Lecture 18. Autocorrelation & Dynamic Models

Saw that may have to worry about autocorrelation of residuals because it biases the standard errors (t values, F tests etc) of OLS estimates - meaning could think things are (in)significant when they are not

Typically model autocorrelation like this

$u_t = \rho u_{t-1} + e_t$ AutoRegressive model of order one – AR(1)

or like this

$u_t = \rho_1 u_{t-1} + \rho_2 u_{t-2} + \rho_3 u_{t-3} + \rho_4 u_{t-4} + e_t$

AutoRegressive model of order four – AR(4)

Test for presence using Breusch-Godfrey $\rho_1=\rho_2=\ldots\rho_1 = 0$

Today:

What to do about autocorrelation (change model, adjust standard errors)

Estimating short and long run effects in dynamic models

More problems when using time series data – stationarity

What to do about stationarity

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- fix up the standard errors

Example: Model Misspecification

Sometimes autocorrelation in residuals can be caused by incorrect functional form in your model or (effectively the same thing) the omission of relevant variables

The data set *gdp.dta* contains quarterly time series data on US GDP growth and inflation over the period 1956:q1 to 2002:q4

A simple regression of inflation rate on 1-period lagged growth rate of GDP gives (using lagged growth to try and deal with possible endogeneity concerns)

```
use "C:\qm2\Lecture 16\gdp.dta", clear

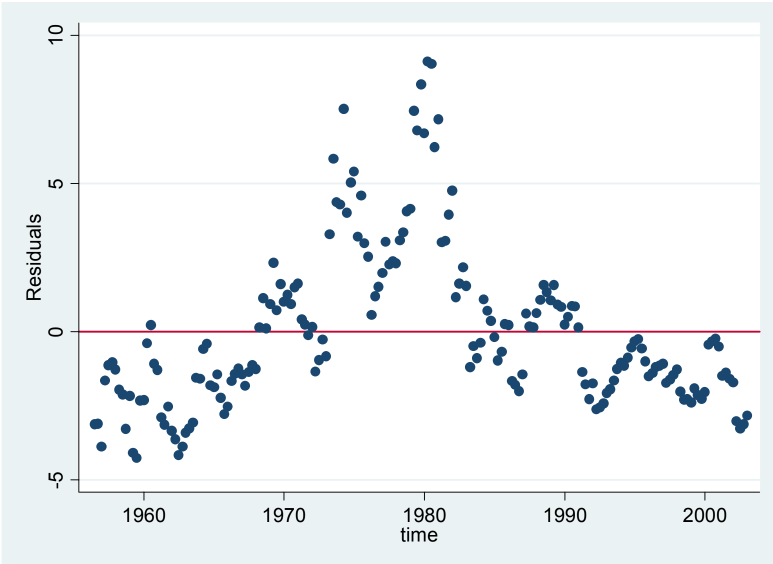
tsset TIME
        time variable:  TIME, 1 to 188

reg usinf DG

      Source |       SS       df       MS              Number of obs =     187
-------------+------------------------------           F(  1,   185) =   13.98
       Model |  110.148815     1  110.148815           Prob > F      =  0.0002
    Residual |  1457.20779   185  7.87679889           R-squared     =  0.0703
-------------+------------------------------           Adj R-squared =  0.0653
       Total |  1567.35661   186  8.42664844           Root MSE      =  2.8066
------------------------------------------------------------------------------
       usinf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          DG |  -.7576151   .2025973    -3.74   0.000    -1.157313    -.357917
       _cons |   4.650681   .2398357    19.39   0.000     4.177516    5.123845
------------------------------------------------------------------------------
. bgtest
Breusch-Godfrey LM statistic:   161.743  Chi-sq( 1)  P-value =  4.7e-37
```

and a graph of the residuals also shows them to be (positively) autocorrelated

```
predict res, resid
two (scatter res time, yline(0) )
```



Now suppose decide to include the 1 period lag of inflation rate on the right hand side

```
. g usinf1=usinf[_n-1]
(1 missing value generated)

reg usinf DG l.usinf

      Source |       SS       df       MS              Number of obs =     187
-------------+------------------------------           F(  2,   184) =  994.55
       Model |  1434.64607     2  717.323035           Prob > F      =  0.0000
    Residual |  132.710539   184  .721252931           R-squared     =  0.9153
-------------+------------------------------           Adj R-squared =  0.9144
       Total |  1567.35661   186  8.42664844           Root MSE      =  .84927
------------------------------------------------------------------------------
       usinf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          DG |  -.0653697   .0633985    -1.03   0.304    -.1904512    .0597117
       usinf |
         L1. |   .9504842   .0221801    42.85   0.000     .9067242    .9942442
       _cons |   .2478525   .1257897     1.97   0.050    -.0003231     .496028
------------------------------------------------------------------------------
```

```
bgtest
Breusch-Godfrey LM statistic:  .2558797  Chi-sq( 1)  P-value =   .613
. predict resa, resid
```

Now the pattern of autocorrelation seems to have become much less noticeable in the new specification compared to the original, (though there still may be endogeneity bias in the OLS estimates of the coefficients)

Can, in principle, manipulate the data to remove autocorrelation from the residuals.

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Suppose you had

$$Y_t = b_0 + b_1 X_t + u_t \qquad (1)$$

and **assumed** AR(1) behaviour in the residuals

$$u_t = \rho u_{t-1} + e_t \qquad (2)$$

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Multiplying (3) by ρ

$$\rho Y_{t-1} = \rho b_0 + \rho b_1X_{t-1} + \rho u_{t-1} \qquad (4)$$

(1) – (4)

On left hand side: $Y_t - \rho Y_{t-1}$

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so

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Or taking ρY_{t-1} to the other side

$$Y_t = b_0 - \rho b_0 + \rho Y_{t-1} + b_1 X_t - \rho b_1 X_{t-1} + u_t - \rho u_{t-1}$$

On left hand side: $Y_t - \rho Y_{t-1}$

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or

$$Y_t = (b_0 - \rho b_0) + \rho Y_{t-1} + b_1 X_t - \rho b_1 X_{t-1} + e_t \qquad (5)$$

On left hand side: $Y_t - \rho Y_{t-1}$

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Since $e_t = u_t - \rho u_{t-1}$ from (2)

then if estimate (5) by OLS there should be no autocorrelation.

On left hand side: $Y_t - \rho Y_{t-1}$

On right hand side: $b_0 - \rho b_0 + b_1 X_t - \rho b_1 X_{t-1} + u_t - \rho u_{t-1}$

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This is called **Feasible Generalised Least Squares** (FGLS)

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So If don’t know exact form of autocorrelation (highly likely) it may be preferable to fix up the OLS standard errors so they are no longer biased but remain inefficient - in the sense that if you knew the precise form of autocorrelation you could write down the exact formula for the standard errors, but you don’t so do the best you can instead.

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Newey-West standard errors do this and are valid in presence of lagged dependent variables and endogenous X variables **if** have large sample (ie fix-up is only valid asymptotically though it has been used on sample sizes of around 50).

