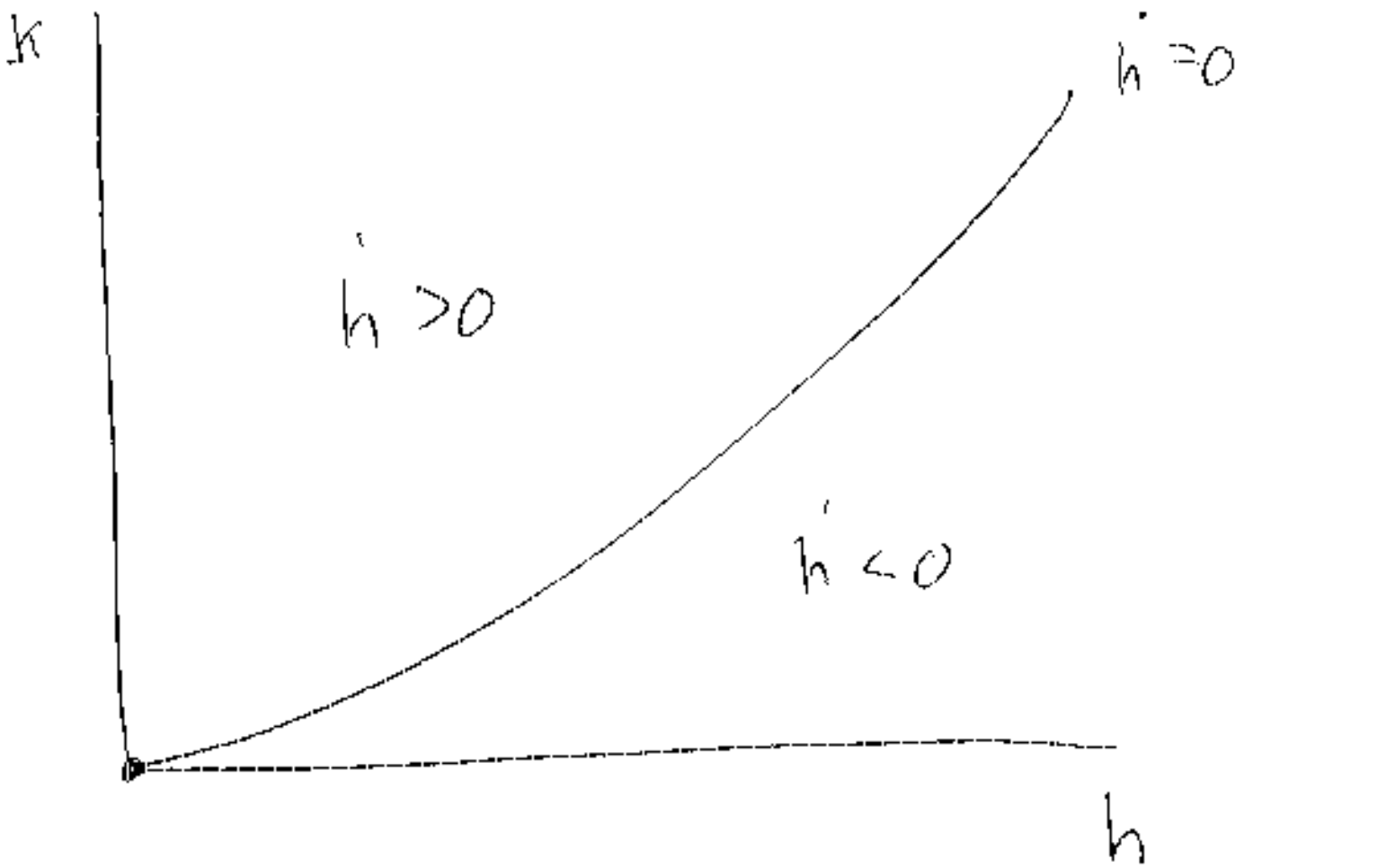
$$\dot{k}(t) = s_K k(t)^{\alpha} h(t)^{\beta} - (n+g) k(t)$$

