Lecture 16. Endogeneity & Instrumental Variable Estimation (continued)

Seen how endogeneity , $Cov(x,u) \neq 0$, can be caused by

Omitting (relevant) variables from the model
Measurement Error in a right hand side variable
Simultaneity (interdependence) between variables

Solution: **instrumental variable** estimation

- Find variable(s) correlated with the problem variable but which does not suffer from endogeneity

So $b_1^{IV} = \dfrac{Cov(Z,y)}{Cov(Z,X)}$ (compare with $b_1^{OLS} = \dfrac{Cov(X,y)}{Var(X)}$)

This process of using extra exogenous variables as instruments for endogenous RHS variables is known as **identification**

If there are no additional exogenous variables outside the original equation that can be used as instruments for the endogenous RHS variables then the equation is said to be **unidentified**

Consider the previous example

$$\text{Price} = b_0 + b_1\text{Wage} + e \qquad (1)$$

$$\text{Wage} = d_0 + d_1\text{Price} + d_2\text{Unemployment} + v \qquad (2)$$

We know Price and Wage are interdependent so can't use OLS

Consider each equation in turn

In (1)

1 Endogenous right hand side variable (Wage)

so we need an instrument – variable that is correlated with wage but not with error term e

In this case we can use unemployment, since

it is not in (1) but is correlated with wages – via (2)

and is uncorrelated with e – *by the assumption of the model*

Equation (1) said to be **(just) identified**.

There is an instrument for Wage (which is Unemployment) so can use IV on equation (1)

Now consider equation (2) in the system

$$\text{Price} = b_0 + b_1\text{Wage} + e \qquad (1)$$
$$\text{Wage} = d_0 + d_1\text{Price} + d_2\text{Unemployment} + v \qquad (2)$$

Would like to estimate effect of prices on wages in (2), but because prices and wages are interdependent, OLS estimates suffer from endogeneity bias

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Can we find an instrument for price this time?

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Now in equation (2)

1 Endogenous rhs variable (Price)
1 Exogenous rhs variables (Unemployment)

Can we find an instrument for price this time?

This time given the model equation (2) is **not** identified. Since there are no available instruments that appear in (1)

so can't use IV

Suppose now that

$$\text{Price} = b_0 + b_1\text{Wage} + e \qquad (1)$$

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and we assume Cov(Productivity, v) = 0 (ie it is exogenous)

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(more instruments - other exogenous variables that do not appear in (1) – in this case unemployment & productivity) than strictly necessary for IV estimation)

If in an over-identified equation there are more instruments (other exogenous variables) than strictly necessary for IV estimation

which instrument to use ?

Again consider the model

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If use unemployment then as before, the IV estimator

$$\hat{b}_{IV} = \frac{Cov(Z, y)}{Cov(Z, X)} = \frac{Cov(Unemp, Price)}{Cov(Unemp, Wage)}$$

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If instead use Productivity then

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Again consider the model

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Which is best?

Both will give unbiased estimate of true value, but likely (especially in small samples) that estimates will be different.

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(Remember though with small samples more efficient to use the **minimum** number of instruments)

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Idea is then to estimate (3) and save the predicted values

$$\hat{wage} = \hat{g}_0 + \hat{g}_1 Unemp + \hat{g}_2 \Pr od$$

and use these predicted values as the instrument for Wage in (1)

The predicted variable $\hat{wage} = \hat{g}_0 + \hat{g}_1 Unemp + \hat{g}_2 \Pr od$

satisfies properties of an instrument since clearly correlated with variable of interest and because it is an average only of exogenous variables, is uncorrelated with the residual e in (1) (by assumption)

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which in the example above becomes

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Note: In many cases you will not have a simultaneous system. More than likely will have just one equation but there may well be endogenous RHS variable(s).

In this case the principle is exactly the same.

- Find additional exogenous variables that are correlated with the problem variable but uncorrelated with the error
("as if" there were another equation in the system).