In absence of autocorrelation we know OLS estimate of variance on any coefficient is

$$Var(\hat{\beta}_{ols}) = \frac{s^2{}_u}{N * Var(X)}$$

In presence of autocorrelation, can show the Newey-West standard errors (unbiased but inefficient) are

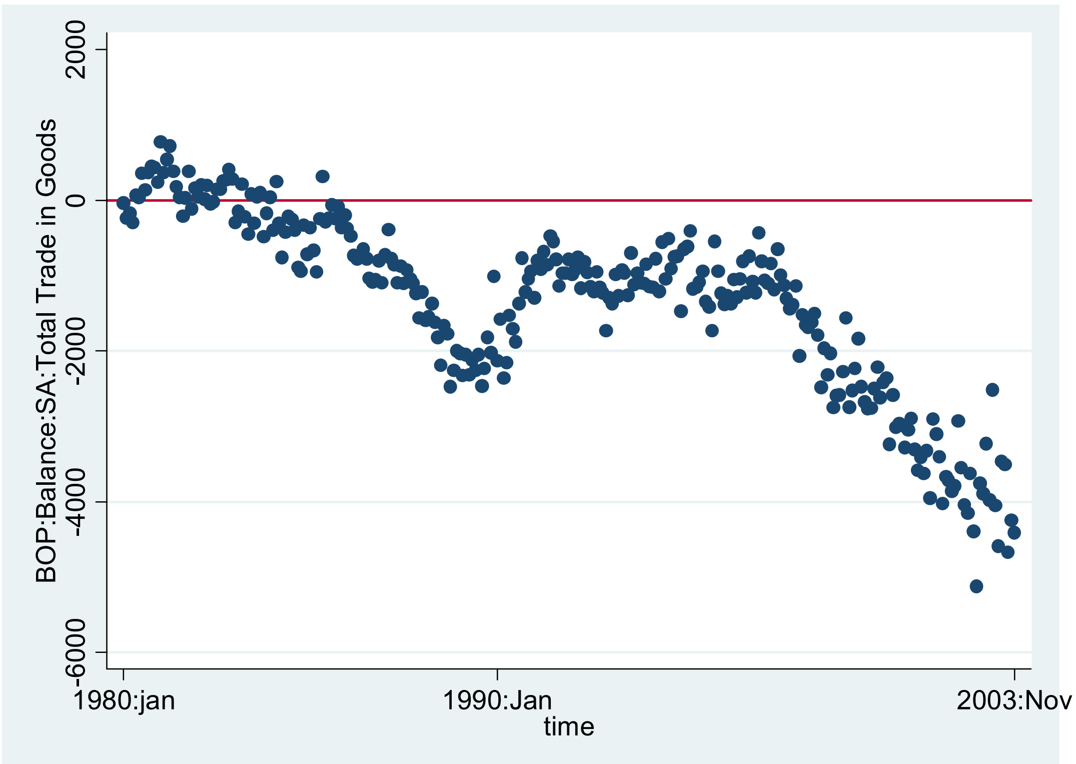
$$Var(\hat{\beta}_{ols}) = \frac{Var(\hat{\beta}_{ols})}{s_u} * v$$

where v is a (complicated) function of the maximum number of lags you believe could be correlated with current residuals (in annual data 1 or 2 lags should be enough)

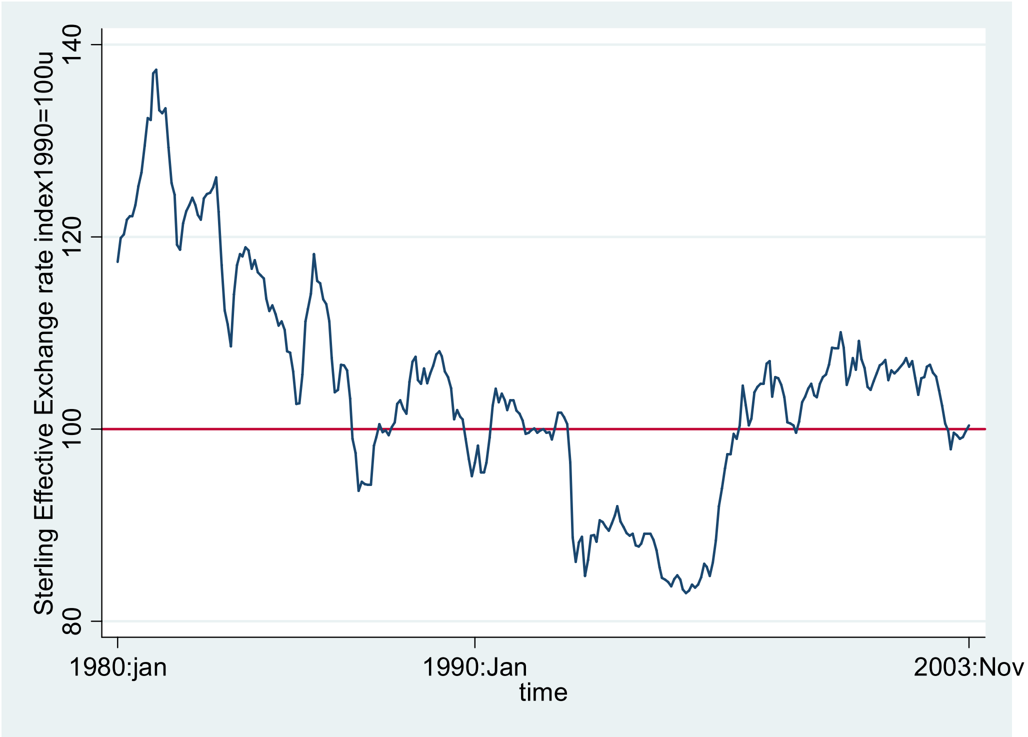
Example

The data set *bop.dta* contains monthly time series data on the UK Balance of Trade in goods (measured in £billion) and the effective exchange rate – a weighted average of sterling's value against a basket of foreign currencies which is centred on the value 100 in January 1990. A value above zero indicates the Balance of Trade is in surplus, a value below zero indicates a deficit. A value > 100 indicates sterling has appreciated, a value<100 indicates that sterling has depreciated

```
two (scatter bop time, yline(0) xlabel(1 121 287, value) )
```



```
two (scatter  xchanger time, yline(100) xlabel(1 121 287, value) )
```



Can see that over time the balance of trade has deteriorated, whilst the value of sterling has generally been high over the same period

To see if the 2 events are related, run a simple regression of the trade balance on sterling's value (lagged by one month to reduce endogeneity concerns)

```
. u bop                             /* read data in */

. tsset year                        /* declare data is time series */
      time variable:  year, 55 to 99

. sort time
. g xchange1=xchange[_n-1]          /* set up 1 period lag of exchange rate */
```

```
. reg bop xchange1
      Source |       SS       df       MS              Number of obs =     286
-------------+------------------------------           F(  1,   284) =   28.08
       Model |  37414954.7     1  37414954.7           Prob > F      =  0.0000
    Residual |   378396350   284  1332381.51           R-squared     =  0.0900
-------------+------------------------------           Adj R-squared =  0.0868
       Total |   415811305   285  1458987.03           Root MSE      =  1154.3
------------------------------------------------------------------------------
         bop |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    xchange1 |   31.56233   5.956083     5.30   0.000     19.83866      43.286
       _cons |  -4646.285   623.5585    -7.45   0.000    -5873.667   -3418.902
```

regressions suggests a high exchange rate is positively correlated with the trade balance
Check for 1st and 12th order autocorrelation (this is monthly data so residuals could be related to last month's value and/or previous 12 month's value)

```
. bgtest,lags(1)
Breusch-Godfrey LM statistic:  249.0974  Chi-sq( 1)  P-value =  4.1e-56

. bgtest,lags(12)
Breusch-Godfrey LM statistic:  250.9979  Chi-sq(12)  P-value =  8.5e-47
```

Results suggest presence of both types of autocorrelation

To fix up standard errors using newey west procedure

```
. newey bop xchange1, lag(1)
Regression with Newey-West standard errors          Number of obs  =       286
maximum lag : 1                                     F(  1,   284)  =     23.81
                                                    Prob > F       =    0.0000
------------------------------------------------------------------------------
             |             Newey-West
         bop |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    xchange1 |   31.56233   6.468508     4.88   0.000     18.83003    44.29463
       _cons |  -4646.285   674.1623    -6.89   0.000    -5973.274   -3319.296
```

and to allow for an AR(12) process

```
. newey bop xchange1, lag(12)
Regression with Newey-West standard errors          Number of obs  =       286
maximum lag : 12                                    F(  1,   284)  =      4.23
                                                    Prob > F       =    0.0406
------------------------------------------------------------------------------
             |             Newey-West
         bop |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    xchange1 |   31.56233   15.34144     2.06   0.041     1.364981    61.75968
       _cons |  -4646.285   1593.395    -2.92   0.004    -7782.646   -1509.923
```

Note the coefficients are unchanged but the standard errors are different, (uncorrected OLS t statistics much larger as expected, particularly when compared to the model that allows for autocorrelation of up to 12 lags)

How can fix up by specifying number of lags, if test is supposed to account for unknown form of autocorrelation? - can be shown that Newey-West test works for unknown forms of autocorrelation as long as number of lags is allowed to rise with number of observations/type of data (monthly, daily etc)

Dynamic Models

Common in time series work to try and include **lags** of explanatory (and dependent) variables in a regression in order to account for belief that the influence of a variable could extend beyond the period in which any change occurred ("persistence" or "inertia")

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The inclusion of lags turns the model from a static one into a dynamic one

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   Takes time to change method of production. Eg supply of a factor may change only with a lag following a shock to the production process.

