Lecture 6: Hypothesis Testing (continued)

Today's lecture

More about t values
$$\hat{t} = \frac{\hat{\beta}_1 - \beta_1^0}{s.e.(\hat{\beta}_1)}$$

Confidence intervals
$$\Pr\left[\hat{\beta}_1 - 1.96 * s.e.(\hat{\beta}_1) \le \beta_1 \le \hat{\beta}_1 + 1.96 * s.e.(\hat{\beta}_1)\right] = 0.95$$

And introduce Type I and Type II error & P values

Some different hypothesis tests

What we know now……

OLS is good

We can use this technique to get estimates of (causal) economic relationships

and

can test estimates against a given view of the world (hypothesis testing)

If the hypothesis is any good then would expect the estimate to be close to the hypothesized value

$$\hat{\beta} \approx \beta$$

the reality of sampling means that would not expect an estimate to be exactly equal to any hypothesized value
(randomness means the stochastic world is not a deterministic world)

But…

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is close enough to zero

allowing for sampling variation

to be consistent with the null hypothesis

Remember all thanks to Student's t distribution introduced by

William Gosset 1976-1937 and developed by Ronald Fisher 1890-1962

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symmetrical about its mean

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has its own set of ***critical values*** which can be taken to define the boundaries of what estimates are consistent with the null hypothesis and what are not
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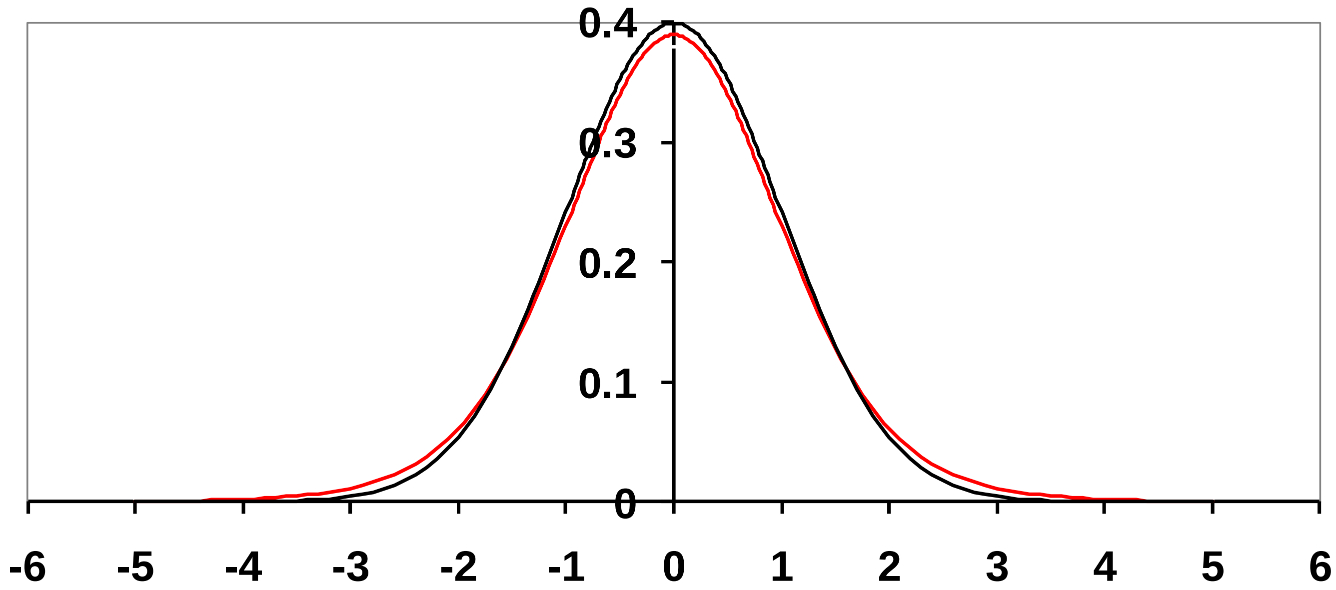
Why does this matter for testing hypotheses?

- It affects the critical values of the t distribution
- More information (more degrees of freedom) means we can impose a tighter range on the acceptance region. Estimates should be that bit closer to zero to be acceptable

When the number of degrees of freedom is large, the *t* distribution looks very much like a normal distribution (and as the number increases, it converges on one).

- This means that “in the limit” – as the sample size N gets very large relative to k - we can use the same critical values as the standard normal to test hypotheses 1.96

- (but before then will be different values)



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(essentially: Big t reject Null, Little t accept null hypothesis)

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- the "level of significance" you choose
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What is a level of significance?