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$Cov(X_t, u_t) \neq 0$ but

$Cov(X_{t-1}, u_t) = 0$

Eg. $GDP_t = b_0 + b_1 Education_t + u_t$

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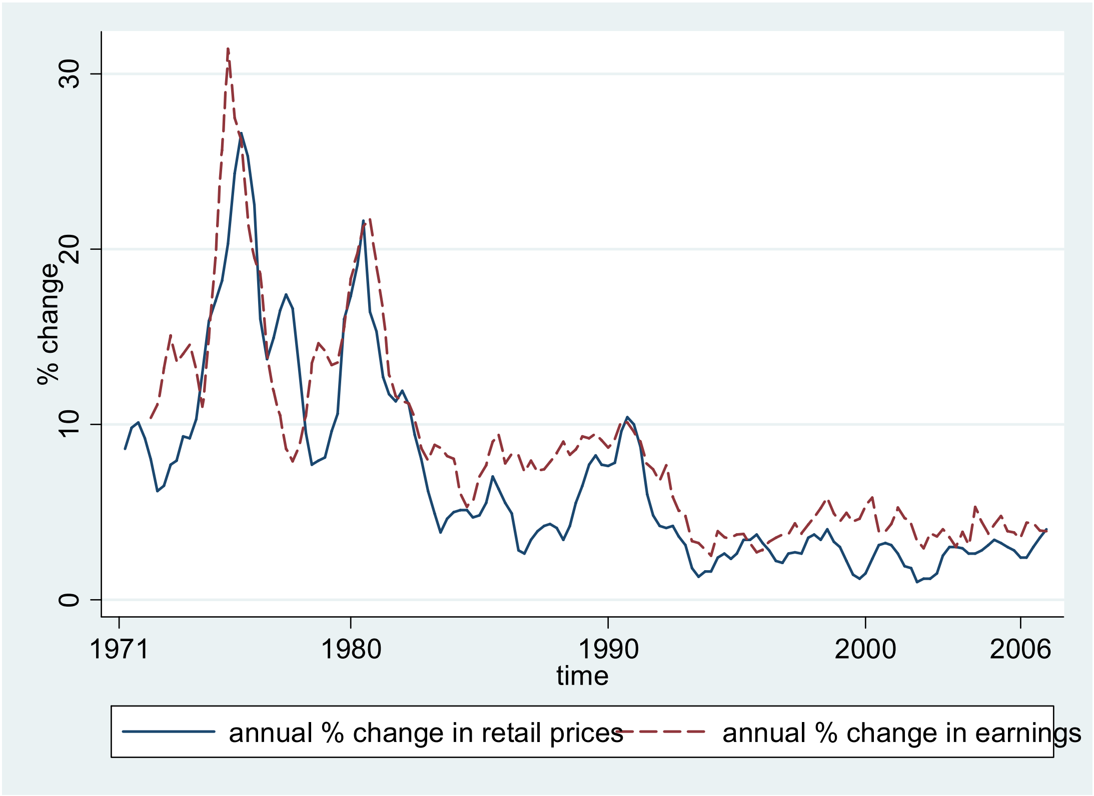
Same argument applies to using lagged values of right hand side variables as instruments

Example

```
two (line  inflation time) (line avearn time, xlabel(1971 1980 1990 2000 2006) ytitle(% change) clpattern(dash))
```



The data set *prod.dta* contains quarterly time series data on wage, price, unemployment and productivity changes

The graph suggests that wages and prices move together over time
and suggests may want to run a regression of inflation on wage changes where by assumption (and nothing else) the direction of causality runs from changes in wages to changes in inflation

```
. reg inf avearn

      Source |       SS       df       MS              Number of obs =     140
```

```
-------------+------------------------------           F(  1,   138) =  569.25
       Model |  3650.00027     1  3650.00027           Prob > F      =  0.0000
    Residual |  884.851892   138  6.41197024           R-squared     =  0.8049
-------------+------------------------------           Adj R-squared =  0.8035
       Total |  4534.85216   139  32.6248357           Root MSE      =  2.5322
------------------------------------------------------------------------------
   inflation |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      avearn |   .8984636   .0376574    23.86   0.000     .8240036    .9729237
       _cons |  -.8945061   .3897212    -2.30   0.023    -1.665103    -.123909
------------------------------------------------------------------------------
```

which suggests an almost one-for-one relation between inflation and wage changes over this period.
However you might equally run a regression of wage changes on prices (where now the implied direction of causality is from changes in the inflation rate to changes in wages)

```
. reg avearn inf

      Source |       SS       df       MS              Number of obs =     140
-------------+------------------------------           F(  1,   138) =  569.25
       Model |   3639.3317     1   3639.3317           Prob > F      =  0.0000
    Residual |  882.265564   138  6.39322872           R-squared     =  0.8049
-------------+------------------------------           Adj R-squared =  0.8035
       Total |  4521.59727   139  32.5294767           Root MSE      =  2.5285
------------------------------------------------------------------------------
      avearn |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
   inflation |   .8958375   .0375473    23.86   0.000     .8215951    .9700799
       _cons |   2.488974   .3351546     7.43   0.000     1.826271    3.151676
------------------------------------------------------------------------------
```

this suggests wages grow at constant rate of around 2.5% a year (the coefficient on the constant) and then each 1 percentage point increase in the inflation rate adds a .89 percentage point increase in wages.