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   Takes time to change method of production. Eg supply of a factor may change only with a lag following a shock to the production process.

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The use of lags allows us to distinguish between the effects of **Long Run and Short Run Multiplier Effects**

Given

$$Y_t = a + b_0X_t + b_1X_{t-1} + b_2X_{t-2} + \ldots + b_kX_{t-k} + u_t$$

(a distributed lag of order k since there are k lags of data)

Given

$$Y_t = a + b_0X_t + b_1X_{t-1} + b_2X_{t-2} + \ldots + b_kX_{t-k} + u_t$$

the coefficient b_0 is said to be the **short-run** multiplier effect, since it captures the immediate effect of any change in X on Y at time t

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$$b_0 = dY_t/dX_t$$

$$\Rightarrow \quad dY_t = b_0 * dX_t$$

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is the effect on Y **at time t** of a change in X at time t-1
and

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is the effect on Y **at time t** of a change in X at time t-1

and

$$\Rightarrow \quad dY_t = b_1 * dX_{t-1}$$

ie Y changes by b_1 times the amount of the change in X at time t-1

and so on for any other lag

Hence for any lagged value of X the impact on Y at time t is given by the coefficient on that lag j

$$b_j = dY_t/dX_{t-j}$$

$$\Rightarrow \qquad dY_t = b_j * dX_{t-j}$$

Given this suppose X were increased by ΔX in every period (eg a new higher level of government spending), then in

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Period j

$$\Delta Y = b_0 \Delta X + b_1 \Delta X + \ldots.\ b_j \Delta X$$

$$= (b_0 + b_1 + \ldots.. + b_j)\, \Delta X$$ (cumulative effects from ALL periods)

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so the overall change in Y given this change in X is (take ΔX to the other side)

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$$\Delta Y = b_0 \Delta X$$

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$$\Delta Y = b_0 \Delta X + b_1 \Delta X = (b_0 + b_1)\ \Delta X$$

:

Period j

$$\begin{aligned}\Delta Y &= b_0 \Delta X + b_1 \Delta X + \ldots.\ b_j \Delta X \\ &= (b_0 + b_1 + \ldots.. + b_j)\ \Delta X\end{aligned}$$

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$$\Delta Y/\Delta X = (b_0 + b_1 + \ldots.. + b_j)$$

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:

Period j

$$\begin{aligned} \Delta Y &= b_0 \Delta X + b_1 \Delta X + \ldots.\ b_j \Delta X \\ &= (b_0 + b_1 + \ldots.. + b_j)\, \Delta X \end{aligned}$$

so the overall change in Y given this change in X is (take ΔX to the other side)

$$\Delta Y / \Delta X = (b_0 + b_1 + \ldots.. + b_j)$$

ie this **sum** of all the b coefficients is the **long-run multiplier** effect of a permanent change in the value of X (the short-run immediate effect b_0 plus all the other effects working through over time)

N.B. 1. Given

$$Y_t = a + b_0X_t + b_1X_{t-1} + b_2X_{t-2} + \ldots + b_kX_{t-k} + u_t$$

When introduce lags this assumes that not just the current value of the X variable is uncorrelated with the residual, but also all past values of X

$$E(u_t / X_t) = 0$$

and

$$E(u_t / X_{t-1}) = 0$$

… $$E(u_t / X_{t-k}) = 0$$

If we assume further that the residuals are also uncorrelated with all **future** values of X and **past** values of X **beyond** lag k

$$E(u_t / X_{t+k+s} \ldots X_t \ldots X_{t-k-s}) = 0$$

this is called **strict exogeneity**

and there may be estimation techniques other than OLS that can be used to estimate dynamic causal effects

Example: The data set *lagdata.dta* contains quarterly information on a firm's investment, (ie), measured in £ and its revenue (cashf) over 24 years. The idea is to estimate the following relationship.

$$Invest_t = a + b_0 * Cashflow_t + u_t$$

To do this read the data in. Then set up time series data in Stata , "time" is the variable in the data set which denotes the period in which the observations on the dependent and explanatory variable was taken. Use the following command.

```
u lagdata

tsset time

Stata responds with

time variable:  time, 1 to 140
```

Now regression, holding back the 1st 8 quarters of data

```
. reg ie cashf   if time>8 & time<105

      Source |       SS       df       MS              Number of obs =      96
-------------+------------------------------           F(  1,    94) =  699.28
       Model |  1.7683e+11     1  1.7683e+11           Prob > F      =  0.0000
    Residual |  2.3770e+10    94   252871543           R-squared     =  0.8815
-------------+------------------------------           Adj R-squared =  0.8802
       Total |  2.0060e+11    95  2.1115e+09           Root MSE      =   15902
------------------------------------------------------------------------------
          ie |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    cashflow |      .8333   .0315121    26.44   0.000      .770732    .8958679
       _cons |   -35688.6    6441.89    -5.54   0.000    -48479.12   -22898.07
------------------------------------------------------------------------------

. bgtest, lag(1)
Breusch-Godfrey LM statistic:  80.03151  Chi-sq( 1)  P-value =  3.7e-19
```

With no lags on cashflow, short-run and long-run multiplier are the same

$$d \text{ invest}/ d \text{ cashflow} = b_0 = 0.83$$

so a £1 increase in revenue generates an immediate (and permanent) 83 pence increase in investment

Note value of Breusch-Godfrey test indicates that there seems to be (1st order) autocorrelation in the data so standard errors are wrongly estimated, (but coefficients are unbiased).

How many lags to include?

Data Mining

– increase the number of lags sequentially until the lagged values start to become insignificant

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Problems:
- may be a limited number of observations in the data set (quite likely if working with annual time series data) which means "degrees of freedom" problems start to set in

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Since T-k ↓ by 1 for every lag added to the model and since variance of OLS coefficient estimates is calculated as

$$Var(\hat{\beta}) = \frac{s_u^2}{T * Var(X)} = \frac{\sum \hat{u}^2 / T-k}{T * Var(X)}$$

the standard error of OLS estimate gets larger as T-k ↓

Could be important for statistical inference (is a variable significant or not?)

Similarly, more lags ↑ risk of multicolinearity, which again increases standard errors and reduces precision of OLS estimates

$$Var(\hat{\beta}_1) = \frac{s^2}{T * Var(X)} * \frac{1}{1 - r^2_{X_t X_{t-s}}}$$

Example: introduce cashflow lagged one quarter as additional explanatory variable in investment model

$$Invest_t = a + b_0*Cashflow_t \; b_1*Cashflow_{t-1} + u_t$$

To generate lags, sort the data by the time variable

```
sort time
gen cash1=cashflow[_n-1]            /* lags cashflow by 1 period */

. regdw ie cashf   cash1 if time>8 & time<105

  Source |       SS       df       MS                  Number of obs =      96
---------+------------------------------               F(  2,    93) =  392.41
   Model |  1.7934e+11     2  8.9672e+10               Prob > F      =  0.0000
Residual |  2.1252e+10    93   228518067               R-squared     =  0.8941
---------+------------------------------               Adj R-squared =  0.8918
   Total |  2.0060e+11    95  2.1115e+09               Root MSE      =   15117
------------------------------------------------------------------------------
      ie |      Coef.   Std. Err.       t     P>|t|       [95% Conf. Interval]
---------+--------------------------------------------------------------------
cashflow |   .2309931   .1839125      1.256   0.212      -.1342206    .5962068
   cash1 |   .6089191   .1834484      3.319   0.001       .2446269    .9732114
   _cons |  -35721.57   6123.845     -5.833   0.000      -47882.31   -23560.83

Durbin-Watson Statistic =  .1145408
```

When introduce lags on cashflow into the model, short-run and long-run multiplier are **not** the same

Short-run multiplier = d invest/ d cashflow = b_0 (as before) = 0.23

Long-run multiplier = $b_0 + b_1 = 0.23 + 0.61 = 0.84$

So long-run effect seems to be much larger than short-run effect, (but agrees with estimate from first regression without lags)

Note while the introduction of lags can sometimes reduce autocorrelation, in this case still appear to get autocorrelation in model.

