

Chow test of Linear Restrictions

A variant of the test of linear restrictions occurs when wish to test whether all the coefficients in the model differ across 2 (or more) sub-groups or time periods

Consider the regression of trhe log of hourly wages on a quadratic in years of experience and dummy variables for whether the individual belongs to a union (1=yes 0=No) and whether the individual is a university graduate

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. reg lhw exper exper2 union grad if female==1
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Source	SS	df	MS	Number of obs = 8691		
Model	599.876585	4	149.969146	F(4, 8686)	=	661.68
Residual	1968.6662	8686	.226648192	Prob > F	=	0.0000
-----				R-squared	=	0.2335
Total	2568.54278	8690	.295574544	Adj R-squared	=	0.2332
-----				Root MSE	=	.47608
lhw	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
exper	.0323483	.001691	19.13	0.000	.0290336	.035663
exper2	-.0006999	.0000381	-18.38	0.000	-.0007745	-.0006253
union	.2628194	.011322	23.21	0.000	.2406256	.2850133
grad	.5735716	.0154846	37.04	0.000	.5432182	.603925
_cons	1.372643	.0165886	82.75	0.000	1.340125	1.40516

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. reg lhw exper exper2 union grad if female==0
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Source	SS	df	MS	Number of obs = 8630		
Model	747.33788	4	186.83447	F(4, 8625)	=	652.59
Residual	2469.30267	8625	.286295962	Prob > F	=	0.0000
-----				R-squared	=	0.2323
Total	3216.64055	8629	.372770953	Adj R-squared	=	0.2320
-----				Root MSE	=	.53507
lhw	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
exper	.0596573	.0017101	34.89	0.000	.0563051	.0630096
exper2	-.0010838	.0000357	-30.33	0.000	-.0011539	-.0010138
union	.0833742	.0124648	6.69	0.000	.0589402	.1078082
grad	.5588589	.0158267	35.31	0.000	.5278348	.5898831
_cons	1.368326	.0179006	76.44	0.000	1.333237	1.403416

Can see that the coefficients - particularly on the returns to experience and union status differ for male and female sub-samples

To test whether these differences are jointly significant use the Chow variant of the test of linear restrictions. Compare the unrestricted RSS with the restricted RSS obtained by pooling the data and imposing the same slope estimates for men and women

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. reg lhw exper exper2 union grad
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Source	SS	df	MS			
Model	1273.78258	4	318.445644	Number of obs = 17321		
Residual	4892.38015	17316	.282535236	F(4, 17316) = 1127.10		
Total	6166.16272	17320	.356014014	Prob > F = 0.0000		
				R-squared = 0.2066		
				Adj R-squared = 0.2064		
				Root MSE = .53154		

lhv	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
exper	.0424958	.0012468	34.08	0.000	.0400519	.0449397
exper2	-.0008041	.0000269	-29.86	0.000	-.0008569	-.0007513
union	.1850738	.0088126	21.00	0.000	.1678003	.2023473
grad	.5948636	.0115904	51.32	0.000	.5721451	.617582
_cons	1.382525	.0127604	108.34	0.000	1.357513	1.407537

In this case $F = \frac{(RSS_{restricted} - RSS_{unrestricted})/q}{RSS_{unrestricted} / N - k} \sim f[q, N - k]$

Becomes $F = \frac{(RSS_{restricted} - RSS_{Female} + RSS_{Male})/q}{RSS_{Female} + RSS_{Male} / N - 2k} \sim f[q, N - 2k]$

(since there are $2K = 2*5 = 10$ unrestricted parameters)

Hence $F = \frac{(4892.38 - (1968.66 + 2469.3))/5}{1968.66 + 2469.3/17321 - 10} \sim f[5, 17321 - 2*5]$

So estimated $F = 353$ and the 5% critical value $F[5, \infty] = 2.21$

Hence reject the null the coefficients are the same for the two sub-sets