Which is right specification?

In a sense both, since wages affect prices but prices also affect wages. The 2 variables are interdependent and said to be **endogenous**. This means that $Cov(X,u) \neq 0$ ie a correlation between right hand side variables and the residuals which makes OLS estimates biased and inconsistent.

Need to **instrument** the endogenous right hand side variables. ie find a variable that is correlated with the suspect right hand side variable but uncorrelated with the error term.

Now it is not easy to come up with good instruments in this example since many macro-economic variables are all interrelated, but one possible solution with time series data is to use **lags** of the endogenous variable. The idea is that while inflation may affect wages and vice versa it is less likely that inflation can influence past values of wages and so they might be used as instruments for wages

Suppose decide to use the 3 and 4 year lag of wages as instruments for wages in the inflation regression

Which instrument to use? Both should give same estimate if sample size is large enough but in finite (small) samples the two IV estimates can be quite different.

```
sort year q             /* important to sort the data before taking lags */

g wlag3=avearn[_n-12]
(16 missing values generated)

g wlag4=avearn[_n-16]
(20 missing values generated)

/* note lose observations when take lags - cant calculate lags of values toward the start of the time period  */
```

Using the 3 year lag as an instrument

```
ivreg inf (avear=wlag3)

Instrumental variables (2SLS) regression

      Source |       SS       df       MS              Number of obs =     128
-------------+------------------------------           F(  1,   126) =  145.52
       Model |   3357.2168     1   3357.2168           Prob > F      =  0.0000
    Residual |  785.683138   126  6.23558046           R-squared     =  0.8104
-------------+------------------------------           Adj R-squared =  0.8088
       Total |  4142.89994   127  32.6212594           Root MSE      =  2.4971
------------------------------------------------------------------------------
   inflation |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      avearn |   1.026884    .085127    12.06   0.000     .8584203    1.195348
       _cons |  -1.790507   .7228275    -2.48   0.015    -3.220961   -.3600522
------------------------------------------------------------------------------
Instrumented:  avearn
Instruments:   wlag3
```

Using the 4 year lag as an instrument

```
ivreg inf (avear=wlag4)

Instrumental variables (2SLS) regression

      Source |       SS       df       MS              Number of obs =     124
-------------+------------------------------           F(  1,   122) =  166.23
       Model |   2192.9109     1   2192.9109           Prob > F      =  0.0000
    Residual |  646.995151   122  5.30323894           R-squared     =  0.7722
-------------+------------------------------           Adj R-squared =  0.7703
       Total |  2839.90605   123   23.088667           Root MSE      =  2.3029
------------------------------------------------------------------------------
   inflation |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      avearn |   1.005013   .0779507    12.89   0.000     .8507014    1.159324
       _cons |   -1.57803   .6190175    -2.55   0.012    -2.803437   -.3526229
------------------------------------------------------------------------------
Instrumented:  avearn
Instruments:   wlag4
```

Both estimates are similar and higher than original OLS estimate So which one?

Best idea (which also gives more efficient estimates ie ones with lower standard errors) is to use **all the instruments** at the same time – at least in large samples

1. Regress endogenous variables (wages) on both instruments (wage$_{t-3}$ and wage$_{t-4}$)

```
. reg avearn wlag4 wlag3

      Source |       SS       df       MS              Number of obs =     124
-------------+------------------------------           F(  2,   121) =   38.00
       Model |  915.451476     2  457.725738           Prob > F      =  0.0000
    Residual |  1457.40163   121  12.0446416           R-squared     =  0.3858
-------------+------------------------------           Adj R-squared =  0.3756
       Total |  2372.85311   123  19.2914887           Root MSE      =  3.4705
------------------------------------------------------------------------------
      avearn |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       wlag4 |   .3527246    .078543     4.49   0.000     .1972279    .5082213
       wlag3 |    .146539   .0778473     1.88   0.062    -.0075803    .3006584
       _cons |   2.909309   .6131674     4.74   0.000     1.695382    4.123235
------------------------------------------------------------------------------
```