Now do data mining and add 6 lags to the model

```
. reg ie cashf   cash1-cash6  if time>8 & time<105

      Source |       SS       df       MS              Number of obs =      96
-------------+------------------------------           F(  7,    88) =  131.17
       Model |  1.8305e+11     7  2.6150e+10           Prob > F      =  0.0000
    Residual |  1.7544e+10    88   199368717           R-squared     =  0.9125
-------------+------------------------------           Adj R-squared =  0.9056
       Total |  2.0060e+11    95  2.1115e+09           Root MSE      =   14120

------------------------------------------------------------------------------
          ie |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    cashflow |   .2572755   .1786268     1.44   0.153    -.0977078    .6122588
       cash1 |    .141559   .2771014     0.51   0.611    -.4091217    .6922398
       cash2 |   .1152186   .2787114     0.41   0.680    -.4386617    .6690988
       cash3 |   .0837524   .2780237     0.30   0.764    -.4687612     .636266
       cash4 |   .0501196   .2789373     0.18   0.858    -.5042097    .6044489
       cash5 |   .1307876   .2789578     0.47   0.640    -.4235823    .6851575
       cash6 |    .092952   .1783286     0.52   0.604    -.2614386    .4473426
       _cons |  -39319.27   5783.234    -6.80   0.000    -50812.23   -27826.31
------------------------------------------------------------------------------

. bgtest, lag(1)
Breusch-Godfrey LM statistic:  90.22796  Chi-sq( 1)  P-value =  2.1e-21
```

Note however there are big changes to estimated coefficients and standard errors when add several lagged cashflow variables, (because of multicolinearity)

Now **none** of cashflow variables is significant and coefficient on $cash_{t-1}$ has changed considerably.

As a result it is harder to estimate the short and long run multipliers accurately

Short-run multiplier now risen to 0.26
Long-run multiplier = 0.25 + 0.14 + 0.12 + 0.08 + 0.05 + 0.13 + 0.09 = 0.86

(Again very similar to first estimate)

Check multicolinearity by looking at correlation coefficients.

```
. corr ie cashf   cash1-cash6  if time>8 & time<105
(obs=96)

         |       ie cashflow    cash1    cash2    cash3    cash4    cash5
---------+---------------------------------------------------------------
      ie |   1.0000
cashflow |   0.9389   1.0000
   cash1 |   0.9446   0.9866   1.0000
   cash2 |   0.9445   0.9668   0.9861   1.0000
   cash3 |   0.9406   0.9481   0.9658   0.9858   1.0000
   cash4 |   0.9348   0.9332   0.9466   0.9655   0.9857   1.0000
   cash5 |   0.9287   0.9223   0.9332   0.9476   0.9664   0.9861   1.0000
   cash6 |   0.9190   0.9128   0.9226   0.9335   0.9478   0.9662   0.9861
```

Can see all cash flow variables are highly colinear

2. Koyck Transformation

rather than estimate a model with a large number of lags can transform data into a more "parsimonious" form

Given a dynamic model

(1) $Y_t = a + b_0X_t + b_1X_{t-1} + b_2X_{t-2} + \ldots + b_kX_{t-k} + u_t$

Assume effect of a change in X recedes over time by an amount λ each period and that this is reflected in size of coefficients such that

(2) $b_k = b_0\lambda^k$ $\qquad 0 < \lambda < 1$

(λ is a fraction so raising fraction to a power ensures effect gets smaller as lag length k increases. The larger the value of λ the slower the speed of adjustment)

We know the long-run multiplier is given by

$$b_0 + b_1 + b_2 + \ldots + b_k = b_0/1- \lambda$$

(ie the sum of an infinite series with constant of multiplication λ)

Sub. (2) into (1)

(3) $Y_t = a + b_0X_t + b_0\lambda\, X_{t-1} + b_2\lambda^2\, X_{t-2} + \ldots + b_k\, \lambda^k X_{t-k}\ + u_t$

If (3) is true at time t it is also true at time t-1, so

(4) $Y_{t-1} = a + b_0X_{t-1} + b_0\lambda\, X_{t-2} + b_0\lambda^2\, X_{t-3} + \ldots + b_0\, \lambda^k X_{t-k-1}\ + u_t$

multiply (4) by λ

(5) $\lambda Y_{t-1} = \lambda a + b0\lambda Xt_{-1} + b_0\lambda^2 X_{t-2} + b_0\lambda^3\, X_{t-3} + \ldots\ b_0\, \lambda^{k+1} X_{t-k-1}\ + \lambda\, u_t$

(3) – (5)

$Y_t - \lambda Y_{t-1} = a - \lambda a + b_0X_t + u_t - \lambda\, u_t$

(6) $Y_t = (a - \lambda a) + b_0X_t + \lambda Y_{t-1} + v_t$ (where $v_t = u_t - \lambda\, u_t$)

ie regress Y_t on a constant, the current level of X and the value of Y lagged 1 period. The coefficients b_0 and λ are what you need to calculate the long run multiplier

- advantage of this is more parsimonious (fewer coefficients to estimate) and so less chance of multicolinearity

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$$Y_t = (a - \lambda a) + b_0 X_t + \lambda Y_{t-1} + v_t$$

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Need to instrument lagged dependent variable if want to use this set-up in this particular example (no need if no autocorrelation)

2. If estimate long-run multiplier as in above example there is no standard error for the estimate which is not helpful

Example: attempt Koyck transformation so that can represent above more parsimoniously(is as current level of cashflow and a lagged dependent variable)

```
. g ie1=ie[_n-1]


. reg ie cashf ie1  if time>8 & time<105

      Source |       SS       df       MS              Number of obs =      96
-------------+------------------------------           F(  2,    93) = 7778.11
       Model |  1.9940e+11     2  9.9702e+10           Prob > F      =  0.0000
    Residual |  1.1921e+09    93  12818348.5           R-squared     =  0.9941
-------------+------------------------------           Adj R-squared =  0.9939
       Total |  2.0060e+11    95  2.1115e+09           Root MSE      =  3580.3

------------------------------------------------------------------------------
          ie |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    cashflow |   .1260826   .0182838     6.90   0.000     .0897747    .1623906
         ie1 |   .8801663    .020972    41.97   0.000       .83852    .9218125
       _cons |  -8096.317   1592.425    -5.08   0.000    -11258.56   -4934.076
------------------------------------------------------------------------------
```

Coefficient on cashflow is short-run multiplier estimate, (0.13)

Coefficient on ie1 is estimate of rate of decay of cashflow effect on investment over time (λ) = 0.88

So long-run multiplier is $b/(1-\lambda) = 0.13/(1-0.88) = 1.08$

Note doesn't solve the other problem of autocorrelation since

```
. bgtest, lag(1)
Breusch-Godfrey LM statistic:  19.23427  Chi-sq( 1)  P-value =  1.2e-05
```

Since estimated chi-squared greater than critical value, reject null of no autocorrelation, conclude that positive autocorrelation exists.

