Mid-Term Answers

1. Given the general linear model y = XB + u where y is n_x1 and X is n_xk and the OLS estimate of B,

$\hat{b}$ = (X'X)$^{-1}$X'y, show that

a) $X'\hat{u} = 0$

(5 marks)

b) E($\hat{b}$) = B

(5 marks)

c) $\hat{b}$ is the Best Linear Unbiased (minimum variance) estimator

(15 marks)

- Answers to all these can be found in your lecture notes

2. The following is taken from a regression of the log of output on the log of labour input and log of capital input. Some of the information has been concealed.

```
      Source |Sum of Squares  df       MS              Number of obs =      27
-------------+------------------------------           F(  ,     ) =
       Model |  14.2115637        7.10578187           Prob > F      =  0.0000
    Residual |   .85163374       .035484739            R-squared     =
-------------+------------------------------           Adj R-squared =  0.9388
       Total |  15.0631975                              Root MSE
------------------------------------------------------------------------------
        logo |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        logl |   .6029994
        logk |   .3757102    .085346     4.40   0.000     .1995648    .5518556
       _cons |   1.170644    .326782     3.58   0.002     .4961988    1.845089
------------------------------------------------------------------------------
```

The variance/co-variance matrix of the parameter estimates is given by

```
            logl       logk
 logl   .0158644
 logk  -.0096162  .00728393
```

Find

the standard error of the estimate of logl,
the estimated t value
the 95% confidence interval around this estimate

(10 marks)

b) Test the hypothesis that there is constant returns to scale

(5 marks)

c) Test the significance of the goodness of fit of the model as a whole

(7 marks)

d) Find the estimated residual variance

(3 marks)

- see attached answer

3. Given the model

$$y = b_1+b_2X_2+b_3X_3+u$$

and the following information

$$X'X = \begin{bmatrix} 30 & 0 & 0 \\ 0 & 10 & 6 \\ 0 & 6 & 4 \end{bmatrix} \qquad X'y = \begin{bmatrix} 120 \\ 16 \\ 9 \end{bmatrix} \qquad \sum_i (Y_i - \bar{Y})^2 = 80.5$$

What is the sample size ? (3 marks)

Find the OLS estimates of

a) the slope coefficients (6 marks)
b) the intercept (3 marks)
c) the residual sum of squares (RSS) (3 marks)
d) the standard error of the slope coefficients, b_2 and b_3 (3 marks)

e) What are "outliers" in data sets? (3 marks)
f) Outline the form of a test that you might use to detect the presence of outliers (4 marks)

- see attached answer

4. The following output is taken from a regression of y on d1 z1 d2 z2 over a 15 year period, where d_1 is an intercept term for the first sub-period of the data, d_2 is an intercept term for the second sub-period of the data, z_1 is the slope estimate for the first sub-period of the data and z_2 is the slope estimate for the second sub-period of the data.

```
regress y d1 d2 z1 z2, noconstant

      Source |       SS       df       MS              Number of obs =      15
-------------+------------------------------           
       Model |  505.000000     4  126.250000           Prob > F      =  0.0000
    Residual |  3.00000000    11  .272727272           R-squared     =  0.9940
-------------+------------------------------           
       Total |      508.00    15  33.9333333
------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          d1 |     -.0625   .4831417    -0.13   0.899    -1.125888    1.000888
          d2 |         .4    .366156     1.09   0.298     -.405904    1.205904
          z1 |      .4375   .0599263     7.30   0.000      .305603     .569397
```

```
          z2 |   .5090909   .0295057    17.25   0.000     .4441493    .5740325
------------------------------------------------------------------------------
```

An alternative regression estimated over the first 10 year sub-period gives

```
regress y z1 if time<=10

      Source |       SS       df       MS              Number of obs =      10
-------------+------------------------------           
       Model |  85.5000000     1  85.5000000           Prob > F      =  0.0000
    Residual |  2.50000000     8  .309090909           R-squared     =  0.9719
-------------+------------------------------           
       Total |       88.00     9  9.77777778
------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          z1 |   .5090909   .0306046    16.63   0.000     .4385167    .5796652
          d1 |         .4   .3797926     1.05   0.323    -.4758033    1.275803
------------------------------------------------------------------------------
```

a) Write down the restricted and unrestricted models in matrix form (5 marks)

a) Do the Chow forecast test of parameter constancy (5 marks)

c) Explain, (without giving formal proof), why the unrestricted residual sum of squares is equivalent to that estimated over the first sub-period. (6 marks)

d) What is the CUSUM test? (6 marks)

e) what is its main advantage over the Chow forecast test? (3 marks)

- see attached answer