Save predicted wage

```
. predict wagehat       /* stata command to save predicted value of dep. var. */
```

2. Include this instead of wages on the right hand side of the inflation regression

```
. reg inf wagehat

      Source |       SS       df       MS              Number of obs =     124
-------------+------------------------------           F(  1,   122) =   59.46
       Model |  930.575733     1  930.575733           Prob > F      =  0.0000
    Residual |  1909.33031   122  15.6502485           R-squared     =  0.3277
-------------+------------------------------           Adj R-squared =  0.3222
       Total |  2839.90605   123   23.088667           Root MSE      =   3.956
------------------------------------------------------------------------------
```

```
   inflation |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     wagehat |   1.008227   .1307504     7.71   0.000     .7493933     1.26706
       _cons |  -1.602087   1.041137    -1.54   0.126    -3.663121    .4589473
```

This gives unbiased estimate of effect of wages on prices.
Compare with original (biased) estimate,
can see wage effect is a little larger, (though standard error of IV estimate is larger than in OLS)

```
. reg inf avearn if e(sample)

      Source |       SS       df       MS              Number of obs =     124
-------------+------------------------------           F(  1,   122) =  417.11
       Model |  2197.24294     1  2197.24294           Prob > F      =  0.0000
    Residual |  642.663105   122  5.26773037           R-squared     =  0.7737
-------------+------------------------------           Adj R-squared =  0.7718
       Total |  2839.90605   123   23.088667           Root MSE      =  2.2952
------------------------------------------------------------------------------
   inflation |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      avearn |   .9622848   .0471169    20.42   0.000     .8690122    1.055557
       _cons |  -1.258218   .4084766    -3.08   0.003    -2.066838   -.4495972
------------------------------------------------------------------------------
```

Stata does all this automatically using the ivreg2 command. Adding “first” to the command will also give the first stage of the two stage least squares regression which will help you decide whether the instruments are weak or not.

```
ivreg2 inf (avearn=wlag3 wlag4), first

First-stage regression of avearn:

Ordinary Least Squares (OLS) regression
--------------------------------------
                                                      Number of obs =      124
                                                      F(  2,   121) =    38.00
                                                      Prob > F      =   0.0000
Total (centered) SS     =  2372.853108                Centered R2   =   0.3858
Total (uncentered) SS   =  9319.731851                Uncentered R2 =   0.8436
Residual SS             =  1457.401632                Root MSE      =      3.5
------------------------------------------------------------------------------
      avearn |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       wlag3 |    .146539   .0778473     1.88   0.062    -.0075803    .3006584
```

```
       wlag4 |   .3527246    .078543     4.49   0.000     .1972279    .5082213
       _cons |   2.909309   .6131674     4.74   0.000     1.695382    4.123235
------------------------------------------------------------------------------
Partial R-squared of excluded instruments:   0.3858
Test of excluded instruments:
  F(  2,   121) =    38.00
  Prob > F      =   0.0000
Summary results for first-stage regressions:
                Shea
Variable      Partial R2      Partial R2       F(  2,   121)    P-value
avearn          0.3858          0.3858             38.00         0.0000

Instrumental variables (2SLS) regression
----------------------------------------
                                                      Number of obs =      124
                                                      F(  1,   122) =   175.29
                                                      Prob > F      =   0.0000
Total (centered) SS     =  2839.906047                Centered R2   =   0.7719
Total (uncentered) SS   =   7221.48998                Uncentered R2 =   0.9103
Residual SS             =  647.6713926                Root MSE      =      2.3
------------------------------------------------------------------------------
   inflation |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      avearn |   1.008227   .0755351    13.35   0.000     .8601806    1.156273
       _cons |  -1.602087   .6014696    -2.66   0.008    -2.780945    -.423228
------------------------------------------------------------------------------
Sargan statistic (overidentification test of all instruments):           0.037
                                                   Chi-sq(1) P-val =   0.84741
------------------------------------------------------------------------------
Instrumented:  avearn
Instruments:   wlag3 wlag4
```

Note that the instruments are jointly significant in the first stage (as suggested by the F value and the R^2)