Important

To get round this, can also show that can estimate long run multiplier **and** its standard error directly from the specification

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(you choose the lag length)

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and the level of X lagged t-j periods

(you choose the lag length)

and can show that coefficient d_{j-1} on X_{t-j} is the long-run multiplier over the whole period

(proof left to problem set)

To generate lags of changes

```
gen cash1=cashflow[_n-1]            /* lags cashflow by 1 period */
gen cash2=cashflow[_n-2]            /* lags cashflow by 2 periods */

gen dc=cashflow-cash1               /* 1st period – 2nd period difference */
gen dc1=cash1-cash2

reg ie dc dc1 cash2 if time>8 & time<105

      Source |       SS       df       MS              Number of obs =      96
-------------+------------------------------           F(  3,    92) =  283.06
       Model |  1.8099e+11     3  6.0330e+10           Prob > F      =  0.0000
    Residual |  1.9608e+10    92   213130254           R-squared     =  0.9023
-------------+------------------------------           Adj R-squared =  0.8991
       Total |  2.0060e+11    95  2.1115e+09           Root MSE      =   14599
------------------------------------------------------------------------------
          ie |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          dc |   .3445294   .1822557     1.89   0.062    -.0174461    .7065048
         dc1 |   .3544745   .1771636     2.00   0.048     .0026124    .7063366
       cash2 |   .8530673   .0293783    29.04   0.000     .7947195     .911415
       _cons |  -37537.75   5950.109    -6.31   0.000    -49355.18   -25720.32
```

and the coefficient on cash2 gives the long run multiplier (compare with estimate above using Koyck transformation)

Stationarity

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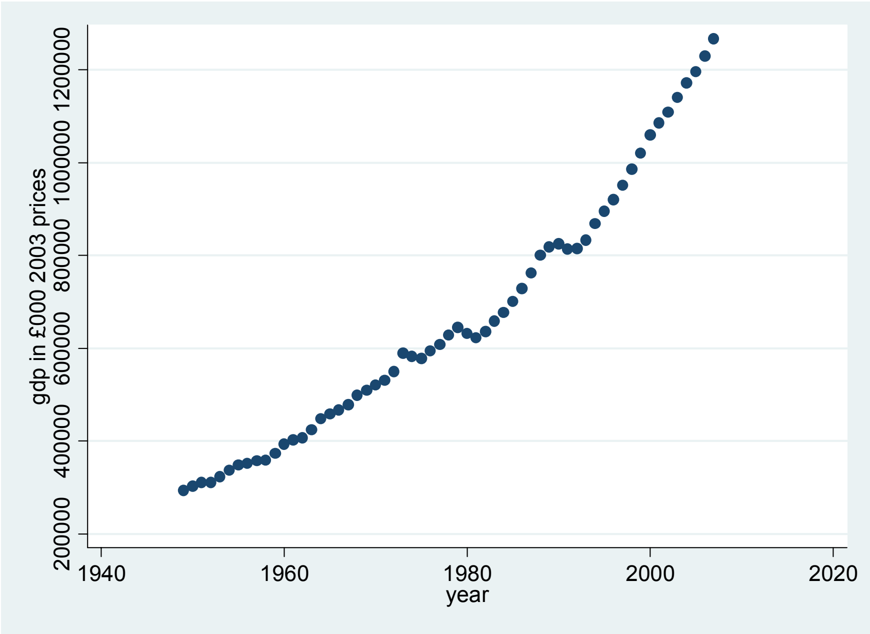
(Essentially any variable that is **trended** is unlikely to be stationary)

Example: A plot of nominal GDP over time using the data set *stationary.dta*

use "E:\qm2\Lecture 17\stationary.dta", clear

```
two (scatter gdp year)
```



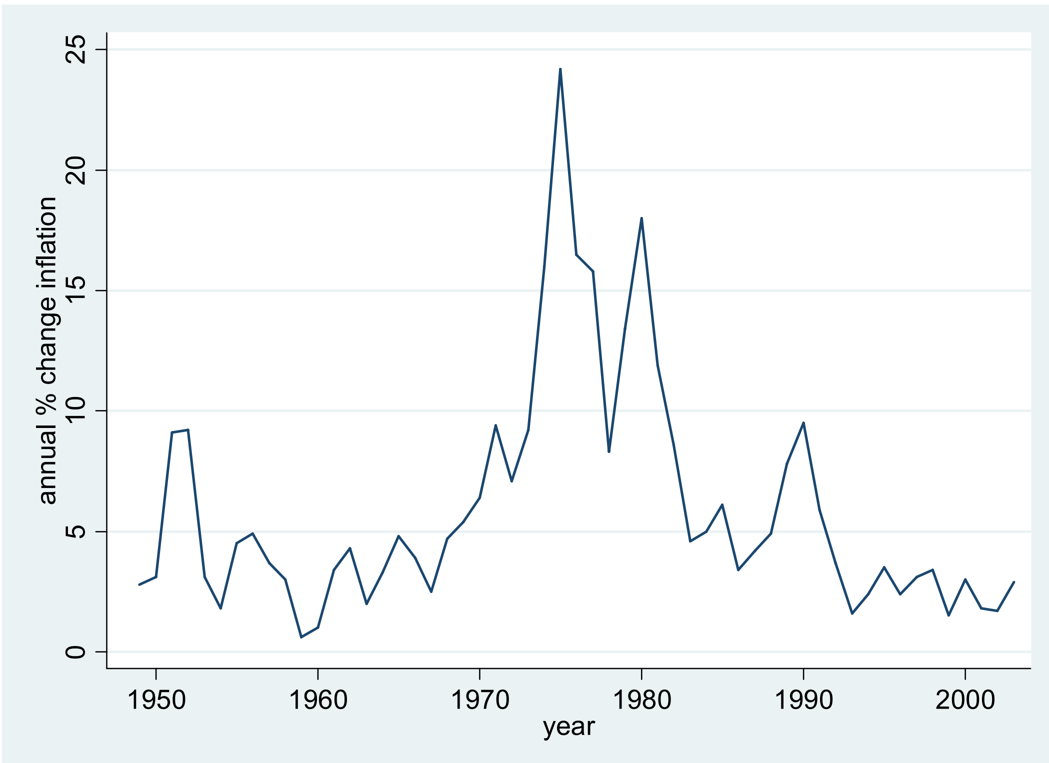
GDP displays a distinct upward trend and so is unlikely to be stationary. Neither its mean value or its variance are stable over time

```
su gdp if year<1980
    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
         gdp |        31    451016.3      109771     293576     644491

su gdp if year>=1980
    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
         gdp |        28    900660.8    196878.9     622722    1266397
```

Some series are already stationary if there is no obvious trend and some sort of reversion to a long run value. The UK inflation rate is one example (from the data set *stationary.dta*)

two (line inflation year)



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(though it is good practice to graph the series anyway)

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- the value of Y today equals last period’s value plus an unpredictable random error e (hence the name) and no other lags

This means that the best forecast of this period’s level is last period’s level.

$$Y_t = Y_{t-1} + e_t$$

$$Y_t = \rho Y_{t-1} + e_t$$

similar then to the AR(1) model used for autocorrelation but with the coefficient ρ set to “1”

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Can also model this by adding a time trend (t=year)

$$Y_t = b_0 + Y_{t-1} + t + e_t$$

– what this means is that a series can be stationary around an upward (or downward) trend

Consequences

Can show that if variables are **NOT** stationary then

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and in non-stationary series the variance of X goes to infinity as T increases

so the 2nd term effectively goes to zero and endogeneity is less of an issue in (long) time series data

Example: Suppose you decide to regress United States inflation rate on the level of British GDP. There should, in truth, be very little relationship between the two (it is difficult to argue how British GDP could really affect US inflation)

If you regress US inflation rates on UK GDP for the period 1956-1979

```
. u gdp_sta
. reg usinf gdp if year<1980 & quarter==1
      Source |       SS       df       MS              Number of obs =      24
-------------+------------------------------           F(  1,    22) =   50.81
       Model |  156.605437     1  156.605437           Prob > F      =  0.0000
    Residual |  67.8141518    22  3.08246144           R-squared     =  0.6978
-------------+------------------------------           Adj R-squared =  0.6841
       Total |  224.419589    23  9.75737343           Root MSE      =  1.7557
------------------------------------------------------------------------------
       usinf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         gdp |   .0001402   .0000197     7.13   0.000     .0000994     .000181
       _cons |  -9.352343   1.945736    -4.81   0.000    -13.38755   -5.317133
```

which appears to suggest a significant positive (causal) relationship between the two. The R^2 is also very high