Remember the confidence interval was defined at the "95% level"

$$\Pr\left[\hat{\beta}_1 - 1.96 * s.e.(\hat{\beta}_1) \leq \beta_1 \leq \hat{\beta}_1 + 1.96 * s.e.(\hat{\beta}_1)\right] = 0.95$$

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Similar logic applies to the significance level of a t test – there will be a range of values where we can be 1-α % confident a set of values should lie

The most common levels of significance of a test are

$\alpha = 0.05$ (5% significance)
$\alpha = 0.01$ (1% significance)

Usual convention is to base a t-test using the critical values at the **5%** level of significance – which is the same thing as defining a 95% acceptance region

(though this may vary a little depending on the size of the sample).

Remember most econometric computer packages routinely report the t value for a ***null hypothesis that that particular coefficient is zero***

(the variable has no effect

$\beta_1 = dy/dx = 0$

- But can test any hypothesized value once you know the formula to calculate a t statistic is

$$\left|\hat{t}\right| = \left|\frac{\hat{\beta}_1 - \beta_1^0}{s.e.(\hat{\beta}_1)}\right| > t_{N-k}^{\alpha}$$

- Example

$$Cons = \beta_0 + \beta_1 Income + u$$

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use "C:\qm2\Lecture 2\consumption_data.dta", clear
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Does this t value lie in acceptance region of the t distribution?
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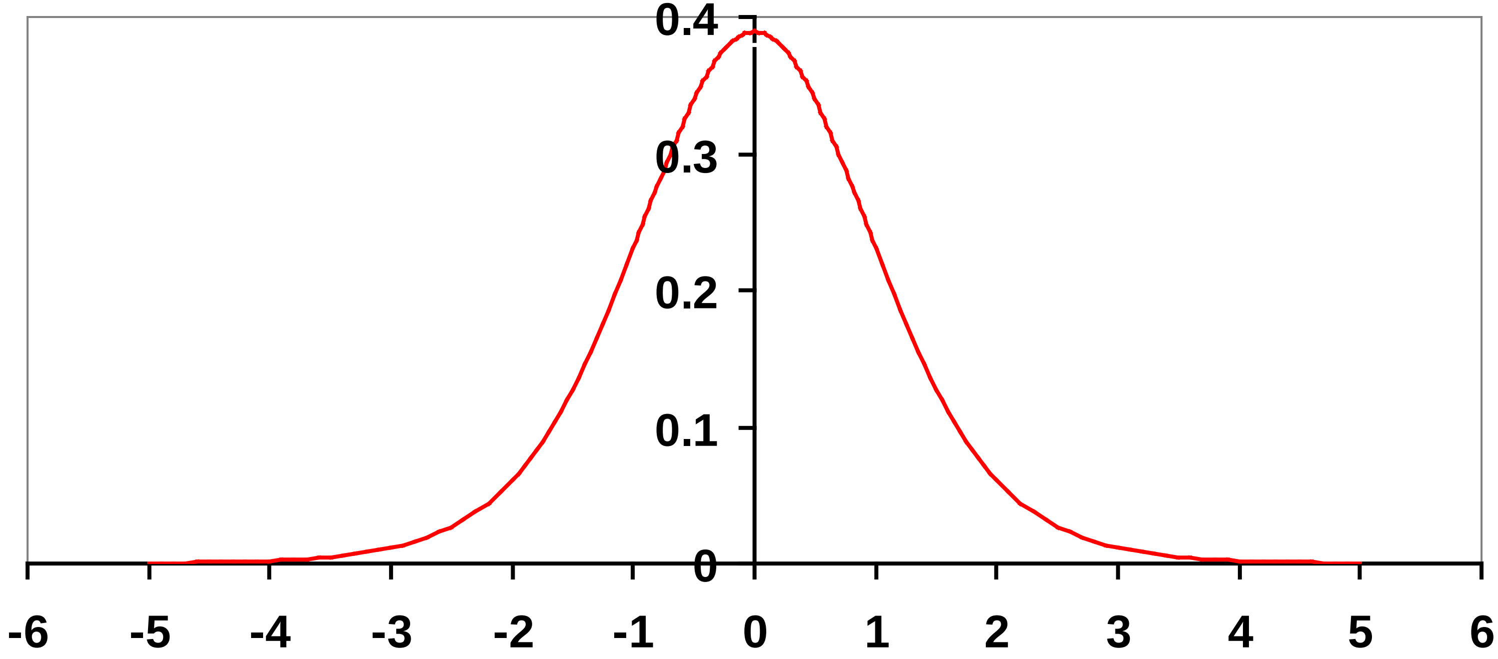
If not it is inconsistent with the null
(estimate too far away from hypothesised value)