Finding instruments is not easy

- in time series data sets you may be able to use **lagged values** of the data as possible instruments since lagged values are less likely to be influenced by current shocks

- in cross sections there may be economic policy interventions that could be plausibly exogenous but correlated with endogenous variable

Eg. Raising of the school leaving age as an instrument for education – exogenous since government policy but is correlated with average level of education)

Weak Instruments

- The two stage least squares approach is also useful in helping illuminate whether the instrument(s) is good or not

Can show that IV estimation strategy may not always be better than biased OLS if the correlation between the instrument and the endogenous variable is weak (at least in small samples)

Sometimes instruments may be statistically significant from zero in the 1st stage and still not good enough.

How do you know if instrument is weak?

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(it should at a minimum be significantly different from zero)

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Cannot use the R^2 from the 1st stage since this could be high purely because of the exogenous variables in the model and not the instruments

Instead there is a rule of thumb (at least in the case of a single endogenous variable) that should only proceed with IV estimation if the F value (strictly **net of the other exogenous variables on the right hand side of the model**) in the test of the goodness of fit of the on the **1st stage** of 2SLS > 10

Can also look at the **partial R^2** which is based on a regression that nets out the effect of the exogenous variable X_2 on both endogenous variable X_1 and the instrument , Z and is obtained from the following regression

1. Regress X_2 on X_1 and save the predicted value $\hat{X}_2$
2. Regress Z on X_1 and save the predicted value $\hat{Z}$
3. Regress $\hat{X}_2$ on $\hat{Z}$

   The R^2 from this regression is the partial R^2 ("partials out" the effect of X_1)

No threshold for the partial R^2 but the higher the value the greater the correlation between instrument and endogenous variable

If there is more than one potentially endogenous rhs variable in your equation the order condition (above) tells us that you will have to find at least one different instrument for **each** endogenous rhs variable (eg 2 endogenous rhs variables requires 2 different instruments).

Again, each instrument should be correlated with the endogenous rhs variable it replaces **net** of the other existing exogenous rhs variables. In this case two stage least squares estimation means predicting a value for each of the endogenous variables based on the instruments.

The examples above are based on models where there is only an endogenous variable on the right hand side of the model

In many cases you will have a combination of exogenous variables (X_1) and endogenous variables (X_2) on the right hand side

$$Y = b_0 + b_1X_1 + b_2X_2 + u$$

In this case the only difference in estimation procedures is to make sure that you include the exogenous variables X_1 at both stages of the two stage estimation process

Example 2: Poor Instruments & IV Estimation

Often a poor choice of instrument can make things much worse than the original OLS estimates.

Consider the example of the effect of education on wages (taken from the data set *video2.dta*). Policy makers are often interested in the costs and benefits of education. Some people argue that education is endogenous (because it also picks up the effects of omitted variables like ability or motivation and so is correlated with the error term).

The OLS estimates from a regression of log hourly wages on years of education suggest that

```
. reg  lhw yearsed

      Source |       SS       df       MS              Number of obs =    4473
-------------+------------------------------           F(  1,  4471) =  496.09
       Model |   160.92557     1   160.92557           Prob > F      =  0.0000
    Residual |  1450.34752  4471  .324389962           R-squared     =  0.0999
-------------+------------------------------           Adj R-squared =  0.0997
       Total |  1611.27309  4472  .360302569           Root MSE      =  .56955
------------------------------------------------------------------------------
         lhw |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     yearsed |   .0558941   .0025095    22.27   0.000     .0509743     .060814
       _cons |   6.163786   .0324931   189.70   0.000     6.100083    6.227488
------------------------------------------------------------------------------
```

1 extra year of education is associated with 7.6% increase in earnings.
If endogeneity is a problem, then these estimates are biased, (upward if ability and education are positively correlated – see lecture notes on omitted variable bias).

So try to instrument instead.
For some reason you choose whether the individual owns a dvd recorder.

To be a good instrument the variable should be a) uncorrelated with the residual and by extension the dependent variable (wages) but b) correlated with the endogenous right hand side variable (education).