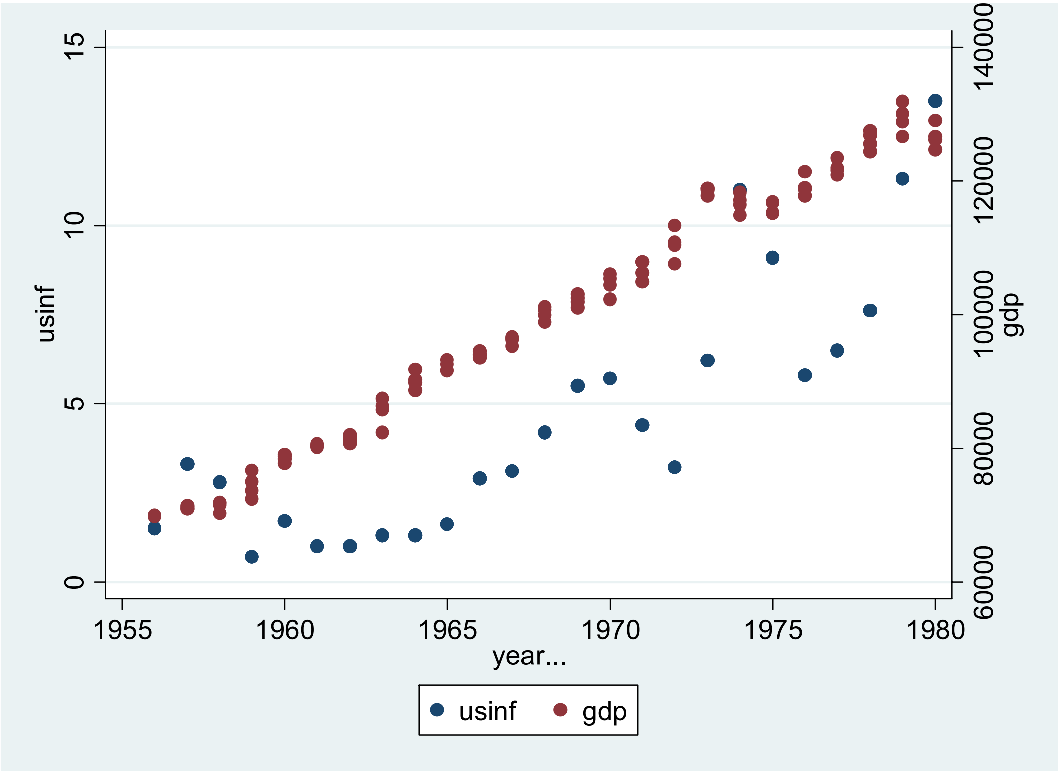
and if you regress US inflation rates on UK GDP for the period 1980-2002

```
. reg usinf gdp if year>=1980 & quarter==1
      Source |       SS       df       MS              Number of obs =      23
-------------+------------------------------           F(  1,    21) =   11.48
       Model |  59.6216433     1  59.6216433           Prob > F      =  0.0028
    Residual |  109.033142    21  5.19205437           R-squared     =  0.3535
-------------+------------------------------           Adj R-squared =  0.3227
       Total |  168.654785    22  7.66612659           Root MSE      =  2.2786
------------------------------------------------------------------------------
       usinf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         gdp |  -.0000589   .0000174    -3.39   0.003     -.000095   -.0000227
       _cons |   13.77226   2.904938     4.74   0.000     7.731107    19.81341
```

this now gives a significant negative relationship and the R^2 is much lower

In truth it is hard to believe that UK GDP has any real effect on US inflation rates. The reason why there appears to be a significant relation is because both variables are trended upward in the 1st period and the regression picks up the common (but unrelated) trends. This is spurious regression

```
twoway (scatter usinf year if year<=1980) (scatter gdp year if year<=1980, yaxis(2))
```



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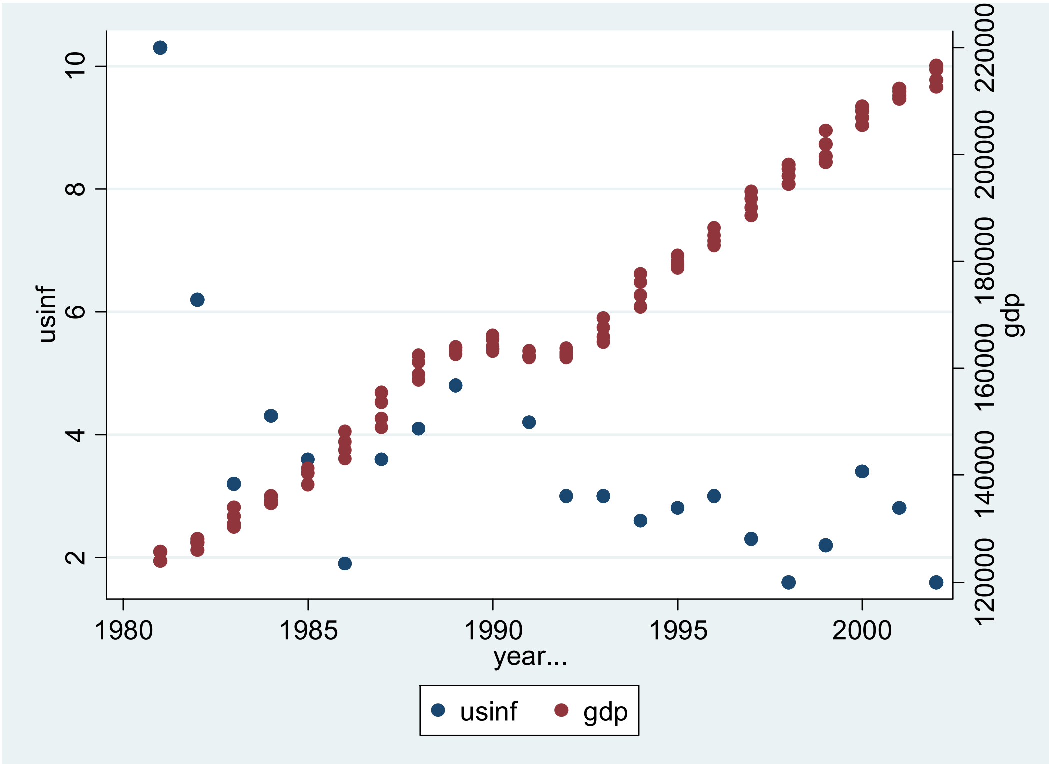
```



Stationarity

Ultimately whether you can sensibly include lags of either the dependent or explanatory variables or indeed the current level of a variable in a regression also depends on whether the time series data that you are analysing are **stationary**

A variable is said to be (weakly) stationary if

1) its mean
2) its variance
3) its autocovariance $Cov(Y_t, Y_{t-s})$ where $s \neq t$

do not change over time

Stationarity is needed if the Gauss-Markov conditions for unbiased, efficient OLS estimation are to be met by time series data

(Essentially any variable that is **trended** is unlikely to be stationary)

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Can show that if variables are **NOT** stationary then

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What to do?

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Eg

If $Y_t = Y_{t-1} + e_t$ is non-stationary

then take Y_{t-1} to the other side to get the difference

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$$Y_t - Y_{t-1} = \Delta Y_t$$

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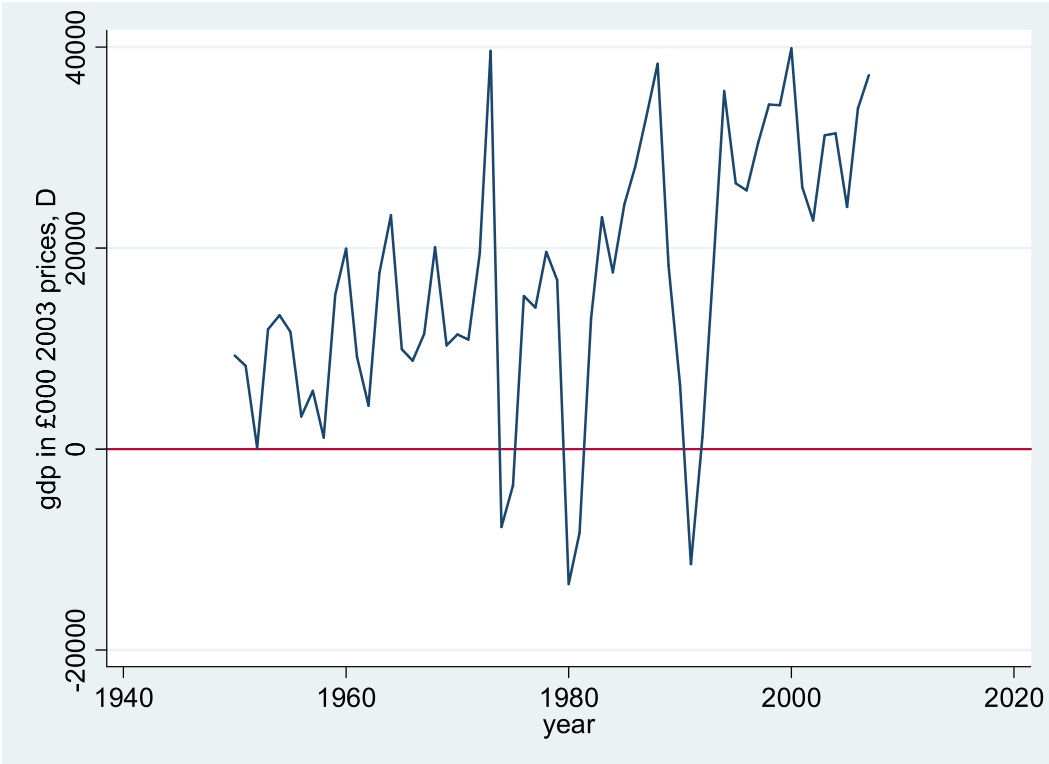
which should be stationary ie random and not trended

- since the differenced variable is just equal to the random error term – which has no trend or systematic behaviour

Example: The % change in gdp looks more likely to be stationary.

use "E:\qm2\Lecture 17\stationary.dta", clear



By inspection it seems there is no trend in the difference of GDP over time (and hence the mean and variance look reasonably stable over time)

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(if the series changes by the same proportional amount in each period then the log of a series changes by the same amount in each sub-period)

In practice not always easy to tell by looking at a series whether it is a random walk (non-stationary) or not.

So need to test this formally

Detection

Given $Y_t = Y_{t-1} + e_t$ is non-stationary (1)

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(can show the variance of Y is constant for (2))

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If so, the data follow a random walk and so the variable is non-stationary

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This is called the **Dickey Fuller Test**

So estimate

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by OLS and accept null of random walk if g is not significantly different from zero.

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and 2.86 if there is a constant in the regression

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and as a general rule only regress variables that are stationary on each other.

If they fail the Dickey-Fuller test then try using **the difference** of that variable instead

Example: To test formally whether the UK house prices are stationary or not