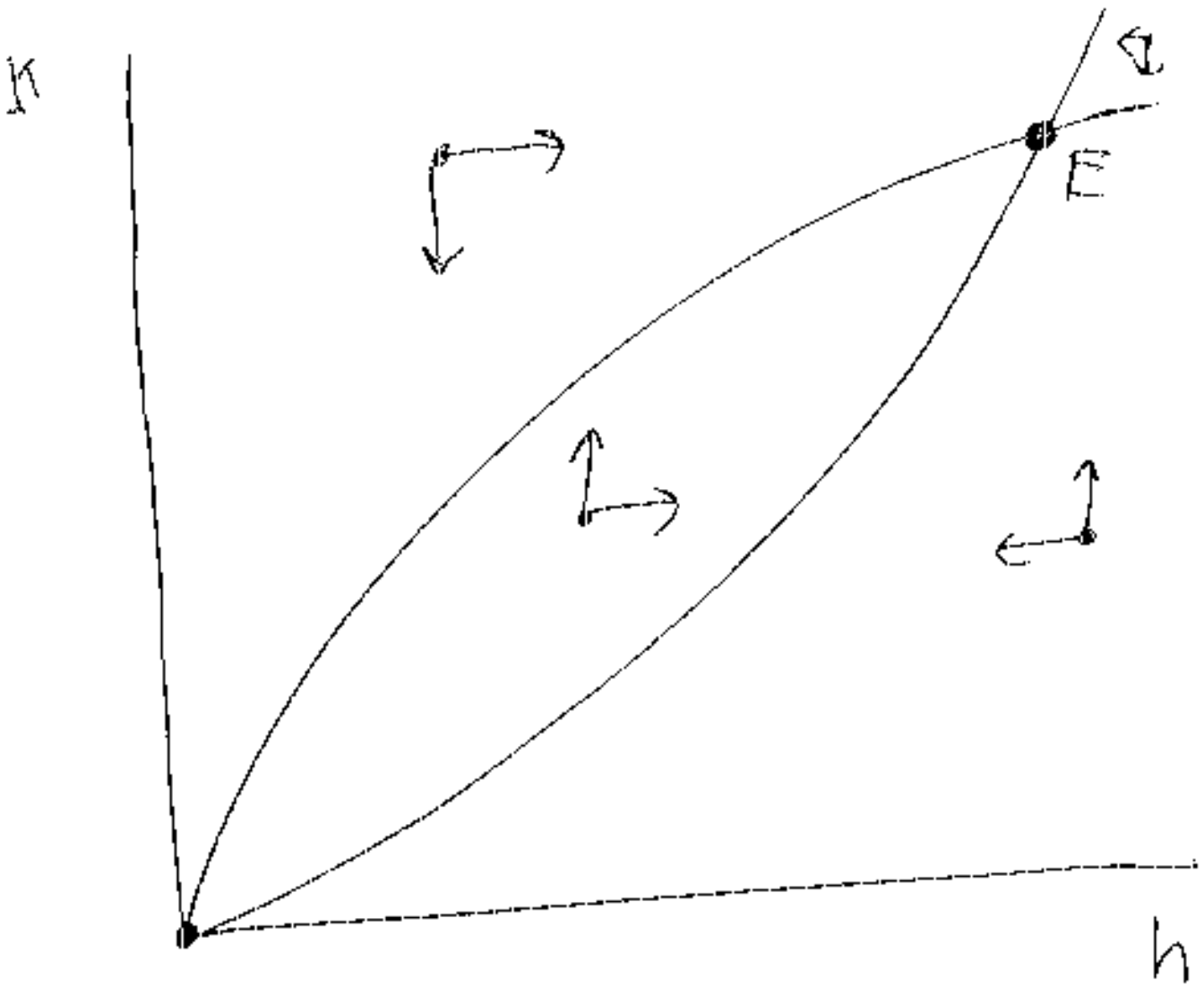
$$\dot{k}(t) = 0 \quad \Leftrightarrow \quad k = \left(\frac{s_K}{n+g}\right)^{1/1-\alpha} h^{\frac{\beta}{1-\alpha}}$$



$$\dot{h}(t) = s_H k(t)^{\alpha} h(t)^{\beta} - (n+g) h(t)$$

$$k = \left(\frac{n+g}{s_H}\right)^{1/2} h^{\frac{1-\beta}{2}} \quad \text{at} \quad \dot{h}(t) = 0$$





E is globally stable

At E

$$s_K (k^*)^{\alpha} (h^*)^{\beta} = (n+g) k^*$$

$$s_H (k^*)^{\alpha} (h^*)^{\beta} = (n+g) h^*$$

$$\ln s_K + \alpha \ln(k^*) + \beta \ln(h^*) = \ln(n+g) + \ln(k^*)$$

$$\ln s_H + \alpha \ln(k^*) + \beta \ln(h^*) = \ln(n+g) + \ln(h^*)$$

$$\ln(k^*) = \frac{1-\beta}{1-\alpha-\beta} \ln(s_K) + \frac{\beta}{1-\alpha-\beta} \ln(s_H) - \frac{1}{1-\alpha-\beta} \ln(n+g)$$

$$\ln(h^*) = \frac{\alpha}{1-\alpha-\beta} \ln(s_K) + \frac{1-\alpha}{1-\alpha-\beta} \ln(s_H) - \frac{1}{1-\alpha-\beta} \ln(n+g)$$

$$\ln(y^*) = \frac{\alpha}{1-\alpha-\beta} \ln(s_K) + \frac{\beta}{1-\alpha-\beta} \ln(s_H) - \frac{\alpha+\beta}{1-\alpha-\beta} \ln(n+g)$$

Solow

$$\ln(y^*) = \frac{\alpha}{1-\alpha} \ln(s_K) - \frac{\alpha}{1-\alpha} \ln(n+g)$$

Other Estimation $\Rightarrow \alpha \approx \beta \approx 1/3$

$$\frac{\alpha}{1-\alpha-\beta} = 1 \qquad \frac{\alpha}{1-\alpha} = .5$$

1. Saving differences have a much bigger impact on output
2. MPK differentials between rich & poor countries are not as large as before.
   Human capital can account for more of the difference

They estimate

$$\ln y_i = a + b\left(\ln(S_{K_i}) - \ln(n_i + 0.05)\right) + c\left[\ln(S_{H_i}) - \ln(n_i + 0.05)\right] + \epsilon_i$$

$$\ln y_i = \underset{(.14)}{7.86} + \underset{(.12)}{.73}\left[\ln(S_{K_i}) - \ln(n_i + 0.05)\right] + \underset{(.07)}{.67}\left[\ln(S_{H_i}) - \ln(n_i + 0.05)\right]$$

$N = 98$ $\quad \bar{R}^2 = .78$ $\quad$ S.E.E. $= .51$

$\alpha = 0.3$[illegible] $\quad \beta = .25$

Not bad