However when you instrument years of education using dvd ownership

```
. ivreg  lhw (yearsed=dvd)

Instrumental variables (2SLS) regression

      Source |       SS       df       MS              Number of obs =    4473
-------------+------------------------------           F(  1,  4471) =    0.16
       Model | -316.945786     1 -316.945786           Prob > F      =  0.6903
    Residual |  1928.21888  4471  .431272394           R-squared     =       .
-------------+------------------------------           Adj R-squared =       .
       Total |  1611.27309  4472  .360302569           Root MSE      =  .65671

------------------------------------------------------------------------------
         lhw |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     yearsed |  -.0404242   .1014589    -0.40   0.690    -.2393338    .1584853
       _cons |   7.367324   1.267809     5.81   0.000     4.881791    9.852856
------------------------------------------------------------------------------
Instrumented:  yearsed
Instruments:   dvd
------------------------------------------------------------------------------
```

The IV estimates are now negative (and insignificant). This does not appear very sensible.
The reason is that dvd ownership is hardly correlated with education as the 1st stage of the regression below shows.

```
ivreg  lhw (yearsed=dvd), first

First-stage regressions
-----------------------

      Source |       SS       df       MS              Number of obs =    4473
-------------+------------------------------           F(  1,  4471) =    3.64
       Model |  41.8959115     1  41.8959115           Prob > F      =  0.0565
    Residual |  51468.2601  4471  11.5115769           R-squared     =  0.0008
-------------+------------------------------           Adj R-squared =  0.0006
       Total |   51510.156  4472  11.5183712           Root MSE      =  3.3929

------------------------------------------------------------------------------
     yearsed |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         dvd |   .4456185   .2335849     1.91   0.056    -.0123235    .9035605
       _cons |   12.02768   .2503714    48.04   0.000     11.53683    12.51853
------------------------------------------------------------------------------
```

Moral: Always check the correlation of your instrument with the endogenous right hand variable. The "first" option on stata's ivreg command will always print the 1st stage of the 2SLS regression – ie the regression above – in which you can tell if the proposed instrument is correlated with the endogenous variable by simply looking at the t value.

Using the rule of thumb (at least in the case of a single endogenous variable) that should only proceed with IV estimation if the F value on the 1st stage of 2SLS > 10.

In the example above this is clearly not the case.

In practice you will often have a model where some but not all of the right hand side variables are endogenous.

$$Y = b_0 + b_1X_1 = b_2X_2 + u$$

where X_1 is exogenous and X_2 is endogenous

The only difference between this situation and the one described above is that you must include the exogenous variables X_1 in **both** stages of the 2SLS estimation process

Example 3. The data set *ivdat.dta* contains information on the number of GCSE passes of a sample of 16 year olds and the total income of the household in which they live. Income tends to be measured with error. Individuals tend to mis-report incomes, particularly third-party incomes and non-labour income. The following regression may therefore be subject to measurement error in one of the right hand side variables.

```
ivreg2 nqfede (inc1=ranki) female, first

First-stage regressions

                                                      Number of obs =      252
                                                      F(  2,   249) =   247.94
                                                      Prob > F      =   0.0000
Total (centered) SS     =  122243.0372                Centered R2   =   0.6657
Total (uncentered) SS   =  382752.6464                Uncentered R2 =   0.8932
Residual SS             =  40863.62602                Root MSE      =       13
------------------------------------------------------------------------------
        inc1 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      female |   .2342779   1.618777     0.14   0.885    -2.953963    3.422518
       ranki |   .2470712   .0110979    22.26   0.000     .2252136    .2689289
```

```
       _cons |   .7722511   1.855748     0.42   0.678    -2.882712    4.427215
------------------------------------------------------------------------------
Partial R-squared of excluded instruments:   0.6656
Test of excluded instruments:
  F(  1,   249) =   495.64
  Prob > F      =   0.0000

IV (2SLS) regression with robust standard errors       Number of obs =     252
                                                       F(  2,   249) =   14.57
                                                       Prob > F      =  0.0000
                                                       R-squared     =  0.1033
                                                       Root MSE      =  3.0711
------------------------------------------------------------------------------
             |               Robust
      nqfede |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        inc1 |   .0450854   .0101681     4.43   0.000     .0250589    .0651119
      female |   1.176652   .3883785     3.03   0.003     .4117266    1.941578
       _cons |   4.753386    .448987    10.59   0.000      3.86909    5.637683
------------------------------------------------------------------------------
Instrumented:  inc1
Instruments:   female ranki
------------------------------------------------------------------------------
```

Note that the exogenous variable “female” appears in both stages

Testing Instrumental Validity (Overidentifying Restrictions)

If you have more instruments than endogenous right hand side variables (the equation is **overidentified** – hence the name for the test) then it is possible to test whether (some of the) instruments are valid – in the sense that they satisfy the assumption of being uncorrelated with the residual in the original model. $Cov(z, u) = 0$

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One way to do this would be, as in the example above, to compute two different 2SLS estimates, one using one instrument and another using the other instrument. If these estimates are radically different you might conclude that one (or both) of the instruments was invalid (not exogenous). If these estimates were similar you might conclude that both instruments were exogenous.