```
. u price_sta

tsset TIME
        time variable:  TIME, 24004 to 24084
                delta:  1 unit

. g dprice=price-price[_n-1]                                 /* creates 1st difference variable */
(1 missing value generated)
. g d2price=dprice-dprice[_n-1]


. reg  dprice l.price

      Source |       SS       df       MS              Number of obs =      80
-------------+------------------------------           F(  1,    78) =    3.32
       Model |  8932482.25     1  8932482.25           Prob > F      =  0.0724
    Residual |   210035668    78  2692764.98           R-squared     =  0.0408
-------------+------------------------------           Adj R-squared =  0.0285
       Total |   218968150    79  2771748.74           Root MSE      =    1641
------------------------------------------------------------------------------
      dprice |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       price |
         L1. |  -.0088124   .0048384    -1.82   0.072     -.018445    .0008202
       _cons |   3012.817   869.2151     3.47   0.001     1282.343    4743.291
------------------------------------------------------------------------------

. dfuller price

Dickey-Fuller test for unit root                   Number of obs   =        80
                           ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)             -1.821            -3.538            -2.906            -2.588
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.3699
```

Since estimated t value < Dickey-Fuller critical value (2.86) can't reject null that null that g= 0 (and b=1) and so original series (ie the level, not the change in prices follows a random walk.
So conclude that house prices are a non-stationary series
If we repeat the test for the 1st difference in prices (ie the change in prices)

```
. reg  d2price l.dprice
      Source |       SS       df       MS              Number of obs =      79
-------------+------------------------------           F(  1,    77) =   29.84
       Model |  67875874.8     1  67875874.8           Prob > F      =  0.0000
    Residual |   175154796    77  2274737.61           R-squared     =  0.2793
-------------+------------------------------           Adj R-squared =  0.2699
       Total |   243030671    78  3115777.83           Root MSE      =  1508.2
------------------------------------------------------------------------------
     d2price |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      dprice |
         L1. |  -.5589261   .1023204    -5.46   0.000    -.7626721   -.3551801
       _cons |   810.6236   225.3341     3.60   0.001     361.9261    1259.321
------------------------------------------------------------------------------
```

Since estimated t value now > Dickey-Fuller critical value (2.86) **reject** null that g= 0 (and b=1) and so new series (ie the change in, not the level of prices) is a stationary series

Should therefore use the change in prices rather than the level of prices in any OLS estimation (same test should be applied to any other variables used in a regression)

Note: stata will do (a variant of) this test automatically – note that the critical values are different since stata includes lagged values of the dependent variable in the test (the augmented Dickey Fuller test)

```
. dfuller dprice, regress
 Dickey-Fuller test for unit root                   Number of obs   =        79
                               ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)             -5.463            -3.539            -2.907            -2.588
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.0000
------------------------------------------------------------------------------
    D.dprice |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      dprice |
         L1. |  -.5589261   .1023204    -5.46   0.000    -.7626721   -.3551801
```