Given a t distribution need to find critical values for acceptance region – and these depend on sample size N and number of coefficients in model k



There are tables that will tell us the critical values

Upper critical values of Student's t distribution with ν degrees of freedom

Probability of exceeding the critical value

ν	0.10	0.05	0.025	0.01	0.005	0.001
1.	3.078	6.314	12.706	31.821	63.657	318.313
2.	1.886	2.920	4.303	6.965	9.925	22.327
3.	1.638	2.353	3.182	4.541	5.841	10.215
4.	1.533	2.132	2.776	3.747	4.604	7.173
5.	1.476	2.015	2.571	3.365	4.032	5.893
6.	1.440	1.943	2.447	3.143	3.707	5.208
7.	1.415	1.895	2.365	2.998	3.499	4.782
8.	1.397	1.860	2.306	2.896	3.355	4.499
9.	1.383	1.833	2.262	2.821	3.250	4.296
10.	1.372	1.812	2.228	2.764	3.169	4.143
11.	1.363	1.796	2.201	2.718	3.106	4.024
12.	1.356	1.782	2.179	2.681	3.055	3.929
13.	1.350	1.771	2.160	2.650	3.012	3.852
14.	1.345	1.761	2.145	2.624	2.977	3.787
15.	1.341	1.753	2.131	2.602	2.947	3.733
16.	1.337	1.746	2.120	2.583	2.921	3.686
17.	1.333	1.740	2.110	2.567	2.898	3.646
18.	1.330	1.734	2.101	2.552	2.878	3.610
19.	1.328	1.729	2.093	2.539	2.861	3.579
20.	1.325	1.725	2.086	2.528	2.845	3.552
21.	1.323	1.721	2.080	2.518	2.831	3.527
22.	1.321	1.717	2.074	2.508	2.819	3.505
23.	1.319	1.714	2.069	2.500	2.807	3.485
24.	1.318	1.711	2.064	2.492	2.797	3.467
25.	1.316	1.708	2.060	2.485	2.787	3.450

26.	1.315	1.706	2.056	2.479	2.779	3.435
27.	1.314	1.703	2.052	2.473	2.771	3.421
28.	1.313	1.701	2.048	2.467	2.763	3.408
29.	1.311	1.699	2.045	2.462	2.756	3.396
30.	1.310	1.697	2.042	2.457	2.750	3.385
31.	1.309	1.696	2.040	2.453	2.744	3.375
32.	1.309	1.694	2.037	2.449	2.738	3.365
33.	1.308	1.692	2.035	2.445	2.733	3.356
34.	1.307	1.691	2.032	2.441	2.728	3.348
35.	1.306	1.690	2.030	2.438	2.724	3.340
36.	1.306	1.688	2.028	2.434	2.719	3.333
37.	1.305	1.687	2.026	2.431	2.715	3.326
38.	1.304	1.686	2.024	2.429	2.712	3.319
39.	1.304	1.685	2.023	2.426	2.708	3.313
40.	1.303	1.684	2.021	2.423	2.704	3.307
41.	1.303	1.683	2.020	2.421	2.701	3.301
42.	1.302	1.682	2.018	2.418	2.698	3.296
43.	1.302	1.681	2.017	2.416	2.695	3.291
44.	1.301	1.680	2.015	2.414	2.692	3.286
45.	1.301	1.679	2.014	2.412	2.690	3.281
46.	1.300	1.679	2.013	2.410	2.687	3.277
47.	1.300	1.678	2.012	2.408	2.685	3.273
48.	1.299	1.677	2.011	2.407	2.682	3.269
49.	1.299	1.677	2.010	2.405	2.680	3.265
50.	1.299	1.676	2.009	2.403	2.678	3.261
51.	1.298	1.675	2.008	2.402	2.676	3.258
52.	1.298	1.675	2.007	2.400	2.674	3.255
53.	1.298	1.674	2.006	2.399	2.672	3.251
54.	1.297	1.674	2.005	2.397	2.670	3.248
55.	1.297	1.673	2.004	2.396	2.668	3.245
56.	1.297	1.673	2.003	2.395	2.667	3.242