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An implicit test of this – that avoids having to compute all of the possible IV estimates - is based on the following idea

Sargan Test   (after Denis Sargan 1924-1996)

Given $y = b_0 + b_1X + u$ and $Cov(X,u) \neq 0$

If an instrument Z is valid (exogenous) it is uncorrelated with u
To test this simply regress u on **all** the possible instruments.

$$u = d_0 + d_1Z_1 + d_2Z_2 + \dots d_lZ_l + v$$

If the instruments are exogenous they should be uncorrelated with u and so the coefficients d_1 .. d_l should all be zero (ie the Z variables have no explanatory power)

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$$u = d_0 + d_1Z_1 + d_2Z_2 + \ldots. d_lZ_l + v$$

If the instruments are exogenous they should be uncorrelated with u and so the coefficients d_1 .. d_l should all be zero (ie the Z variables have no explanatory power)

Since u is never observed have to use a proxy for this. This turns out to be the residual from the 2SLS estimation estimated using all the possible instruments

$$\hat{u}^{2sls} = y - \hat{b}_0^{2sls} - \hat{b}_1^{2sls} X$$

(since this is a consistent estimate of the true unknown residuals)

So to Test Overidentifying Restrictions

1. Estimate model by 2SLS and save the residuals
2. Regress these residuals on ***all*** the exogenous variables (including those X_1 variables in the original equation that are not suspect)

   $\hat{u}^{2sls} = d_0 + b_1X_1 + d_1Z_1 + d_2Z_2 + \ldots\ d_lZ_l + v$

   and save the R^2

3. Compute $N*R^2$

4. Under the null that all the instruments are uncorrelated then

   $N*R^2 \sim \chi^2$ with L-k degrees of freedom

   (L is the number of instruments and k is the number of endogenous right hand side variables in the original equation)

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Note that can only do this test if there are more instruments than endogenous right hand side variables (in just identified case the residuals and right hand side variables are uncorrelated by construction)

Also this test is again only valid in large samples

Example: using the *prod.dta* file we can test whether some of the instruments ($wage_{t-3}$ and $wage_{t-4}$) are valid instruments for wages

1st do the two stage least squares regression using all the possible instruments

```
ivreg inf (avearn=wlag3 wlag4)
```

Now save these 2sls residuals

```
. predict ivres, resid
```

and regress these on all the exogenous variables in the system (remember there may be situations where the original equation contained other exogenous variables, in which case include them here also)

```
. reg ivres wlag3 wlag4

      Source |       SS       df       MS              Number of obs =     124
-------------+------------------------------           F(  2,   121) =    0.02
       Model |  .193389811     2  .096694905           Prob > F      =  0.9821
    Residual |  647.478005   121   5.3510579           R-squared     =  0.0003
-------------+------------------------------           Adj R-squared = -0.0162
       Total |  647.671395   123   5.2656211           Root MSE      =  2.3132
------------------------------------------------------------------------------
       ivres |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       wlag3 |   .0096316    .051888     0.19   0.853    -.0930943    .1123575
       wlag4 |  -.0085304   .0523517    -0.16   0.871    -.1121743    .0951136
       _cons |  -.0073752   .4086975    -0.02   0.986    -.8164997    .8017493
```

The test is $N*R^2 = 124*0.0003 = 0.04$ which is $\sim \chi^2$ (L-k = 2-1)

(L=2 instruments and k=1 endogenous right hand side variable)

Since $\chi^2_{(1)}{}^{critical} = 3.84$, estimated value is below critical value so **accept** null hypothesis that some of the instruments are valid

Can obtain these results automatically using the command:

```
overid
```

```
Tests of overidentifying restrictions:
Sargan N*R-sq test        0.037  Chi-sq(1)    P-value = 0.8474
Basmann test              0.036  Chi-sq(1)    P-value = 0.8492
```