```
      _cons |   810.6236   225.3341     3.60   0.001     361.9261    1259.321
------------------------------------------------------------------------------
```

p value is <.05 so again reject null that g= 0 (and b=1)

COINTEGRATION

If economic data have to be differenced in order to avoid the problems of spurious regressions it becomes harder to interpret the coefficients from a differenced equation as anything other than the effect of the *change in X* on the *change* in Y

$$\Delta Y_t = b_0 + b_1 \Delta X_t + u_t$$

When we might like to find the effect on the *level* of Y of a change in the *level* of X

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(If we can only estimate a model in changes it is hard to find out the long run (steady state) effect when X is constant ie $\Delta X=0$)

COINTEGRATION

If economic data have to be differenced in order to avoid the problems of spurious regressions it becomes harder to interpret the coefficients from a differenced equation as anything other than the effect of the *change in X* on the *change* in Y

$$\Delta Y_t = b_0 + b_1 \Delta X_t + u_t$$

When we might like to find the effect on the *level* of Y of a change in the *level* of X

$$Y_t = \beta_0 + \beta_1 X_t + u_t$$

(If we can only estimate a model in changes it is hard to find out the long run (steady state) effect when X is constant ie $\Delta X=0$)

However if a long-run relationship exists, the 2 variables are said to be **cointegrated.** The trick is to try and tease out the long-run relationship.

variables with a common trend are also said to be cointegrated

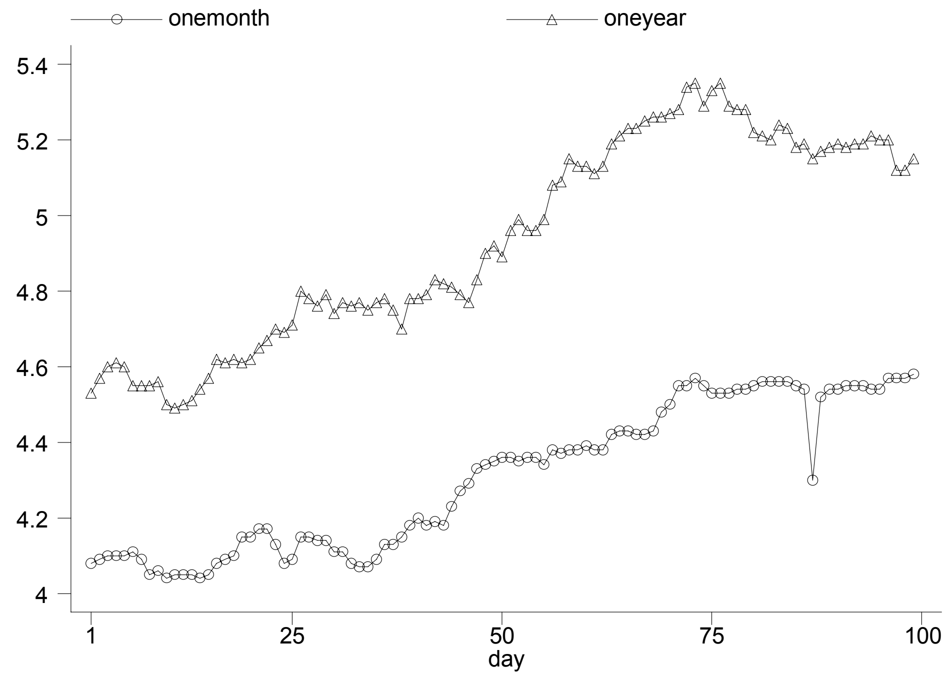
If 2 non-stationary variables have a ***common trend*** then we can net it out (like in the Dickey-Fuller test) by using some function of the difference in the 2 series

$Y_t - \delta X_t$ where δ is some constant

This difference term will, in general, be stationary and so can be added to a model

$$\Delta Y_t = b_0 + b_1 \Delta X_t + b_2 (Y_t - \delta X_t) \quad u_t$$

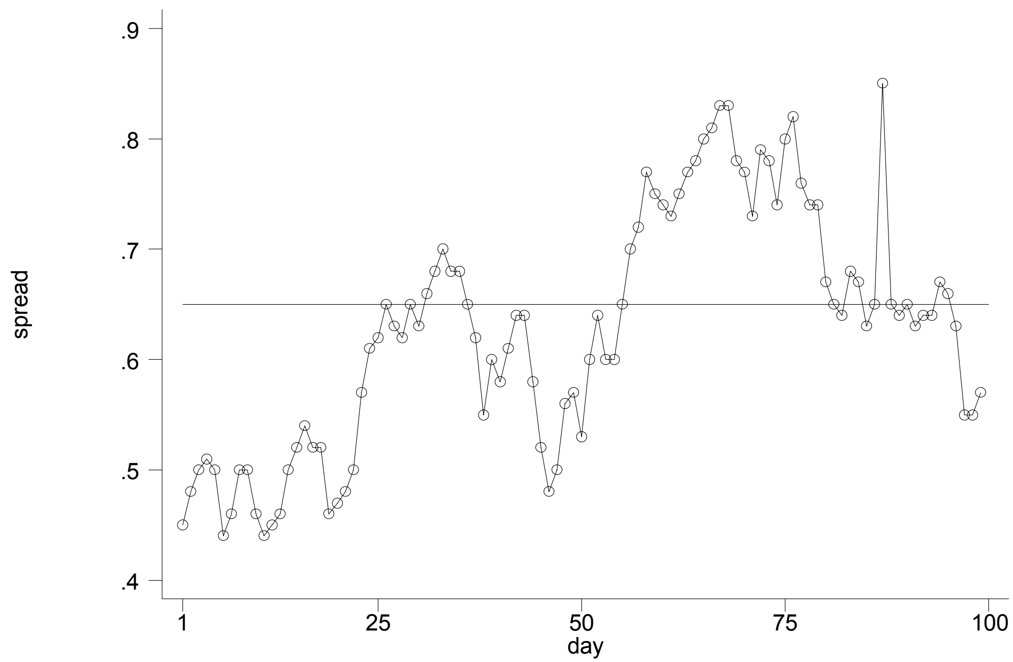
The usefulness of this procedure lies in the fact that the coefficient b_2 is called the **error correction effect** and it can be used to assess the extent of movement of y to its long run equilibrium value following a change in X



This is a graph of the level of UK one-year and one-month interest rates. Long-term interest rates have a higher return to compensate for the longer lending period. Can see over time however that the interest rates tend to move in the same direction

The difference between the 2 series is called the *spread*

Would not expect the spread to be trended over any significant length of time (otherwise it would be worth shifting all funds into the more favourable asset)



It appears that the spread is centred around .65 % age points over time

In this case the 2 interest rates have a common trend and so are cointegrated

If using time series variables you must ensure that the series are either stationary or that the variables are cointegrated.

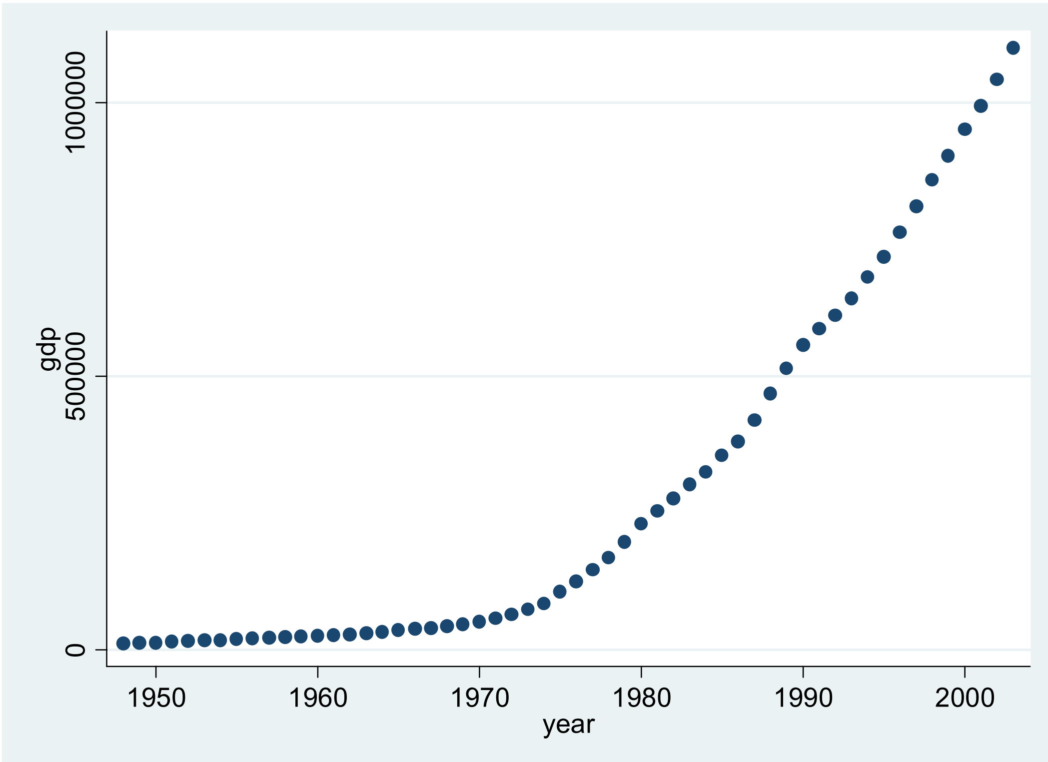
Another way to do this is to look at the behaviour of the residual from the model

$$Y_t = b_0 + b_1X_t + u_t$$

Suppose Y band X are non-stationary, but related. If so then any residual should be not be systematic (ie random)

Some series are captured better by an **exponential trend** (since the series appears to grow exponentially). A longer time series of nominal GDP for example looks like this

```
two (scatter gdp year)
```



(this is again consistent with the idea that the percentage growth in the variable is the same in each year)

you can model this by regressing the log of the series on a time trend

$$Log(GDP)_t = b_0 + b_1 Time + u_t$$

and $dLn(GDP)/dTime = b_1$

= % change in GDP for unit increase in Time (ie 1 year)
= annual % growth rate in GDP

```
. reg lgdp year

      Source |       SS       df       MS              Number of obs =      56
-------------+------------------------------           F(  1,    54) = 3149.93
       Model |  123.799553     1  123.799553           Prob > F      =  0.0000
    Residual |  2.12232554    54  .039302325           R-squared     =  0.9831
-------------+------------------------------           Adj R-squared =  0.9828
       Total |  125.921878    55   2.2894887           Root MSE      =  .19825
------------------------------------------------------------------------------
        lgdp |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        year |   .0919893    .001639    56.12   0.000     .0887033    .0952754
       _cons |  -170.0515   3.238013   -52.52   0.000    -176.5433   -163.5597
```

so the annual nominal growth in GDP is around 9%