88.	1.291	1.662	1.987	2.369	2.633	3.185
89.	1.291	1.662	1.987	2.369	2.632	3.184
90.	1.291	1.662	1.987	2.368	2.632	3.183
91.	1.291	1.662	1.986	2.368	2.631	3.182
92.	1.291	1.662	1.986	2.368	2.630	3.181
93.	1.291	1.661	1.986	2.367	2.630	3.180
94.	1.291	1.661	1.986	2.367	2.629	3.179
95.	1.291	1.661	1.985	2.366	2.629	3.178
96.	1.290	1.661	1.985	2.366	2.628	3.177
97.	1.290	1.661	1.985	2.365	2.627	3.176
98.	1.290	1.661	1.984	2.365	2.627	3.175
99.	1.290	1.660	1.984	2.365	2.626	3.175
100.	1.290	1.660	1.984	2.364	2.626	3.174
∞	1.282	1.645	1.960	2.326	2.576	3.090

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then ask whether the estimated t value lies above or below this value (in absolute terms)

If it is above then lies outside acceptance region (inconsistent with null)

If it is below then lies within acceptance region (and is consistent with null)

Since estimate t=77.26

Then $\left|\hat{t}\right| = \left|\dfrac{\hat{\beta}_1 - \beta_1{}^0}{s.e.(\hat{\beta}_1)}\right| > t^{\alpha}_{N-k}$ reject the null – that the variable has no effect - at the 95% significance level.

77.26 > 2.017

Note that the choice of a 95% significance level is somewhat arbitrary

You will also sometimes see the term "**size**" used instead of "significance level".

Unfortunately this can be confusing, since to increase the significance level of the test from 95% to 99% usually means to reduce the size of the test ie go from 5% to 1% level.

Also some texts refer to 95% levels which is the same as a 5% level (95% acceptance probability, 5% rejection probability)

Be warned

Most econometric software routinely reports the t value based on a null hypothesis that a single coefficient is zero

(and remember a null of zero effectively implies that the effect of that variable is zero since

$Cons = \beta_0 + \beta_1 Income + u$

that the slope estimate $\dfrac{\partial Cons}{\partial Income} = \beta_1 = 0$

(a change in income has NO effect on consumption)

However the principle of using a t value to compare the difference between the estimate and the hypothesized value applies to ANY hypothesized value

So suppose we have a new null

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So look across the row with 43 degrees of freedom

Upper critical values of Student's t distribution with ν degrees of freedom

Probability of exceeding the critical value

ν	0.10	0.05	0.025	0.01	0.005	0.001
43.	1.302	1.681	2.017	2.416	2.695	3.291

to find that the critical value that puts 2.5% of the distribution in the top tail
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This time $\left|\hat{t}\right| = \left|\dfrac{\hat{\beta}_1 - \beta_1^0}{s.e.(\hat{\beta}_1)}\right| > t^{\alpha}_{N-k}$

So accept the null – that the variables true effect is 0.9 - at the 5% significance Level 1.43 < 2.017

Thing to remember

Whether we accept or reject a hypothesis is partly dependent on the choice of significance levels of a test

(this may open up possibilities for manipulation – not a good thing)

By **increasing** the size of the test (a goes from 1% to 5% for example)

- equivalent to **reducing** the significance level from 99% to 95%

we **reduce** the acceptance region for a given t estimate (and increase the range of estimate that fall in the rejection region)

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Since rarely know the costs of these errors, have to settle on critical values which balance the two errors

- which is why the 5% level of significance is the one usually used.

It is true however that there is some **trade-off** between the sample size and the significance level of the test.

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Then ↑N → ↓ $s.e.(\hat{\beta}_1)$

and hence because $\hat{t} = \dfrac{\hat{\beta}_1 - \beta_1^0}{s.e.(\hat{\beta}_1)}$ this will ↑ estimated t value

There are no rules on this but might want to think about the following guidelines:

N in the low 10's use a =10%
N in the 100's use a =5%
N in the 10000's use a =1%

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More formally if in the regression output

$p <$ chosen level of α (say 0.05 = 5%)

reject the null hypothesis

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Moral: Significance and size of effect are both important. When reporting the effect of coefficients

1. Check variable is statistically significant from zero
2. If it is then (and only then) discuss the size of the effect as implied by the regression coefficient

Confidence Intervals Revisited

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> Key point here is to remember it is a 2-tailed test which is why in the notation the significance level a =0.05 is divided by 2 to allow 5% of the distribution to appear outside the acceptance region with 2.5% in either tail)