Single point in time

Owners - N of them
Workers - Unlimited pool
Owners earn α
Workers earn β
Rebel leadership - $R = aN$ of them

$$\max_M \left(\frac{M}{N}\right)^\lambda \left(\frac{\alpha N}{aN}\right) - \frac{\beta M}{aN} - \text{Expected income per rebel leader}$$

M, number of soldiers hired
$\left(\frac{M}{N}\right)^\lambda$, probability of taking over country
βM, cost of soldiers hired

We can multiply the whole thing through by a constant, aN, and the solution to the problem will not change:

$$\max_M \left(\frac{M}{N}\right)^\lambda \alpha N - \beta M$$

Differentiating with resppect to M and setting this derivative equal to zero we get:

$$\lambda \left(\frac{M}{N}\right)^{\lambda-1} \frac{\alpha N}{N} = \beta$$

$$M^* = N \left(\frac{\alpha\lambda}{\beta}\right)^{\frac{1}{1-\lambda}}$$

Probability of overthrow

$$\left(\frac{\alpha\lambda}{\beta}\right)^{\frac{\lambda}{1-\lambda}}$$

1. Increasing in λ - technology of revolution
2. Decreasing in β - workers wage
3. Increasing in α - owners income

Also - number of rebels is proportional to the number of owners

Income per rebel leader is:

$$\left(\frac{\alpha\lambda}{\beta}\right)^{\frac{\lambda}{1-\lambda}}\left(\frac{\alpha}{a}\right)-\frac{\beta}{a}\left(\frac{\alpha\lambda}{\beta}\right)^{\frac{1}{1-\lambda}}$$

Expected income per owner is:

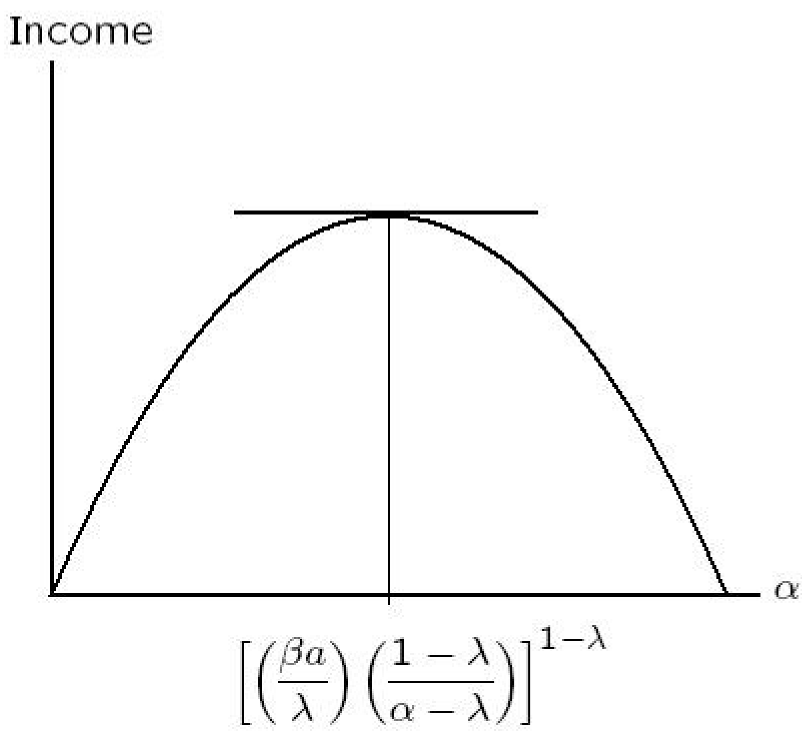
$$\left[1-\left(\frac{\alpha\lambda}{\beta}\right)^{\frac{1}{1-\lambda}}\right]\alpha=\alpha-\left(\frac{\lambda}{\beta}\right)^{\frac{1}{1-\lambda}}\alpha^{\left(\frac{2-\lambda}{1-\lambda}\right)}$$

Take the derivative with respect to α of expected income per owner and we get:

$$1-\left(\frac{\lambda}{\beta}\right)^{\frac{1}{1-\lambda}}\left(\frac{2-\lambda}{1-\lambda}\right)\alpha^{\left(\frac{1}{1-\lambda}\right)}$$

This starts at zero, reaches a maximum at:

$$\alpha=\frac{\beta}{\lambda}\left(\frac{1-\lambda}{2-\lambda}\right)^{1-\lambda}$$

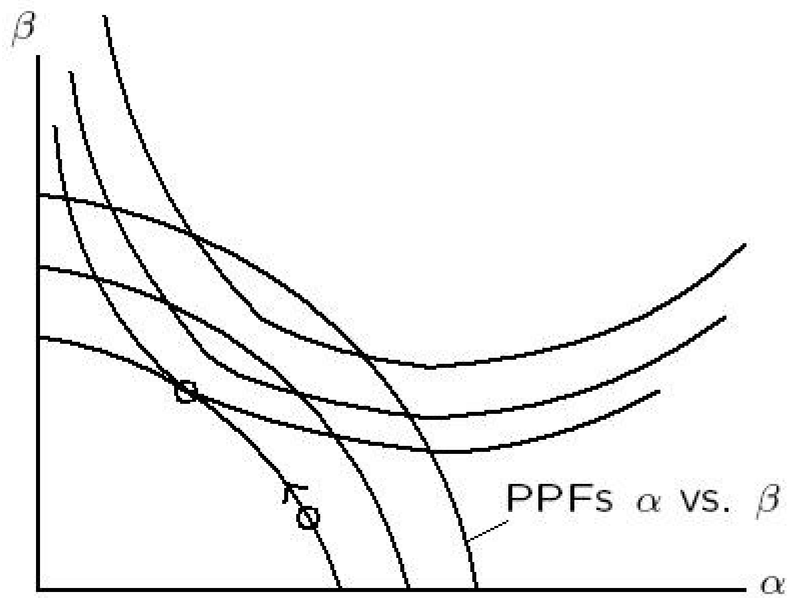


Owners would not want income to grow too high.

Worker income must grow to allow owner income to grow.

This is because larger α turns owners into bigger targets for the rebellion.

$I_0(\alpha, \beta) -$ Increasing in β
Increasing then decreasing in α



The curves labeled "PPFs" in the picture give the possibilities for trading off owner income against worker income. In other words, they represent a "technology of redistribution." The curves labeled "owner indifference curves" give the preferences of owners. They take into account the fact that large income disparities between owners and workers feeds the rebellion.

This is why these indifference curves eventually turn upwards.

If α is higher relative to β than value that maximizes the expected income of owners, then owners redistribute income to workers.

Suppose that the technology of redistribution improves as the country grows richers. This is captured in figure 2 by the way the PPFs have increasingly large negative slopes as they shift out. Larger negative slopes mean that decreases in α bring larger increases in β. In other words, we assume that as the economy gets richer then owners get more "bang for their buck" in redistribution.

In this case the ratio of β to α increases over time. This means that inequality and the probability of a sucessful revolution decreases over time.